Explore or Exploit? Labor Market Frictions and the Innovation Choice*

Richard T. Thakor†

December 2023

Abstract

This paper theoretically examines innovation investment incentives by firms facing labor market frictions. A firm can choose the degree of innovation of its projects: non-innovative (exploitative) investments that continue existing ideas, incrementally innovative investments, or truly novel innovations that involve uncertain technologies. In an optimal contracting setting with moral hazard, unknown managerial talent, and asymmetric information about project quality, there are two main results. First, constrained-efficient contracts generate rents for managers that represent agency costs for firms, and these vary across different project types. Second, labor market frictions have a non-monotonic effect on the innovation choice. Innovation occurs when there are both low and high labor market frictions, with incremental innovation occurring with low labor frictions and uncertain novel innovation occurring with high labor frictions. Exploitative investments occur when there are intermediate degrees of labor market frictions.

Keywords: R&D Investments; Innovation; Explore; Labor Markets

JEL Classification: D81, D82, D83, G31, G32, J23, J24, O31, O32

*I thank Xuelin Li, Anjan Thakor, and participants at the North American Summer Meeting of the *Econometric Society* and the Junior Innovation Economics Conference for helpful comments and discussions. I alone am responsible for any errors.

†University of Minnesota and MIT LFE. E-mail: rthakor@umn.edu

1 Introduction

Firms often have to choose whether to invest in familiar projects with well-known business designs and technologies, or new innovative projects. For example, Apple faced a choice of whether to invest in the new technology for its first iPhone, or to invest in existing technology with more conventional applications. Similarly, a drug development company can invest in new drugs or in "me-too" generic drugs that are substitutes for existing drugs. March (1991) referred to this choice between innovation and pursuing the familiar as the "exploration-exploitation" tradeoff. Furthermore, conditional on innovating, a firm may also choose the *degree* of innovation—whether to explore brand-new and uncertain technologies or to create small improvements in existing technologies (e.g. Dewar and Dutton (1986)). For instance, a drug development company can invest in an incremental drug or in a novel drug, which may use a new mechanism of action or have an entirely different chemical makeup compared to previous drugs (e.g. Krieger, Li, and Papanikolaou (2022)).[1] While incremental innovation can play a role in supporting economic growth, excessive incremental innovation that crowds out novel innovation can hinder growth (e.g. Aghion et al. (2001)) and also prevent the development of new technologies that can benefit society.[2] Thus, this choice is important not only for understanding the innovation process, but also for the broader economy.

This innovation choice is likely to be affected by conditions in the labor market because innovation often depends on the firm's ability to hire skilled workers and replace them with those with different skills if necessary. The business press often touts the importance of lowering labor market frictions to spur innovation.[3] The reasoning is simple: labor markets

[1] Along similar dimensions, a drug development firm may choose to develop a drug that may marginally improve the efficacy of existing treatments for a particular disease, or it may target diseases for which there are few/no treatments, such as rare/orphan diseases (see, e.g., Lo and Thakor (2022)).

[2] For example, innovation has been noted to be critical for goals such as addressing climate change (e.g. Barnett et al. (2023)) and developing treatments against many diseases (e.g. Lo and Thakor (2022)).

[3] For example, economist Mark Curtis at the Atlanta Fed's macroblog states: "A decline in these rates [job creation and destruction] could indicate less innovation or less labor market flexibility, both of which are likely to retard economic growth" (Ferro (2014)).

which are more rigid—that is, there are higher costs of hiring and firing workers—are viewed as being less efficient and thus inimical to the free flow of labor that presumably facilitates innovation. However, the theory and empirical evidence on this effect is mixed. For example, Acharya, Baghai, and Subramanian (2013) find that *more* stringent labor laws that restrict employee termination encourage innovation.[4] This raises the question: how is the firm's choice between exploitation, incremental innovation, and novel innovation affected by labor market frictions?

In this paper, I address these questions by theoretically examining how a firm chooses between exploitation (harvesting well-known technologies), incremental innovation, and novel innovation in an environment in which labor market frictions affect this choice. The labor market friction I focus on is fluidity, which is viewed as a proxy for search frictions. This follows Shimer (2001) who views greater labor market fluidity as greater ease of recruiting talent by firms. I model it as the cost of replacing an incumbent manager with a new one, which can be viewed more generally as the cost faced by firms and managers in matching up with each other.[5] When this cost is low, managerial (labor market) fluidity is high and job reallocation rates will also be high as firms will be quicker to fire workers and hire their replacements.

This question is analyzed with a model of the firm's choice between exploitation and incremental or novel innovation in a setting with risk neutrality, unknown managerial talent, and moral hazard. Agency costs and managerial replacement costs stemming from labor market frictions interact to influence the firm's choice. Labor market fluidity affects the

[4] This is consistent with the theoretical view, following Arrow (1962), that the knowledge generated by the innovation process may spill over to competing firms when skilled workers move, and thus investment in innovation will be underprovided since owners cannot fully capture the rewards from it. Chen et al. (2023) develop a model in which knowledge capital generated by intangible investment accrues to skilled workers, thus improving their outside options and increasing the probability of going to another firm, but incentives to produce effort are reduced if they are prevented from leaving.

[5] This is akin to a "firing tax", such as that in Europe, but it can include training costs for replacement workers (as in Acemoglu (1997)) in addition to the cost of search and matching frictions. There are many ways in which labor market fluidity has been measured empirically (see Molloy et al. (2016)). My definition is perhaps closest to one measure used by Hyatt and Spletzer (2013), namely that it is related to "hires and separations". When firms face a high cost of replacing managers, labor market fluidity is low in the sense that there are few separations and new hires.

magnitude of agency costs associated with the firm's project choices, but it affects these costs differently across different projects, so a change in labor market fluidity impacts the firm's project choice.

The model has two time periods. In each period, the firm decides whether to exploit, incrementally innovate, or explore novel innovation. The manager is delegated the task of searching for a good project of the type stipulated by the firm in each period. To find such a project, the manager must exert privately-costly effort. If his effort yields no fruit, the manager can choose to not fund a project or request funding for an always-available bad project that yields him a private benefit. The manager's *a priori* unknown talent determines the success probability of a good project; no one (including the manager) knows this talent. The firm designs optimal wage contracts for the two periods to resolve the twin moral hazard problems of inducing the manager to work hard to find a good project in each period and coaxing him to never seek funding for a bad project. It also instructs him on the type of project to search for.

The differences between exploitation, incremental innovation, and novel innovation are modeled on the basis of two dimensions: the potential value of learning by doing or exploring (a hallmark of exploration) and possible uncertainty about project quality (risk associated with innovation). Exploitation is simply an extension of what the firm is currently doing, so it involves neither the value of learning nor project quality uncertainty. The payoff distribution of exploit projects is thus i.i.d. for projects in each period. Incremental innovation involves the value of learning by doing but has no project-quality uncertainty, since the quality of the innovation has been previously established. Thus, the expected payoff of this project is higher in the second period than in the first, as long as the firm continues with its incumbent manager (who learned from working on the project in the first period). Novel innovation involves both the value of learning by doing and project-quality uncertainty, as the technology or basis of the innovation has not been proven.

The observed outcome of the first-period project reveals information about the manager's talent, and also about project quality in the case of innovation. If the information is adverse, the firm can fire the manager and find a replacement for the second period. But whether it will do so depends on labor market fluidity. When this fluidity is high, the replacement cost is low and the manager is fired. However, low fluidity can lead to the manager being retained even after first-period failure.

There are two central results. First, optimal (constrained-efficient) contracts generate rents for the manager, but these rents—which are agency costs for the firm—vary across the exploit, incremental innovation, and novel innovation projects. Second, the relationship between exploration and labor market fluidity is non-monotonic. Innovation occurs both when labor market fluidity is high and when it is low, with exploitation occurring for intermediate values of fluidity. However the nature of innovation varies depending on labor market fluidity—incremental innovation occurs when labor market fluidity is high and novel innovation occurs when labor market fluidity is low. One implication of this is that as labor markets become more fluid, incremental innovation become relatively more ubiquitous.

The intuition for the heterogeneity in managerial rents and agency costs across project types is that the information revealed by the project outcome of the first period as well as the value of retaining the manager for the second period vary across the different types of projects. This means the constrained-efficient wage contracts are also different for the the three types of projects. When labor market fluidity is high, the manager is fired following first-period project failure regardless of project type. This leads to the highest agency costs for novel innovation because now the manager can be fired even when he works hard to find a good project, simply due to project-quality uncertainty. Exploitation and incremental innovation have the same agency costs, but incremental innovation is preferred due to its value of learning by doing. For intermediate values of labor market fluidity, the manager is fired for first-period failure with exploitation but not with innovation. The lack of a firing threat with innovation makes it more costly to motivate the manager to search for a good project, so agency costs are the lowest with exploitation, and this is what the firm chooses.

When labor market fluidity is low, the manager is not fired following failure, regardless of project type. Thus, all project types are associated with high agency costs, so novel innovation is preferred due to its highest option value.

An interesting implication of the analysis is related to the *equilibrium* relationship between project choice and employee turnover. The conventional wisdom is that innovation provides greater termination protection than exploitation, due to human capital that employees accrue by investing in innovation (e.g. Chen et al. (2023)). What the analysis here shows is that in equilibrium, exploitation and incremental innovation involve the same employee turnover, whereas novel innovation involves *no* turnover.

This paper is related to the literature on optimal contracting for innovation. Similar to this paper, Holmstrom (1989) and Aghion and Tirole (1994) propose that failure must be tolerated to a greater extent with innovation due to the noisier performance assessment with innovation. Hellmann (2007) and Hellmann and Thiele (2008) examine incentives for innovation in a multi-tasking principal-agent model. Bolton, Scheinkman, and Xiong (2006) focus on how investors' investment horizons affect firms' choices of project duration. Starks, Venkat, and Zhu (2017) provide empirical evidence of a clientele effect which manifests itself in long-horizon investors preferring firms that emphasize environmental and social responsibility more. In contrast to these papers which focus on the contracting aspect of innovation, this paper focuses both on the process of innovation and on optimal contracting for incremental and novel innovation as well as exploitation. In this respect, it is more closely related to Manso (2011), who models the firm's innovation process and examines the tradeoff between exploration and exploitation. It differs from Manso (2011) in its examination of exploitation, incremental, and novel innovation and the interaction between project choice, multi-tasking moral hazard, and learning about managerial talent in an optimal contracting setting. The fact that the firm is learning both about the innovation project and about the manager in this model generates new results about the firm's decision of whether to pursue exploit or explore, and the degree of innovation to choose in the latter case, depending on labor market fluidity.

Also relevant is the literature on the economics of innovation, which includes Arrow (1969) and Schumpeter (1934). March (1991) emphasized the role of learning by doing in innovation and introduced the exploration versus exploitation tradeoff.[6] While learning about innovation quality also plays a role in my model, an important role is also played by learning about the manager and the *interaction* between these two forms of learning. A related paper is Acemoglu (2010), which develops a macroeconomic model in which labor scarcity may encourage or discourage innovation depending on the nature of technology, and its impact on the marginal product of labor. Acemoglu's (2010) focus is on labor as a factor input and technological change in a macroeconomic framework, while I highlight a different set of channels in an optimal contracting framework related to learning about managers and hirings/firings.

Other papers have examined imitation and innovation as alternative forms of exploration, including Aghion et al. (2001), Benoit (1985), and Benhabib, Perla, and Tonetti (2014). The analysis here differs in that it focuses on the relationship between project choice and labor market fluidity in an optimal contracting setting. The explore-exploit choice also appears in models in which the focus is on the financing of innovation. Gomes, Gottlieb, and Maestri (2016) study financial contracting between an investor and a firm that is privately informed about its payoffs from exploration and exploitation. Kerr and Nanda (2014) review this literature. Unlike these papers, the focus here is not on how innovation is financed.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes optimal contracting with exploitation. Section 4 analyzes optimal contracting with innovation. Section 5 discusses implications. Section 6 concludes.

[6] Other papers that emphasize the role of learning in innovation include Aghion (2002), Bhattacharya, Chatterjee, and Samuelson (1986), Moscarini and Smith (2001), and Roberts and Weitzman (1981). Also see Krieger (2021), Krieger, Li and Thakor (2022), and Frankel et al. (2023) for more recent contributions in the context of drug development.

2 Model

2.1 Agent Preferences

All agents in the model are risk neutral, and the riskless interest rate is zero. There are three dates, divided into two periods: the first period begins at $t = 0$ and ends at $t = 1$, while the second period begins at $t = 1$ and ends at $t = 2$. There are $N > 1$ unlevered firms in the economy, which have funds to invest in projects. Each firm has a Chief Executive Officer (CEO), who (faithfully) represents the interests of the firm's owners/shareholders, and a manager that maximizes expected utility over consumption; the manager faces a consumption discount factor of $\delta \in (0, 1)$. Shareholders value consumption at all dates equally.

2.2 Investment Choice

In each period, each firm faces a choice of project type: exploitation, incremental innovation, and novel innovation. That is, the firm can: (i) make an investment at $t = 0$ in either a familiar project utilizing existing technology that is labeled "exploit" (E); (ii) make an investment at $t = 0$ in a project that is incrementally innovative (C);[7] or (iii) making an investment at $t = 0$ in exploring a novel, breakthrough innovation (I). The first-period project investment occurs at $t = 0$ and the project payoff occurs at $t = 1$. The second-period project investment occurs at $t = 1$ and the project payoff occurs at $t = 2$. For any project type, the investment that is required is 1 and it is irreversible, i.e. the liquidation value of any project prior to its cash flow realization is zero.

In each period, the CEO decides whether the manager will be asked to search for E, C, or I. At $t = 0$, after the CEO has chosen E, C, or I, the manager must choose effort $e \in \{0, 1\}$ to search for a good (positive-NPV, denoted as G) project. The (private) cost of

[7] I use C to designate incremental innovation as "copycat" innovation.

effort for the manager is:

$$\hat{\psi}(e) = \begin{cases} \psi > 0 & \text{if } e = 1 \\ 0 & \text{if } e = 0 \end{cases} \tag{1}$$

In each period, regardless of project type and duration, a choice of $e = 1$ means that the manager finds a good project with probability $p \in (0, 1)$, and a choice of $e = 0$ means the probability of finding a good project is zero.[8] If a good project is not found, the manager always has a bad (negative-NPV, denoted as B) project available. At $t = 1$, the manager is again asked to choose from E, C, and I, and can again choose $e \in \{0, 1\}$. In each period, the manager privately observes whether he found a good project or not, and then decides whether to request funding or not. The CEO costlessly observes the manager's choice of E, C, or I and decides whether to approve the funding request.

2.3 Exploitation, Incremental Innovation, or Novel Innovation

The fundamental difference between exploitation (E) and innovation (C and I) is that E represents a well-known technology where there is no value of learning, whereas C and I involve projects that carry with them value from learning. This is reflected in the assumption that, holding managerial quality fixed, the probability distribution of the payoff of the good project is the same in both periods with E, whereas the project payoff is stochastically higher in the second period than in the first with C and I if the first-period project is successful and the firm continues with the manager.[9]

While both C and I reflect the value of learning by doing typically associated with innovation, there is a difference between C and I. For the G project, there is no project quality uncertainty with C—the firm is merely taking an existing product idea and improving on it—the quality of the project being improved upon is already known. However, as in

[8]For simplicity, the probability of finding G is independent of whether the project is E or I and whether it has a long or a short horizon. It is plausible to assume instead that good I projects are harder to find. This will increase the firm's wage cost with I relative to its cost with E and shrink the region of K values over which I is chosen (see *Figure 2*), but the results will be qualitatively the same.

[9]This is meant to capture manager-specific learning about the innovative project.

Manso (2011), novel innovation (I) involves project quality uncertainty. The probability is $r \in (0, 1)$ that a G project of type I will be a "hit" (H) with a high payoff distribution, and the probability is $1-r$ that it will be a "flop"/failure (F) with a payoff of zero with probability one. The second difference is that the passage of time and any associated learning has no impact on the payoff distribution with E. That is, with E, the project in the first period has the same payoff distribution as the project in the second period. In contrast, with I, the second-period project has a higher expected payoff than the first-period project, conditional on investing in I in the first period and continuing with the first-period manager.[10] This assumption implies that continuing with the same manager has greater value with innovation than with exploitation, which contributes in the analysis to a higher effective cost of firing the manager with innovation.

To provide an example, consider the context of drug development. A biopharma firm may exploit by producing a generic drug, which is designed to be effectively the same as an already-approved drug (i.e. the same therapeutic category, mechanism of action, efficacy, etc.), and thus there is no learning involved. The firm could alternatively invest in incremental innovation, which may involve applying an existing mechanism of action or a modification of a known chemical compound to a different therapeutic category. While the efficacy and other attributes of the therapy to the new therapeutic category have not been established—the drug must go through clinical trials—the underlying technology is well known. For instance, it is common for drugs to be used for multiple indications—Botox has applications in treating migraines as well as for cosmetics. Finally, the firm could invest in novel innovation, which could involve treating a disease with few existing treatments or an entirely unproven mechanism of action or chemical. In drug development, the U.S. Food and Drug Administration (FDA) has designations for orphan drugs (drugs targeting rare diseases) and Breakthrough Therapies (drugs that treat serious diseases and represent a substantial improvement over existing therapies). Krieger, Li, and Papanikolaou (2022)

[10] This is meant to capture manager-specific learning about the innovative project.

provide empirical evidence that novel drugs are riskier but more economically valuable than other drugs, consistent with the assumption that the novel drug project may enable higher payoffs if successful.[11]

2.4 Manager Types and Project Payoffs

There are M managers, where $M > N$ (number of firms), with N and M being integers. This means that firms design contracts to minimize the rents provided to managers, subject to managers' participation constraints. The manager's type (ability), represented $\tau \in \{T, U\}$, affects the payoff distributions of projects. If $\tau = T$, a manager is "talented", and if $\tau = U$, a manager is "untalented". Let y_t represent the project payoff at date t. Moreover, it costs the firm $K > 0$ to fire and replace a manager (I discuss this more in Section 2.9).

Exploit (E): The good E project requires an investment of 1 at date $t \in \{0, 1\}$ and pays off $R > 1$ at date $t = 1$ with probability $\tilde{q}(\tau)$ (which is dependent on the manager's ability) and pays off 0 with probability $1 - \tilde{q}(\tau)$, with:

$$\tilde{q}(\tau) = \begin{cases} 1 & \text{if } \tau = T \\ q \in (0.5, 1) & \text{if } \tau = U \end{cases} \tag{2}$$

The first-period good E project and the second-period good E project have the same payoff distribution.

It is common knowledge that $\Pr(\tau = T \text{ at date } t) = \theta_t \in (0, 1)$—agents therefore start out with a common prior θ_0 at $t = 0$ regarding the manager's initial type. The bad E project pays off R with probability $b \in [0, q)$ and 0 with probability $1 - b$, where b does not depend on the manager's ability. This means that the bad project managed by any manager is worse than the good project managed by even the untalented manager. Furthermore, it

[11] Grabowski and Vernon (1990) and Grabowski, Vernon, and DiMasi (2002) also show that "blockbuster" drugs achieve much higher returns than other drugs.

is assumed that $M_T \equiv \theta_0 M > N$, where M_T is the (expected) number of talented managers. This means there are enough talented managers to fully staff all firms.

Incremental Innovation (C): Like E, the probability distribution of the payoff of the good C project for the first-period project is a payoff of R with probability $\tilde{q}(\tau)$ and 0 with probability $1 - \tilde{q}(\tau)$, where $\tilde{q}(\tau)$ is given by (2). If there is no investment in the first period, then the payoff distribution of the second-period project is the same as that of the first-period project. However, if there is investment in the first-period project, and the first-period manager continues, then the second-period project pays off $R_C > R$ with probability $\tilde{q}(\tau)$ and 0 with probability $1-\tilde{q}(\tau)$. This reflects the value of "learning by doing" with innovation. It makes it more costly for the firm to fire the manager with incremental innovation than with exploitation.[12]

Define $q_0 \equiv \theta_0 + [1 - \theta_0]\, q$ as the prior (at $t = 0$) probability of project success, given the (common) prior belief θ_0 that $\tau = T$. It will be assumed that $qR_C < q_0 R$. From (2), note that a good E project managed by a talented manager succeeds almost surely, so a payoff $y_1 = 0$ at $t = 1$ leads to the posterior belief that the manager is untalented ($\tau = U$). Thus, q is the posterior probability of success in the second period following $y_1 = 0$ at $t = 1$. This means that the assumption that the expected payoff on C in the second period with the incumbent manager following first-period failure (qR_C) is less than the expected payoff on E (or C) in the second period with a new manager ($q_0 R$), implying that the firm would prefer to fire the worker following first-period failure if $K = 0$. If this inequality were reversed, the firm would retain the manager and invest in C in the second period regardless of K, which would sever the link between the firm's choice and labor market fluidity.

[12]This value of learning exists regardless of the first-period project outcome, as long as investment occurs in the project at $t = 0$. Empirical evidence suggests that failure may have a substantial learning effect; see, e.g., Krieger (2021) and Krieger et al. (2022). The assumption that failure produces a larger learning effect will only strengthen my results.

Innovate (I): The first-period good I project that is a hit (H) pays off R at $t = 1$ with probability $\tilde{q}_I(\tau)$ and 0 with probability $1 - \tilde{q}_I(\tau)$. The second-period good I project pays off $R_I > R$ at $t = 2$ with probability $\tilde{q}_I(\tau)$ and 0 with probability $1 - \tilde{q}_I(\tau)$ if it is H, the firm had invested in I at $t = 0$, and the first-period manager continues. As with the good C project, this specification captures the synergy value of managerial experience/learning with exploration that comes from the manager becoming more familiar with I. This benefit is not available if there is no investment in the first period. That is, the second-period project has the same payoff distribution as the first-period good I project if there is no investment in I in the first period. Here

$$\tilde{q}_I(\tau) = \begin{cases} 1 & \text{if } \tau = T \text{ and project is } H \\ q & \text{if } \tau = U \text{ and project is } H \\ 0 & \text{if project is } F \end{cases} \tag{3}$$

Thus, a talented manager succeeds almost surely with a hit (H) project. If the manager is untalented, the innovation succeeds with probability q if it is H or F. If the novel innovation is a flop (F), the manager always fails regardless of talent.

To capture the value of learning with I, it is assumed that the quality of the good I projects is perfectly intertemporally correlated. Thus, if the first-period good I project is of quality $i \in \{H, F\}$, then the second-period good I project has the same quality i. This applies only if the manager selects *good* I projects in both periods. A bad project has the same payoff distribution regardless of whether it is I or E. Thus means that the outcome on the first-period I project is informative about project quality when it comes to the second-period I project.

The firm can switch from one type of project in the first period to another type in the second period. *Project switching* essentially applies only to the second-period project. Based on the first-period project outcome, if the firm decides to switch projects, it means that it chooses $i \in \{E, C, I\}$ in the first period and then $j \neq i$, where $j \in \{E, C, I\}$, in the second period. It is assumed that the firm will switch projects in the second period only if it is strictly better off from switching than from sticking to its first-period choice.

As in Manso (2011), the above specification implies that the expected value of novel innovation at the outset is less than that of exploitation. This follows since the expected value of I in the first period, evaluated at prior beliefs, is rq_0R. However, conditional on success (for both I and E), I has a higher expected value than E in the second period. That is,

$$rR_I > R > \underline{r}_1^I R_I \tag{4}$$

$$r\left[R + R_I\right] > R + R_C \tag{5}$$

where $\underline{r}_1^I = \frac{[1-\theta_0][1-q]r}{[1-\theta_0][1-q]r+[1-r]}$ is the posterior belief that I is H after $y_1 = 0$ is observed at $t = 1$. Thus, (4) implies that, having chosen I and realized $y_1 = R$ in the first period, the firm will retain the manager and continue with I in the second period, and following $y_1 = 0$, the firm will drop I in the second period. (5) implies that, conditional on $\tau = T$, the long-run value of I exceeds even the long-run value of C. Thus, one difference between this specification and the previous literature is that the *expected* value of *long-term* novel innovation exceeds the value of both incremental innovation and exploitation. This is meant to reflect the benefit of the paired manager-firm learning and innovation-specific human capital developed over time. Companies that have been pioneers in breakthrough innovation have consistently touted the benefit of developing interorganizational processes and managerial training and experience in organizational innovation success.[13]

Viewed at $t = 0$, the expected payoffs on all good projects exceed the project investment of 1 plus the cost of managerial effort ψ (i.e. they all have positive NPV ex ante), and the expected payoffs on all bad projects are less than the project investment of 1.

[13]See, for example, Hamel (2006).

2.5 Informational Assumptions

The manager's ability is initially unknown, but information becomes revealed about it to all agents over time. The manager's search effort choices at each date are privately known only to the manager, but it is common knowledge (based on the CEO's directive) whether he is innovating (C or I) or exploiting (E) and searching for a good project. The manager also privately observes whether he found a good project or not, and also privately observes whether the project he is requesting funding for is good or bad.

The payoff on the first-period good project, $y_1 \in \{R, 0\}$, is observed by all at $t = 1$, and the payoff on the second-period good project—$y_2^E \in \{0, R\}$, $y_2^C \in \{0, R, R_C\}$, $y_2^I \in \{0, R, R_I\}$— is observed by all at $t = 2$.

2.6 Parametric Restrictions

I impose three restrictions on the exogenous parameters to focus on the cases of interest (the mathematical expressions for these are in the Appendix):

(Restriction 1) The option value of investing in I is greater than the option value of investing in C.

(Restriction 2) The manager's effort disutility is high enough to generate incremental agency costs from removing the threat of firing or firing the manager excessively (relative to the first best) that exceed the option value of I relative to E.

(Restriction 3) Novel Innovation (I) is sufficiently risky in the sense that r, the probability of a hit (H), is sufficiently low.

Restriction 1 is a natural way to distinguish between novel and incremental innovation. Restriction 2 captures the idea that the principal driver of the firm's choice of project is agency costs, and that the option values of I and C are essentially "tie breakers". Otherwise, if the option value of (novel) innovation is sufficiently high, then such innovation is all the firm would do. Finally, Restriction 3 ensures that the riskiness of the novel innovation provides a sufficiently valuable shield to the manager from being fired.

2.7 Wage Contracts

The manager's wage in each period can only be based on observable outcomes at the end of the period.

For the first-period project, the manager's wage, paid at $t = 1$, is W_1^x, where $x \in \{n, h, l\}$, with $x = h$ representing $y_1 = R$ and $x = l$ representing $y_1 = 0$. For the second-period project, the manager's wage is $W_2^z(x)$, paid at $t = 2$, where x is the first-period outcome and the second-period outcome is $z \in \{n, h, l\}$, with $z = h$ representing $y_2 = R$ or R_C or R_I, and $z = l$ representing $y_2 = 0$. Note that $W_2^z(x)$ is a function of the outcome x on the first-period project that is observed at $t = 1$. Thus, $x \in \{n, y_1\}$, depending on whether there was no investment at $t = 0$ (n), there was investment at $t = 0$ and y_1 was observed at $t = 1$. All wages are constrained to be non-negative.

2.8 The CEO's Choices

At $t = 0$, in addition to determining the project type she wants the manager to search for, the CEO offers the manager a wage contract W_1^x. The wage contracts for exploit (E) will be denoted by W with the appropriate subscripts and superscripts, for incremental innovation they will be denoted by $\overline{W}$, and for novel innovation they will be denoted by $\underline{W}$. and for explore (I) they will be denoted by $\underline{W}$.

At $t = 1$, the CEO decides whether to retain the manager for the second period or fire him. If the manager is retained, the CEO offers a second-period contract $W_2^x(z_1)$, $\overline{W}_2^z(x)$, or $\underline{W}_2^z(x)$, depending on the second-period project type.

2.9 Manager's Reservation Utility and Firm's Firing Cost

The manager's reservation utility in each period is 0. In each period, the CEO makes the manager a wage contract offer that is take-it-or-leave-it. If the contract satisfies the manager's participation constraint, then the manager accepts the contract.

It costs the firm $K \in \left(0, \overline{K}\right)$, where $\overline{K}$ is an upper bound, to fire and replace the manager. This cost K reflects the transactions costs of searching for, hiring, and training a new manager. This cost varies in the cross-section of firms and its impact on the firm's choice between novel innovation, incremental innovation, and exploitation will be examined. It is viewed as a measure of labor market fluidity—greater fluidity means a lower K. To ensure that it may sometimes make sense for the firm to retain a failed manager at $t = 1$ with E, assume that

$$qR - \psi \left[\frac{p\delta q + \{1 - p\delta\} b}{p\delta[q - b]} \right] > 1 \tag{6}$$

We will see later that the second term on the left-hand side of (6) is equal to the expected cost of compensating a failed manager under the optimal contract with E.[14]

2.10 Equilibrium

The focus is on subgame perfect equilibria—in each period, the CEO designs contracts that maximize firm value over the remaining dates. The CEO solves for optimal wage contracts in both periods that will be offered to the manager, taking into account project type (E, C, or I), and rationally anticipates the manager's choices related to search effort and the timing of funding requests, in addition to her decision about replacing the manager in the second period. The CEO then compares firm values in all cases, and decides whether to go for novel innovation, incremental innovation, or exploitation.

[14]Basically this condition says that continuing with a failed manager is viable for the firm in the sense of yielding an expected payoff net of wages that exceeds the project investment. But, of course, replacing a failed manager with a new manager may be better, depending on K.

2.11 Notation

The posterior probabilities of success for various second-period projects depend on what project was chosen in the first period and what was learned from the first-period outcome. To capture this, the subsequent analysis uses notation which is as follows. Throughout we will use x to refer generically to the first-period outcome and z to refer to the second period outcome.

When the firm invests in E at $t = 0$, the posterior belief that the manager is T at $t = 1$, conditional on observing outcome $x \in \{h, l, n\}$ at $t = 1$ (where h means $y_1 = R$, l means $y_1 = 0$, and n means no project was proposed for funding), is θ_1^x; the probability of success of E in the second period is $q_1^x(E)$, and of I in the second period is $q_1^x(I)$. When the firm invests in C at $t = 0$, the posterior belief that the manager is T at $t = 1$ is $\overline{\theta}_1^x$; the probability of success with $i \in \{E, C, I\}$ in the second period is $\overline{q}_1^x(i)$. When the firm invests in I in the first period, the posterior belief that the manager is T at $t = 1$ is $\underline{\theta}_1^x$, and the probability of success in the second period is $\underline{q}_1^x(i)$, with $i \in \{E, C, I\}$. *Table 1* summarizes the notation for the success probabilities.

Table 1: Second-period Project Success Probabilities

1st Period Project	Second-period Project Success Probability		
	E	C	I
E	$q_1^x(E)$	$q_1^x(C)$	$q_1^x(I)$
C	$\overline{q}_1^x(E)$	$\overline{q}_1^x(C)$	$\overline{q}_1^x(I)$
I	$\underline{q}_1^x(E)$	$\underline{q}_1^x(C)$	$\underline{q}_1^x(I)$

2.12 First Best

In the first best case, the manager's search effort is observable, and the quality of the project (whether it is good or bad, not whether I is H or F) is observable to the CEO. The manager's ability is unknown to all at $t = 0$, and there is no firing cost. Thus, the CEO will instruct the manager to choose $e = 1$, pay him a fixed wage of ψ, and ask him to search for a good

I project at $t = 0$. If the manager finds a good project, funding is provided; otherwise, no investment is made at $t = 0$. Then at $t = 1$, the manager is retained if $y_1 = R$ and fired if $y_1 = 0$. In the second period, if the manager is retained, he is again paid a fixed wage of ψ, instructed to choose $e = 1$ and search for a good I project. If he is fired at $t = 1$, the replacement manager is also paid a fixed wage ψ and asked to choose $e = 1$ and search for a good E or C project.

Funding is provided only if the manager finds a good project. Note that the first best in terms of project choice is the same whether the manager's ability is known or unknown. As long as effort is observable, the manager is always paid a fixed wage ψ—since effort disutility is not ability-dependent—and asked to choose I at $t = 0$.

3 Results for Exploit (E) and Incremental Innovation (C)

In this section, I analyze the model for E and C. For E, I begin with the second period and then move to the first period contracts.

3.1 Second Best Contracts when Manager Searches for a Good E Project in the First Period

In the second best, the manager's search effort choice and the quality of the project for which funding is requested are not observable. The model is solved by backward induction. So first the optimal wage contract offered at $t = 1$ for the second period is solved for. Note that if the manager searched for E in the first period, we know that the manager will be asked to search for E in the second period, regardless of the first-period outcome, since the payoff distribution of E is the same in both periods.

3.1.1 Second-period Contract

At $t = 1$, the posterior belief that the manager is T is given by θ_1. If there was no investment at $t = 0$, then clearly $\theta_1 = \theta_0$. If there was investment and the first-period S project failed, then the posterior belief is:

$$\theta_1^l = \Pr\left(\tau = T \mid y_1 = 0\right) = 0 \tag{7}$$

If $y_1 = R$, then the posterior belief is

$$\begin{aligned} \theta_1^h &= \Pr\left(\tau = T \mid y_1 = R\right) \\ &= \frac{\theta_0}{\theta_0 + \left[1 - \theta_0\right] q} \end{aligned} \tag{8}$$

If the manager is retained, his second-period wage contract is a triplet $\left\{W_2^n\left(z_1\right), W_2^h\left(z_1\right), W_2^l\left(z_1\right)\right\}$, where z_1 if the outcome on the first-period project, $W_2^n\left(z_1\right)$ is what the manager is paid if he does not request second-period funding, $W_2^h\left(z_1\right)$ is his wage if a project is invested in and it pays off R at $t = 2$, and $W_2^l\left(z_1\right)$ is his wage if the project pays off 0. This contract must satisfy two incentive compatibility (IC) constraints and a managerial participation constraint.

The first IC constraint is that the manager prefers to choose $e = 1$:

$$\delta p\left\{q_1^h(E)W_2^h\left(z_1\right) + \left[1 - q_1^h(E)\right]W_2^l\left(z_1\right)\right\} + \delta\left[1 - p\right]W_2^n\left(z_1\right) - \psi \geq \delta W_2^n\left(z_1\right) \tag{9}$$

where

$$q_1^h(E) \equiv \theta_1^h + \left[1 - \theta_1^h\right] q \tag{10}$$

Since $\theta_1^l = 0$, it is straightforward that $q_1^l(E) = q$. The second IC constraint is that if the manager does not find a good project, he will not request funding:

$$bW_2^h\left(z_1\right) + [1 - b]W_2^l\left(z_1\right) \leq W_2^n\left(z_1\right) \tag{11}$$

The manager's participation constraint is:

$$\delta p \left\{ q_1^h(E) W_2^h(z_1) + \left[1 - q_1^h(E)\right] W_2^l(z_1) \right\} + \delta \left[1 - p\right] W_2^n(z_1) - \psi \geq 0 \tag{12}$$

The optimal wage contract in the second period is characterized below.

Lemma 1: *If the manager searched for an E project in the first period that was funded and had $y_1 = R$ at $t = 1$, then the manager is retained for the second period and the optimal second-period wage contract is:*

$$W_2^h(R) = \frac{\psi}{p \left[q_1^h(E) - b\right] \delta} \tag{13}$$

$$W_2^l(R) = 0 \tag{14}$$

$$W_2^n(R) = \frac{b\psi}{p \left[q_1^h(E) - b\right] \delta} \tag{15}$$

The manager's participation constraint is slack under the optimal contract and the manager earns a rent equal to $W_2^n(R)$, with utility value $\delta W_2^n(R)$.

Two points are worth noting. First, it is clear that the higher $W_2^l(R)$ is, the more costly it is for the firm to ensure satisfaction of the IC constraint (9). So, given the zero lower bound constraint on wages, it is efficient to set $W_2^l(R) = 0$. Second, to ensure satisfaction of the IC constraint (11), the manager must be paid a wage even when he does not request project funding. Absent this wage, the manager will request funding even for a bad project.

The reason why the manager earns a rent is that he has to be motivated both to work hard to find a good project and also to not request funding for a bad project. Thus, the combination of the manager's private information about his own effort choice *and* the quality of the project for which he is requesting funding generates an efficiency wage in this multi-tasking setting that provides an informational rent for him.[15]

[15] See Holmstrom and Milgrom (1991).

The next lemma characterizes the optimal second-period contract when the manager searched for S in the first period but did not find a good project and thus did not request funding.

Lemma 2: *If the manager searched for an E project at $t = 0$ but did not request funding for it, he is retained in the second period and the optimal second-period wage contract is:*

$$W_2^h(n) = \frac{\psi}{p[q_0 - b]\delta} \tag{16}$$

$$W_2^l(n) = 0 \tag{17}$$

$$W_2^n(n) = \frac{b\psi}{p[q_0 - b]\delta} \tag{18}$$

where $q_0 = q_1^n(E) \equiv \theta_0 + [1 - \theta_0]\, q$. The manager's participation constraint is slack and he earns a rent of $W_2^n(n)$.

The structure of contracts is the same as in Lemma 1, with q_1^h replaced by the prior belief q_0, since a lack of investment in the first period leads to no revision of beliefs about managerial ability.

Lemma 3: *If $y_1 = 0$ is observed at $t = 1$, the manager is fired and replaced with a new manager for the second period if K is small enough, and retained otherwise. If the manager is retained, the optimal second-period wage contract is:*

$$W_2^h(0) = \frac{\psi}{p\left[q_1^l(E) - b\right]\delta} \tag{19}$$

$$W_2^l(0) = 0 \tag{20}$$

$$W_2^n(0) = \frac{b\psi}{p\left[q_1^l(E) - b\right]\delta} \tag{21}$$

where $q_1^l(E) = q$. The manager's participation constraint is slack and he earns a rent of $W_2^n(0)$.

There are two reasons why the firm would like to fire a manager when $y_1 = 0$. One is that his expected talent is lower than that of a new manager, and the other is that he earns a higher rent than a new manager: notice that $W_2^n(0) > W_2^n(n)$. What deters the firms from firing the manager is K. When K is low, the manager is fired; otherwise he is retained.

3.1.2 First-period Contract

Optimal Contract if Firm Intends to Fire Failing Manager: The first-period contract is a triplet $\left\{W_1^n(R), W_1^h(R), W_1^l(R)\right\}$. Using the logic used in proving Lemma 1, it can be shown that $W_1^l = 0$. Thus, this contract is one that minimizes the firm's expected wage bill subject to two IC constraints and one participation constraint. The first IC constraint is that the manager chooses $e = 1$ at $t = 0$:

$$pq_0\left[W_1^h + \delta W_2^n(R)\right] + [1-p]\left[W_1^n + \delta W_2^n(n)\right] - \psi \geq W_1^n + \delta W_2^n(n) \tag{22}$$

In writing this constraint, it is recognized that the manager is maximizing his expected utility over two periods in making his first-period choice and that he will get fired at $t = 1$ if $y_1 = 0$, so there is no second-period rent for him to extract in this case. The second IC constraint is that the manager will not request funding for a bad project:

$$b\left[W_1^h + \delta W_2^n(R)\right] \leq W_1^n + \delta W_2^n(n) \tag{23}$$

The manager's participation constraint is that:

$$pq_0\left[W_1^h + \delta W_2^n(R)\right] + [1-p]\left[W_1^n + \delta W_2^n(n)\right] - \psi \geq 0 \tag{24}$$

This leads to the following result:

Proposition 1: *The optimal first-period wage contract when the manager can be fired is as* *follows:*

$$W_1^h = \frac{\psi}{p\left[q_0 - b\right]} - \delta W_2^n\left(R\right) \tag{25}$$

$$W_1^l = 0 \tag{26}$$

$$W_1^n = \frac{b\psi}{p\left[q_0 - b\right]} - \delta W_2^n\left(n\right) \tag{27}$$

With this wage contract, the manager chooses $e = 1$ to search for E in the first period, requests first-period funding only if he finds a good project, and is retained in the second period if he requested first-period funding for E and experienced $y_1 = R$ or if he did not request first-period funding. If the manager is retained in the second period, the wage contract he receives is described in Lemmas 1 and 2.

Optimal Contracts if the Firm Intends to Retain Failing Manager: The analog of (22) is

$$p\left\{q_0\left[\hat{W}_1^h + \delta W_2^n\left(R\right)\right] + \left[1 - q_0\right]\left[\delta W_2^n\left(0\right)\right]\right\} + \left[1 - p\right]\left\{\hat{W}_1^n + \delta W_2^n\left(n\right)\right\} - \psi \geq \hat{W}_1^n + \delta W_2^n\left(n\right) \tag{28}$$

and the analog of (23) is:

$$b\left\{\hat{W}_1^h + \delta W_2^n\left(R\right)\right\} + [1 - b]\delta W_2^n(0) \leq \hat{W}_1^n + \delta W_2^n(n) \tag{29}$$

where the $(\hat{\cdot})$ on the first-period wages denotes that these correspond to the no-firing contract. We now have:

Proposition 2: *The optimal first-period wage contract when the manager is always retained after the first period is:*

$$\hat{W}_1^h = \frac{\psi}{p\left[q_0 - b\right]} - \delta W_2^n\left(R\right) + \delta W_2^n\left(0\right) \tag{30}$$

$$\hat{W}_1^l = 0 \tag{31}$$

$$\hat{W}_1^n = \frac{b\psi}{p\left[q_0 - b\right]} - \delta W_2^n\left(n\right) + \delta W_2^n\left(0\right) \tag{32}$$

In contrast to the standard intuition that a risk-averse manager who is provided insurance against termination at the end of the first period will earn a lower first-period wage than a manager not provided such insurance, here the manager earns a *higher* first-period wage when he is not fired for failure. The intuition for this result is that the removal of the threat of being fired makes it more attractive for the manager to propose the bad project when he does not have a good project. Thus, the manager has to be provided a higher wage for not proposing a project (compare (27) and (32)). But this then increases the challenge of inducing the manager to work hard to find a good project. So his wage for achieving first-period project success ($\hat{W}_1^h$) has to go up as well (compare (25) and (30)). As a consequence, insurance against being fired allows the manager to earn a higher rent.

3.2 Second-Best Contracts when the Manager is Asked to Search for C at $t = 0$

3.2.1 Second-period Contracts

Lemma 4: *If the manager searched for a C project in the first period that was funded and had $y_1 = R$ at $t = 1$, then the manager is retained for the second period. If the project was funded and had $y_1 = 0$ at $t = 1$, then the manager is fired if K, the cost to fire the manager, is low enough, and retained otherwise. If the manager did not seek first-period funding, then he is retained for the second period. When the manager is retained, her is asked to search for a good C project in the second period. The second-period wage contract (for first-period outcome $x \in \{R, n, 0\}$ is:*

$$\overline{W}_2^h(x) = \frac{\psi}{p\left[\overline{q}_1^x(C) - b\right]\delta} \tag{33}$$

$$\overline{W}_2^l(x) = 0 \tag{34}$$

$$\overline{W}_2^n(x) = \frac{b\psi}{p\left[\overline{q}_1^x(C) - b\right]\delta} \tag{35}$$

where

$$\overline{q}_1^h(C) = \theta_1^h + \left[1 - \theta_1^h\right] q \tag{36}$$

$$\overline{q}_1^n(C) = q_0 = \theta_0 + \left[1 - \theta_0\right] q \tag{37}$$

$$\overline{q}_1^l(C) = q \tag{38}$$

The manager earns a rent of $\overline{W}_2^n(x)$*.*

This now leads to:

Proposition 3: *Suppose the manager is instructed to choose* C *at* $t = 0$*. The optimal first-period contract when the manager can be fired at* $t = 1$ *is as follows:*

$$\overline{W}_1^h = \frac{\psi}{p\left[q_0 - b\right]} - \delta \overline{W}_2^n(R) \tag{39}$$

$$\overline{W}_1^l = 0 \tag{40}$$

$$\overline{W}_1^n = \frac{b\psi}{p\left[q_0 - b\right]} - \delta \overline{W}_2^n(n) \tag{41}$$

If the manager is always retained after the first period, the optimal contracts are the same as those in Proposition 2. If the firm chooses C *in the first period, it always chooses* C *in the second period.*

The contracts have the same structure as for the E project because the incentive compatibility problem in both periods is the same. The reason why a firm that chooses C in the first period never switches projects is as follows. If there is investment in the first period and the project succeeds, then there is a payoff-enhancement benefit from continuing with the same manager and investing in C in the second period. If the project fails and the manager is retained, there is a payoff-enhancement (conditional on success) from continuing with C

in the second period, so this always dominates asking the retained manager to search for E or I in the second period. If there is no investment at $t = 0$, the second-period project payoff distribution with C is the same as with a new E or I project. So there is no benefit in switching.

3.2.2 Inefficiency of Wage Deferral

Until now, it has been assumed that deferring until $t = 2$ the manager's wage that is payable at $t = 1$ is not allowed. It will be shown now that such a deferral is inefficient.

Lemma 5: *Deferring the manager's compensation at* $t = 1$ *until* $t = 2$ *is inefficient.*

The intuition is as follows. Suppose the manager was asked to search for L at $t = 0$. If the manager's wage is paid at $t = 2$ instead of $t = 1$, then there are two possibilities. One is that the deferred wage is simply added to the manager's second-period wage in each state, in which case it has no impact on the manager's incentives on either the first-period project or the second-period project. In this case, the deferral is simply inefficient because the manager prefers consumption at $t = 1$ over consumption at $t = 2$, all else being equal. Further, the wage deferral cannot improve on the incentives provided by the optimal contract derived for L in the previous analysis, since that is the least-cost contract to incentivize the manager to work hard and propose only the good project; such a contract cannot be improved upon by making the manager's payoff contingent on a future project.

So deferral can only improve second-period incentives. But any optimal contract requires that the manager be paid nothing for a failed project. Thus, all of the wage deferral must be spread out over the manager's second-period wage for success on the second-period project or his wage for not proposing a second-period project. However, this cannot improve incentives on the second-period project since we solved for the optimal contract with a zero payoff for project failure. Therefore, wage deferral fails to improve incentives and leads to a higher wage cost.[16]

[16] Anderson, Bustamante, and Zervos (2018) show another reason why a wage deferral may be ineffi-

4 Results for Novel Innovation (I)

Recall that with the I project chosen at $t = 0$, the firm has two choices at $t = 1$: (i) it can replace the manager as it can with E and C, and (ii) it can replace the I project with the E or the C project. The second choice is moot when the firm invests in E or C at $t = 0$ since there is no uncertainty about the quality of the project itself, once managerial talent is controlled for. With I, however, project quality is uncertain even if managerial talent is known. Hence, the option to abandon I at $t = 1$ is potentially valuable.

4.1 Belief Revision

Suppose the manager chooses an I project at $t = 0$. How should beliefs be revised when the project outcome is observed at $t = 1$? Recall that the notation $\underline{\theta}_1^l$ and $\underline{\theta}_1^h$ is used to denote the posterior beliefs at $t = 1$ with I chosen at $t = 0$.

Now,

$$\begin{aligned}\underline{\theta}_1^l &= \Pr\left(T \mid y_1 = 0\right) \\ &= \frac{\theta_0[1-r]}{[1-r] + r\,[1-\theta_0]\,[1-q]}\end{aligned} \tag{42}$$

and

$$\begin{aligned}\underline{\theta}_1^h &= \Pr\left(T \mid y_1 = R_S\right) \\ &= \frac{\theta_0}{\theta_0 + q\,[1-\theta_0]} = \theta_1^h\end{aligned} \tag{43}$$

Comparing (42) to (7) and (43) to (8), we see immediately that while success with I is just as informative about the manager's type as success with E, failure with I is a more noisy indicator of the manager's type than failure with E.

cient—the likelihood of growth-induced managerial turnover.

4.2 Managerial Replacement and Optimal Contracts

This section examines how the firm makes its optimal firing/retention decision and the effect this has on optimal contracts with I.

Lemma 6: *Let $K^*(i)$ be the value of K such that the firm is indifferent between firing and retaining the manager conditional on observing $y_1 = 0$ at $t = 1$ when it invests in project $i \in \{E, C, I\}$ at $t = 0$. Then when the firm invests in $i \in \{E, C, I\}$ at $t = 0$, it prefers to fire the manager after $y_1 = 0$ is observed at $t = 1$ if $K < K^*(i)$ and retains him if $K \geq K^*(i)$. Moreover, $K^*(I) < K^*(E) > K^*(C)$.*

The intuition is as follows. $K^*(C) < K^*(E)$because continuing with the same manager in the second period leads to a higher payoff conditional on success than in the first period with C but not with E. This greater benefit of retaining the manager leads the firm to retain a failed manager for a bigger set of K values with C than with E.

In comparing $K^*(I)$ and $K^*(E)$, the firm drops the I project in the second period if it fails in the first period, so a direct payoff benefit of retention does not exist with I as it does with C. Failure in the first period with either I or E leads the firm to invest in E in the second period. Thus, the intuition for $K^*(I) < K^*(E)$ comes directly from the fact that failure at $t = 1$ with I can be either because the manager is untalented or because the project is a flop. This added uncertainty about project quality permits a bigger set of K values for which a failing manager is retained in the second period compared to E. This means that greater noise in inference about the manager's type acts as a "protective shield" for the manager in case the first-period project fails, and the threshold replacement cost for retaining the manager is lower if K varies in the cross-section of firms in the industry.[17] It is assumed throughout that $K^*(E) < \overline{K}$.

[17]Whether $K^*(I) < K^*(C)$ depends on the exogenous parameter values. If R_C is large enough, then $K^*(C) < K^*(I)$. If r is small, so that the posterior assessment of the manager's ability at $t = 1$ even after $y_1 = 0$ with I is high enough, then $K^*(I) < K^*(C)$.

Now we compute $\underline{\theta}_1^l$ and $\underline{\theta}_1^h$:

$$\underline{\theta}_1^h = \Pr\left(\tau = T \mid y_1 = R\right) = \theta_1^h \tag{44}$$

$$\begin{aligned}\underline{\theta}_1^l &= \Pr\left(\tau = T \mid y_1 = 0\right) \\ &= \frac{\theta_0\left[1-r\right]}{\theta_0\left[1-r\right]+\left[1-\theta_0\right]\left\{1-rq\right\}}\end{aligned} \tag{45}$$

Comparing $\underline{\theta}_1^h$ and $\underline{\theta}_1^l$ to $\overline{\theta}_1^h$ and $\overline{\theta}_1^l$, it is clear that the inferences about the manager's types are now noisier.

How does the firm update its beliefs about the quality of the I project? Suppose it invested in I at $t = 0$. Then

$$\underline{r}_1^h = \Pr\left(\text{project } H \mid y_1 = R\right) = 1 \tag{46}$$

$$\underline{r}_1^l = \Pr\left(\text{project } H \mid y_1 = 0\right) = \frac{r[1-q]\left[1-\theta_0\right]}{r[1-q]\left[1-\theta_0\right]+\left[1-r\right]} \tag{47}$$

It is useful to write down posterior beliefs at $t = 1$ when the outcome, x, of the first-period I project has been observed. These posteriors refer to the probability of success of the second-period project $i \in \{E, C, I\}$ undertaken at $t = 1$, given by $\underline{q}_1^x(i)$ (see Table 1 for the notation):

$$\underline{q}_1^h(i) = \underline{\theta}_1^h + \left[1-\underline{\theta}_1^h\right] = q_1^h(i)\,\forall i \in \{E, C, I\} \tag{48}$$

$$\underline{q}_1^l(i) = \underline{\theta}_1^l + \left[1-\underline{\theta}_1^l\right] q\,\forall i \in \{E, C\} \tag{49}$$

$$\underline{q}_1^l(I) = \underline{r}_1^l \underline{q}_1^l(E) \tag{50}$$

where $\underline{r}_1^l$ is given in (47) and $\underline{\theta}_1^h$ and $\underline{\theta}_1^l$ are given in (44) and (45). In all cases, $\underline{q}_1^n(i) = q_0$ $\forall i \in \{E, C, I\}$;see Lemma 2 for q_0.

Lemma 7: *Suppose the firm chose I at $t = 0$, assuming that the manager is retained* *even if* $y_1 = 0$. *The optimal second-period wage contracts for Exploit (E) and Incremental Innovation (C) are* ($\forall i \in \{E, C\}$)*:*

$$W_2^h(x) = \frac{\psi}{p\delta\left[\underline{q}_1^x(i) - b\right]} \tag{51}$$

$$W_2^l(x) = 0 \tag{52}$$

$$W_2^n(x) = \frac{b\psi}{p\delta\left[\underline{q}_1^x(i) - b\right]} \tag{53}$$

where h *means* $y_2 = R$, l *means* $y_2 = 0$, n *means no second-period funding was requested,* x *is the first-period outcome, and* $\underline{q}_1^x(i)$ *is given in (48)-(50). If the manager is fired following* $y_1 = 0$, *the new manager's contracts are the same as (51)-(53), with* $\underline{q}_1^x(i)$ *replaced by* q_0. *For the Novel Innovation (I), the second-period contracts are:*

$$\underline{W}_2^h(x) = \frac{\psi}{p\delta\left[\underline{q}_1^x(i) - b\right]} \tag{54}$$

$$\underline{W}_2^l(x) = 0 \tag{55}$$

$$\underline{W}_2^n(x) = \frac{b\psi}{p\delta\left[\underline{q}_1^x(i) - b\right]} \tag{56}$$

The structure of the contracts in this lemma is similar to that in Lemma 1. What varies is the posterior belief about second-period success, which is described in Table 1. The next result characterizes the optimal first-period wage contracts.

Lemma 8: *Suppose a firm that invests in* I *at* $t = 0$ *fires the manager if* $y_1 = 0$ *at* $t = 1$ *and retains him otherwise, and it invests in* I *at* $t = 1$ *if* $y_1 = R$ *but invests in* E *at* $t = 1$ *otherwise. Then the optimal first-period contracts are as follows for* I*:*

$$\underline{W}_1^h = \frac{\psi}{p\left[rq_0 - b\right]} - \delta W_2^n(R) \tag{57}$$

$$\underline{W}_1^l = 0 \quad (58)$$

$$\underline{W}_1^n = \frac{b\psi}{p\,[rq_0 - b]} - \delta W_2^n(n) \quad (59)$$

where $W_2^n(R)$ *is defined in (15) and* $W_2^n(n)$ *is defined in (18).*

The contracts above are similar to those provided in Proposition 3.

The next result is straightforward but useful.

Lemma 9: *The following hold:*

1. *If the firm fires the manager at* $t = 1$ *and replaces him with a new manager, it will always invest in an* E *project in the second period,if it invested in* E *or* I *at* $t = 0$*, and continue with* C *if it chose* C *in the first period. A firm that invested in* E *or* C *in the first period will never invest in* I *in the second period.*

2. *If the firm invested in an* I *project at* $t = 0$*, it will not switch project types in the second period if at* $t = 1$ *it observes* $y_1 = R$*. If the firm instructed the manager to search for* I *at* $t = 0$ *and no investment was made in the first period, then the firm will retain the manager but switch to* E *in the second period.*

This lemma indicates that both project switching and managerial firing will occur only after a bad first-period outcome is observed at $t = 1$. Moreover, project switching occurs only from I to E, never the other way around. The reason, of course, is the value of learning. Innovation (both C and I) is possibly more profitable than exploitation for the firm only after the firm and the manager have been involved in exploration for a period. Thus, if there was no learning via exploration in the first period, it does not pay to do it in the second period. This is also why managerial replacement or no first-period investment with I is always followed by investment in E in the second period—the value of learning is lost when the manager is fired or when the firm does not invest in I in the first period.

4.3 Optimal Project Choice Based on Labor Market Fluidity

How will the firm choose its projects at $t = 0$ and $t = 1$, and how is this choice affected by K? This is now examined. Three ranges of K are examined: low, intermediate, and high.

Proposition 4: *The firm's project choices are as follows:*

1. *When $K < K^*(I)$, the firm asks the manager to search for C in the first period and C also in the second period regardless of the first-period outcome. The manager is fired at $t = 1$ if $y_1 = 0$ and retained otherwise.*

2. *When $K \in [K^*(C), K^*(E)]$, the firm asks the manager to search for E in the first period and also in the second period regardless of the first-period outcome. The manager is fired at $t = 1$ if $y_1 = 0$ and retained otherwise.*

3. *When $K \geq K^*(E)$, the firm asks the manager to search for I in the first period. the manager is retained regardless of the first-period outcome, but the firm continues with I in the second period only if $y_1 = R$; otherwise the firm switches to E in the second period.*

This proposition shows that the relationship between innovation and labor market fluidity is non-monotonic. Innovation occurs both when labor market fluidity is high (K low) and when labor market fluidity is low (K high). For intermediate values of K, exploitation is optimal. However, the nature of innovation is different at the two extremes of labor market fluidity. When fluidity is high, exploration occurs through incremental innovation, whereas it occurs through novel innovation when fluidity is low.

The intuition is as follows. When $K < K^*(C)$, the firm fires the manager at $t = 1$ if $y_1 = 0$, regardless of whether the first-period project was E, C, or I. In this case, agency costs of wage contracting are highest with I and the same for E and C. The reason is that the manager can be fired with I even when he is talented and proposes a good I project, simply because the project happens to not be a hit (H). This uncertainty reduces his search incentive, so he needs to be paid more for success and hence also more for not proposing anything. Since the agency costs of contracting are the same for C and E, C is preferred because of the option value of continuing with the incumbent manager and reaping the benefit of a higher second-period payoff (due to learning by doing).

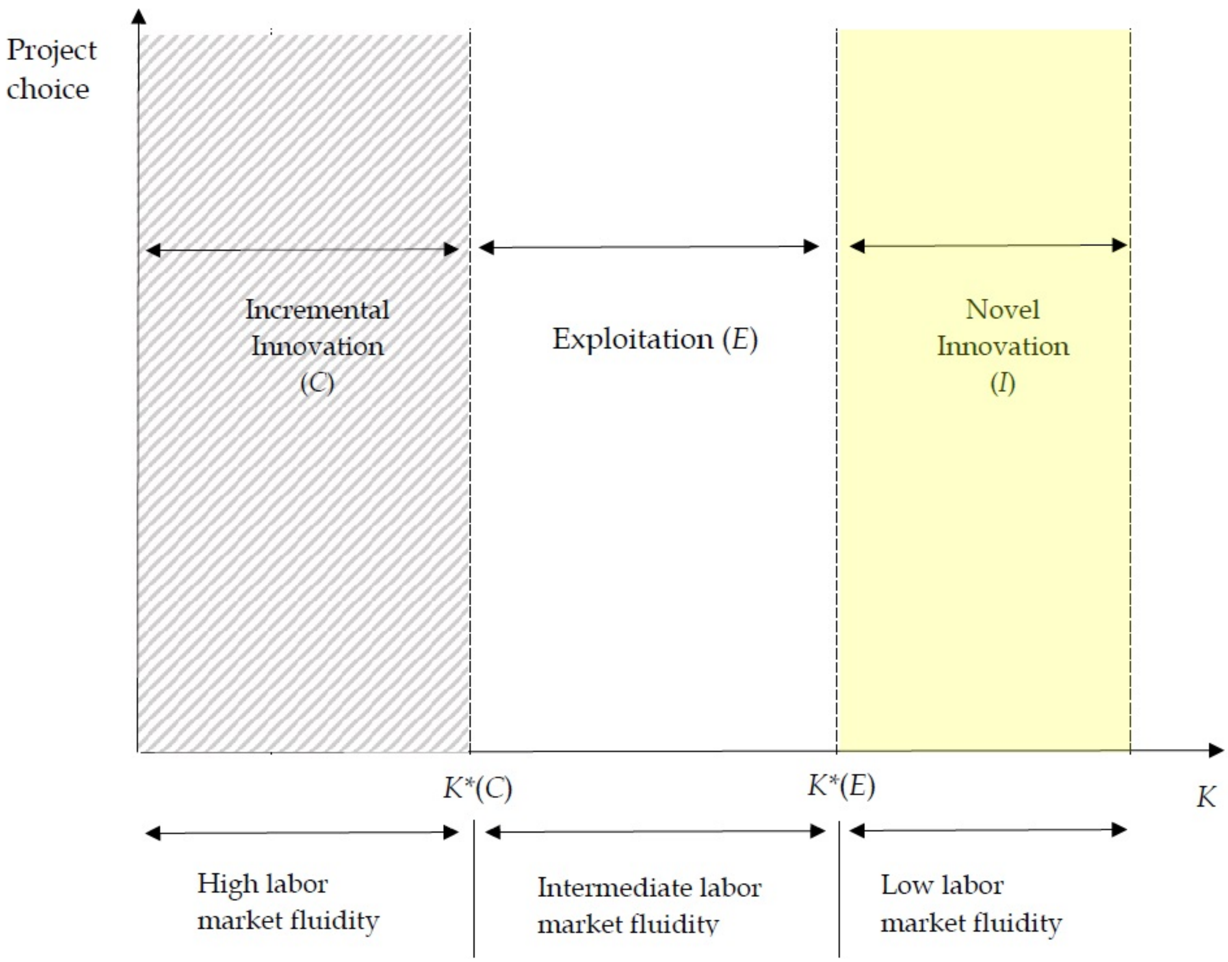
When $K \in [K^*(C), K^*(E)]$, the manager is fired when $y_1 = 0$ if the firm chose to search for E in the first period but not if it chose to search for C. The lack of a firing threat increases the agency costs associated with C relative to E, so E is preferred. Note that since $K^*(I) < K^*(E)$, the same argument applies to a comparison of I and E. Thus, E is optimally chosen.

When $K \geq K^*(E)$, the cost of managerial replacement is the highest and labor market fluidity the lowest, so the manager is not fired when $y_1 = 0$, regardless of which project he searched for in the first period. Now I gains an advantage relative to the other twp types of projects. Because a manager experiencing $y_1 = 0$ could be either untalented or unlucky to have picked a project that was not a hit, the (commonly-held) posterior belief about his ability at $t = 1$, conditional on $y_1 = 0$, is higher with I than with C or E. This means in the second period the manager who failed in the first period with I feels more confident about his ability than does a manager who failed with C or E. This makes it less expensive to motivate the manager to search for a good second-period project with I than with C or E. Hence, agency costs are now lowest with I. *Figure 1* visualizes this intuition.

5 Interpretation and Applications

These results have implications for two strands of the literature: (i) that on the firm's "exploration-exploitation" tradeoff, and area pioneered by March (1991); and (ii) that on factors that encourage innovation.

Figure 1: The Firm's Choice of Explore vs. Exploit



5.1 Incremental or Novel Innovation or Exploitation

The analysis shows that K has an impact on the firm's choice of whether to innovate or exploit an existing technology. K can be interpreted in various ways. One is that it is affected by labor laws—the more restrictive they are in terms of firing employees, the higher will be the K the firm faces. Another interpretation is that K may depend on the level of skill or education required of the manager, with higher requirements implying a higher K. Thus, firms that need very specialized knowledge may face a higher K. Along these lines, the I project may also be associated with a higher K than the E project.

Firm size could also influence K. The empirical evidence on this is mixed. Dube, Freeman, and Reich (2010) document that employee replacement costs are higher for larger firms in California. However, Blatter, Muehlemann, Schenker, and Wolter (2015) provide evidence of substantial and increasing marginal hiring costs for Swiss workers that can be reduced through internal training of unskilled workers, which suggests economies of scale that favor large firms. Regardless of these differences, it appears that these costs are substantial and are higher when labor markets are less fluid (e.g. Amberger and Eeckhout (2017)).

Contrary to popular belief, having high labor market fluidity does not necessarily encourage innovation. Surprisingly, innovation occurs at both ends of the labor market fluidity continuum, with novel innovation occurring when labor market fluidity is the lowest and incremental innovation occurring when labor market fluidity is the highest.

6 Conclusion

This paper has developed a theory of the firm's choice between developing novel innovation, incrementally innovating, and exploiting existing technologies in a setting with learning about managerial ability, moral hazard, and project quality uncertainty with innovation. Firms solve for optimal compensation contracts and managers extract rents in equilibrium because they must be incentivized to work hard to find good projects and also to not propose bad ones. The firm's choice of project type is driven both by the first-best (no agency costs) values of projects as well as the rents that must be surrendered to managers with the different project choices, given optimal contracts. In equilibrium, labor market fluidity affects the firm's choice. Novel innovation thrives when the labor market has low fluidity, incremental innovation thrives when the labor market has high fluidity, and exploitation occurs in the middle.

These results imply that firms that find it easier to tap the labor market to find replacement managers at a relatively low or intermediate cost—such as large firms—are more likely to engage in incremental innovation and exploitation projects. It is the small firms—those that encounter high costs of replacing managers—that engage in novel innovation.

References

[1] Acemoglu, Daron. "Training and innovation in an imperfect labour market." *The Review of Economic Studies* 64, no. 3 (1997): 445-464.

[2] Acemoglu, Daron. "When does labor scarcity encourage innovation?." *Journal of Political Economy* 118, no. 6 (2010): 1037-1078.

[3] Acharya, Viral V., Ramin P. Baghai, and Krishnamurthy V. Subramanian. "Labor laws and innovation." *The Journal of Law and Economics* 56, no. 4 (2013): 997-1037.

[4] Aghion, Philippe. "Schumpeterian growth theory and the dynamics of income inequality." *Econometrica* 70, no. 3 (2002): 855-882.

[5] Aghion, Philippe, Christopher Harris, Peter Howitt, and John Vickers. "Competition, imitation and growth with step-by-step innovation." *The Review of Economic Studies* 68, no. 3 (2001): 467-492.

[6] Aghion, Philippe, and Jean Tirole. "The management of innovation." *The Quarterly Journal of Economics* (1994): 1185-1209.

[7] Amberger, Korie, and Jan Eeckhout. "Labor Market Liquidity." (2017).

[8] Anderson, Ronald W., M. Cecilia Bustamante, Stéphane Guibaud, and Mihail Zervos. "Agency, firm growth, and managerial turnover." *The Journal of Finance* 73, no. 1 (2018): 419-464.

[9] Arrow, Kenneth J. "Classificatory notes on the production and transmission of technological knowledge." *The American Economic Review* 59, no. 2 (1969): 29-35.

[10] Barnett, Michael, William Brock, Lars Peter Hansen, Ruimeng Hu, and Joseph Huang. "A Deep Learning Analysis of Climate Change, Innovation, and Uncertainty." arXiv preprint arXiv:2310.13200 (2023).

[11] Benoit, Jean-Pierre. "Innovation and Imitation in a Duopoly." *The Review of Economic Studies* 52, no. 1 (1985): 99-106.

[12] Benhabib, Jess, Jesse Perla, and Christopher Tonetti. "Catch-up and fall-back through innovation and imitation." *Journal of Economic Growth* 19 (2014): 1-35.

[13] Bhattacharya, Sudipto, Kalyan Chatterjee, and Larry Samuelson. "Sequential research and the adoption of innovations." *Oxford Economic Papers* 38 (1986): 219-243.

[14] Blatter, Marc, Samuel Muehlemann, Samuel Schenker, and Stefan C. Wolter. "Hiring costs for skilled workers and the supply of firm-provided training." *Oxford Economic Papers* 68, no. 1 (2015): 238-257.

[15] Chen, Yuchen, Xuelin Li, Richard T. Thakor, and Colin Ward. "Appropriated growth." Available at SSRN 4244951 (2023).

[16] Dewar, Robert D., and Jane E. Dutton. "The adoption of radical and incremental innovations: An empirical analysis." *Management Science* 32, no. 11 (1986): 1422-1433.

[17] Dube, Arindrajit, Eric Freeman, and Michael Reich. "Employee replacement costs." (2010).

[18] Ferro, Shane, "A Major Long-term Change in the Labor Market is Bad News for the U.S. Economy", *Business Insider*, November 22, 2014.

[19] Frankel, Alexander P., Joshua L. Krieger, Danielle Li, and Dimitris Papanikolaou. Evaluation and Learning in R&D Investment. No. w31290. National Bureau of Economic Research, 2023.

[20] Gomes, Renato, Daniel Gottlieb, and Lucas Maestri. "Experimentation and project selection: Screening and learning." *Games and Economic Behavior* 96 (2016): 145-169.

[21] Grabowski, Henry, and John Vernon. "A new look at the returns and risks to pharmaceutical R&D." *Management Science* 36, no. 7 (1990): 804-821.

[22] Grabowski, Henry, John Vernon, and Joseph A. DiMasi. "Returns on research and development for 1990s new drug introductions." *Pharmacoeconomics* 20, no. 3 (2002): 11-29.

[23] Hamel, Gary. "The why, what, and how of management innovation." *Harvard business review* 84, no. 2 (2006): 72.

[24] Harris, Milton, and Bengt Holmstrom. "A theory of wage dynamics." *The Review of Economic Studies* 49, no. 3 (1982): 315-333.

[25] Hellmann, Thomas. "When do employees become entrepreneurs?." *Management science* 53, no. 6 (2007): 919-933.

[26] Hellmann, Thomas, and Veikko Thiele. "Incentives and innovation: A multitasking approach." *American Economic Journal: Microeconomics* 3, no. 1 (2011): 78-128.

[27] Holmstrom, Bengt. "Agency costs and innovation." *Journal of Economic Behavior & Organization* 12, no. 3 (1989): 305-327.

[28] Holmstrom, Bengt, and Paul Milgrom. "Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design." *JL Econ. & Org.* 7 (1991): 24.

[29] Hyatt, Henry R., and James R. Spletzer. "The recent decline in employment dynamics." *IZA Journal of Labor Economics* 2, no. 1 (2013): 5.

[30] Kerr, William R., and Ramana Nanda. "Financing Innovation." *Annual Review of Financial Economics* 7, no. 1 (2015).

[31] Krieger, Joshua L. "Trials and terminations: Learning from competitors' R&D failures." *Management Science* 67, no. 9 (2021): 5525-5548.

[32] Krieger, Joshua, Danielle Li, and Dimitris Papanikolaou. "Missing novelty in drug development." *The Review of Financial Studies* 35, no. 2 (2022): 636-679.

[33] Krieger, Joshua L., Xuelin Li, and Richard T. Thakor. "Find and replace: R&D investment following the erosion of existing products." *Management Science* 68, no. 9 (2022): 6552-6571.

[34] Lo, Andrew W., and Richard T. Thakor. "Financing biomedical innovation." *Annual Review of Financial Economics* 14 (2022): 231-270.

[35] Manso, Gustavo. "Motivating innovation." *The Journal of Finance* 66, no. 5 (2011): 1823-1860.

[36] March, James G. "Exploration and exploitation in organizational learning." *Organization science* 2, no. 1 (1991): 71-87.

[37] Molloy, Raven, Riccardo Trezzi, Christopher L. Smith, and Abigail Wozniak. "Understanding declining fluidity in the US labor market." *Brookings Papers on Economic Activity* 2016, no. 1 (2016): 183-259.

[38] Moscarini, Giuseppe, and Lones Smith. "The optimal level of experimentation." *Econometrica* 69, no. 6 (2001): 1629-1644.

[39] Roberts, Kevin, and Martin L. Weitzman. "Funding criteria for research, development, and exploration projects." *Econometrica: Journal of the Econometric Society* (1981): 1261-1288.

[40] Schumpeter, Joseph A. "Change and the Entrepreneur." *Essays of JA Schumpeter* (1934).

[41] Shimer, Robert. "The impact of young workers on the aggregate labor market." *The Quarterly Journal of Economics* 116, no. 3 (2001): 969-1007.

[42] Stiglitz, Joseph E., and Andrew Weiss. "Incentive effects of terminations: Applications to the credit and labor markets." *The American Economic Review* 73, no. 5 (1983): 912-927.

Appendix

A.1 Parametric Restrictions

(Restriction 1)

$$r\overline{R}_I^h - R > r\overline{\overline{R}}_I^h - R > 0 \tag{R.1}$$

where

$$\overline{R}_I^h \equiv pq_1^h R_I + \left[1 - pq_1^h\right] R$$

$$\overline{\overline{R}}_I^h \equiv pq_1^h \left[R_I - R_C\right] + R$$

(Restriction 2)

$$\psi > \max\left\{pA_1A_2,\ A_3^{-1}pq_0\left[r\overline{R}_I^h - R\right]\right\} \tag{R.2}$$

where

$$A_1 \equiv p\left\{q_0pq_1\left[R_C - R\right] + [1-q_0]\, p\left\{qR_C - q_0R\right\}\right\}$$

$$A_2 \equiv \frac{p[q-b]}{b\left[pq_0 + [1-p]\right]}$$

$$A_3 \equiv \frac{prq_0 + [1-p]b}{p\left[rq_0 - b\right]} - \frac{[1-p]b + pq_0}{p\left[q_0 - b\right]} + \frac{bpq_0[1-r]}{p\left[q_1^h - b\right]}$$

(Restriction 3)

$$r\left[B_1 + B_2\right] - B_3 < q_0B_2 + B_4 \tag{R.3}$$

where

$$B_1 \equiv \frac{\psi}{p\left[rq_0 - b\right]} - \frac{\psi\left[p\underline{q}_1^l + [1-p]b\right]}{p\left[\underline{q}_1^l - b\right]}$$

$$B_2 \equiv pq_1^h W_2^h(R) + [1-p]W_2^h(R)$$

$$B_3 \equiv \frac{\psi}{p\left[q_0 - b\right]} - \frac{\psi\left[pq + [1-p]b\right]}{p\left[q - b\right]}$$

$$B_4 \equiv \frac{\psi\left[pq + [1-p]b\right]}{p\left[q - b\right]} - \frac{\psi\left[p\underline{q}_1^l + [1-p]b\right]}{\left[\underline{q}_1^l - b\right]p}$$

A.2 Proofs

Proof of Lemma 1: As argued in the text, it is optimal to set $W_2^l(R) = 0$. Next, note that (9) and (11) will be binding in equilibrium. Solving these simultaneously yield (13) and (15). With this solution, (12) is clearly satisfied. Finally, since (9) holds and $W_2^n(R) > 0$, it follows that (12) is slack, which means the manager earns a rent equal to $W_2^n(R)$. It is clear that the manager will be retained since $\theta_1^h > \theta_0$ and $W_2^n(R) < W_2^n(n)$. ■

Proof of Lemma 2: Since not investing at $t = 0$ leads to beliefs about managerial ability remaining unchanged at $t = 1$, the contract in Lemma 2 is identical to that in Lemma 1 with q_1^h replaced by q_0. This yields (16)–(18). It is obvious the manager will not be fired since he is identical to a new replacement, and firing him costs the firm K. ■

Proof of Lemma 3: The structure of optimal contracts follows from the earlier proofs, with the posterior probability of success with E in the second period with a failed manager being $q_1^l(E) = q$. The two things that need to be verified are that: (i) the firm will get a positive NPV from its second-period project; and (ii) the manager will be fired if K is low and not otherwise.

Consider (i). Given the rent to be paid to a failed manager under the optimal contract, the project NPV is (recalling that $q_1^l(E) = q$):

$$qR - \psi - \frac{b\psi}{p\delta[q-b]} > 1 \tag{A.1}$$

which is positive given (6). Now consider (ii). It will be optimal to fire the manager if the gain in value from hiring a new manager exceeds K (recalling $\theta_1^l = 0$):

$$[\overline{q}_0 - q]\, R_S + \frac{b\psi}{p\delta}\left\{[q-b]^{-1} - [q_0 - b]^{-1}\right\} > K \tag{A.2}$$

This will hold when K is small and not otherwise. ■

Proof of Proposition 1: $W_1^l = 0$ follows from earlier arguments. (25) and (27) are obtained by solving (22) and (23) as simultaneous equations because both constraints are binding at the optimum. ■

Proof of Proposition 2: $\hat{W}_1^l = 0$ follows from earlier arguments. $\hat{W}_1^h$ and $\hat{W}_1^n$ can be obtained by recognizing that (28) and (29) are binding constraints and solving them as simultaneous equations. ■

Proof of Lemma 4: Similar to the proofs of Lemmas 1, 2, and 3. ■

Proof of Proposition 3: Similar to the proof of Proposition 1. ■

Proof of Lemma 5: Suppose $\delta = 1$, and assume that the manager is paid $\{W_L^h, W_L^l, W_L^n\}$ according to the contract described in Proposition 2 at $t = 2$ instead of $t = 1$. Given that $\hat{W}_L^l = 0$, one possibility for the firm is to implement the deferral scheme by paying the manager $W_2^h(x)+W_1^x$ if the second-period project succeeds, $W_2^l(x)+W_1^x$ if the second-period project fails, and $W_2^n(x)+W_1^x$ if the manager did not request funding for the second-period project, where $x \in \{h, n\}$ on the first-period project. It is clear that doing this will have no effect on the manager's incentives with respect to either the first-period or the second-period project. Of course, to maximize the effectiveness of incentives, we know that the manager should be paid 0 at $t = 2$ if the second-period project fails. To achieve this, the deferral can pay the manager $W_2^h(x) + [W_1^x/q_1^x]$ if the second-period project succeeds and $W_2^n(x) + [W_1^x/q_1^x]$ if no funding was requested for the second-period project, where $x \in \{h, n\}$. However, we derived the cheapest way to incentivize the manager to choose $e = 1$ and propose only the good project in the second period when we solved for the subgame-perfect second-period contract. So we cannot improve on second-period incentives by paying the manager more. Further, the manager's incentives on the first-period contract also cannot be improved by this deferral since beliefs follow a martingale and the manager's choices on L at $t = 0$ do not affect the success probability of S chosen at $t = 1$.

Thus, with $\delta = 1$, wage deferral cannot improve on the outcome with the wage paid at $t = 1$. This means that with $\delta < 1$, wage deferral leads to a strictly higher expected wage cost (with no improvement in incentives). ■

Proof of Lemma 6: Let us first compare C and E. In both cases, observing $y_1 = 0$ leads to $\theta_1^l = 0$. Thus, the posterior belief about the manager's ability is identical in both cases. In the case of C, continuing with the manager for the second period leads to a second-period project payoff conditional on success of $R_C > R$, whereas the second-period project payoff conditional on success with E is R. Thus, the benefit of continuing with the first-period manager is greater with C than with E. This means $K^*(C) < K^*(E)$.

Now compare I with C. With I, observing $y_1 = 0$ leads to $\underline{\theta}_1^l > 0$ (see (42)). Moreover, the second-period expected payoff conditional on success with I, rR_I, exceeds R_C, the second-period payoff conditional on success with C. This implies a higher value of continuing with the same manager with I than with C. Thus, $K^*(I) < K^*(C)$. ■

Proof of Lemma 7: The structure of the contracts follows the logic of the earlier proofs, with two key differences. One is that the posterior belief $\underline{q}_1^x(i)$ is formed based on the project chosen at $t = 0$, its outcome at $t = 1$, and the new project searched for at $t = 1$. The other is that the posterior belief after investing in I at $t = 0$ also reflects uncertainty about project quality. ■

Proof of Lemma 8: The optimal contract for I mirrors that for E in Proposition 1, except that the prior probability of first-period project success is rq_0 instead of q_0, reflecting the additional project quality uncertainty with I. ■

Proof of Lemma 9: (1) Once the manager is fired, the value of a second-period I project is the same as the value of second-period E project, conditional on the I project being H. However, if the manager is fired, it is because of $y_1 = 0$. In both cases, the posterior probability that I is H is less than r, the prior probability. Since $rR < R$, E strictly dominates I at $t = 1$, and C and E are equivalent. Thus, a firm that invested in E or C will continue with that project in the second period.

(2) If $y_1 = R$ at $t = 1$, then the firm does not fire the manager and the posterior belief about project quality r_1^h (given by (46)), where $r_1^h > r$. Using (5), it follows that the expected value of I conditional on success, $r_1^h R_C$, in the second period exceeds the expected value of either C or E conditional on success. Hence, the firm does not switch from I to C or E. If no investment is made

at $t = 0$, then nothing is learned about managerial talent, so he is retained at $t = 1$, but then the second period payoff conditional on success is rR for I and R for E, so the firm switches to E in the second period. ■

Proof of Proposition 4: For the purposes of the proof, let us define the option value of I relative to E, which is the expected value of the second-period option to invest in I minus the expected loss on the first-period investment in I, as :

$$\triangle V_I^E = prq_0 \left[pq_1^h \{R_I - R\} \right] - pq_0 R[1 - r] \tag{A.3}$$

Clearly, $\partial \triangle V_I^E / \partial r > 0$, so it will be assumed that r is large enough to ensure $\triangle V_I^E > 0$. Defining:

$$\overline{R}_I^h \equiv pq_1^h R_I + \left[1 - pq_1^h \right] R \tag{A.4}$$

we have

$$\triangle V_I^E = pq_0 \left\{ r\overline{R}_I^h - R \right\} \tag{A.5}$$

Thus, (R.1), which guarantees that $r\overline{R}_I^h > R$, ensures that $\triangle V_I^E > 0$.

Similarly, the net option value of I over C is:

$$\begin{aligned} \triangle V_I^C &= prq_0 \left[pq_1^h \{R_I - R_C\} \right] - pq_0 R[1 - r] \\ &= pq_0 \left[r\overline{\overline{R}}_I^h - R \right] \end{aligned} \tag{A.6}$$

where

$$\overline{\overline{R}}_I^h = pq_1^h \{R_I - R_C\} + R \tag{A.7}$$

(R.1) guarantees that $r\overline{\overline{R}}_I^h > R$, so $\triangle V_I^C > 0$. The option value of C relative to E is (assuming the manager is fired after $y_1 = 0$):

$$\triangle V_C^E = pq_0 \left[pq_1^h \{R_C - R\} \right] \tag{A.8}$$

It is clear that $\triangle V_C^E > 0$.

As mentioned earlier in the text, throughout the proof it will be assumed that agency costs principally determine the firm's choice of project type. Thus, the assumed parametric restrictions are such that the option values above are small relative to agency costs, and the option values determine project choice only when agency costs are (roughly) equal. The different cases are now considered.

(1) $K < K^*(C)$:

In this case, the manager is fired following $y_1 = 0$ for all projects; I, C, and E. The second-period contracts are all the same in terms of agency costs since the firm switches to E if $y_1 = 0$ with I in the first period (see (4)), and stays with C in the second period if it invested in C in the first period. The reason why staying with C is optimal is that following $y_1 = R$ in the first period, it is optimal to retain the manager ($q_1^h > q_0$) and following $y_1 = 0$, the manager is fired but the second-period expected payoff on C is the same as that on E. Given that the second-period contracts are identical in terms of agency costs, we can compare just the first-period contracts.

Now the first-period expected wage cost with I when the manager is fired after $y_1 = 0$:

$$\mathbb{E}\left[W_I^f\right] = prq_0\underline{W}_1^h + [1-p]\underline{W}_1^n \tag{A.9}$$

where

$$\begin{aligned}\underline{W}_1^h &= \frac{\psi}{p\left[rq_0 - b\right]} - \delta\underline{W}_2^n(R)\\ &= \frac{\psi}{p}\left\{\frac{1}{rq_0 - b} - \frac{b}{q_1^h - b}\right\}\end{aligned} \tag{A.10}$$

$$\underline{W}_1^n = \frac{b\psi}{p}\left\{\frac{1}{rq_0 - b} - \frac{b}{q_0 - b}\right\} \tag{A.11}$$

The expected first-period wage cost with C when the manager is fired after $y_1 = 0$:

$$\mathbb{E}\left[W_C^f\right] = pq_0\underline{W}_1^h + [1-p]\underline{W}_1^n \tag{A.12}$$

where

$$\underline{W}_1^h = \frac{\psi}{p\left[q_0 - b\right]} - \frac{b\psi}{p\left[\overline{q}_1^h - b\right]} \tag{A.13}$$

$$\underline{W}_1^n = \frac{b\psi}{p\left[q_0 - b\right]} - \frac{b\psi}{p\left[q_0 - b\right]} = 0 \tag{A.14}$$

Thus,

$$\mathbb{E}\left[W_C^f\right] = pq_0\underline{W}_1^h \tag{A.15}$$

It will now be shown that $\mathbb{E}\left[W_I^f\right] > \mathbb{E}\left[W_C^f\right]$.

$$\begin{aligned}\mathbb{E}\left[W_I^f\right] - \mathbb{E}\left[W_C^f\right] =& prq_0\left\{\frac{\psi}{p\left[rq_0 - b\right]} - \frac{b\psi}{p\left[\overline{q}_1^h - b\right]}\right\} + [1-p]\left\{\frac{b\psi}{p\left[rq_0 - b\right]} - \frac{b\psi}{p\left[q_0 - b\right]}\right\} \\ & - pq_0\left\{\frac{\psi}{p\left[q_0 - b\right]} - \frac{b\psi}{p\left[\overline{q}_1^h - b\right]}\right\} \\ =& \frac{\psi}{p\left[rq_0 - b\right]}\left\{prq_0 + [1-p]b\right\} + \frac{b\psi pq_0[1-r]}{p\left[\overline{q}_1^h - b\right]} - \frac{\psi\left[[1-p]b + pq_0\right]}{p\left[q_0 - b\right]}\end{aligned} \tag{A.16}$$

Now note that

$$\frac{\partial\left\{\frac{[1-p]b + pq_0 r}{p[rq_0 - b]}\right\}}{\partial r} < 0 \tag{A.17}$$

This means that

$$\frac{[1-p]b + pq_0}{p\left[q_0 - b\right]} < \frac{[1-p]b + pq_0 r}{p\left[rq_0 - b\right]} \tag{A.18}$$

which proves that

$$\mathbb{E}\left[W_I^f\right] - \mathbb{E}\left[W_C^f\right] > 0 \tag{A.19}$$

Now for C to be preferred to I, we need

$$\mathbb{E}\left[W_I^f\right] - \mathbb{E}\left[W_C^f\right] > pq_0\left[r\overline{\overline{R}}_I^h - R\right] \tag{A.20}$$

which is guaranteed by (R.2).

It is obvious that the first-period contracting costs are identical with C and E. Thus, the option value of C, $\triangle V_C^E$, meams that C is chosen in both periods.

(2) $K \in [K^*(C), K^*(E)]$:

It will first be shown that E dominates C and then that E dominates I. First note that since the manager is retained with C even after $y_1 = 0$, the option value of C relative to E:

$$\triangle\hat{V}_C^E = p\left\{q_0\left[pq_1^h\left[R_C - R\right]\right] + \left[1 - q_0\right]p\left\{q_1^l R_C - q_0 R\right\}\right\} < \triangle V_C^E \tag{A.21}$$

since $q_1^l R_C < q_0 R$.

Now the first-period contract with C and no firing after $y_1 = 0$ is (using Proposition 2):

$$\overline{W}_{f1}^h = \frac{\psi}{p\left[q_0 - b\right]} - \delta W_2^n(R) + \delta W_2^n(0) \tag{A.22}$$

$$\overline{W}_{f1}^n = \frac{b\psi}{p\left[q_0 - b\right]} - \delta W_2^n(R) + \delta W_2^n(0) \tag{A.23}$$

where

$$W_2^n(0) = \frac{b\psi}{p\left[q_1^l(E) - b\right]\delta} \tag{A.24}$$

$$W_2^n(n) = \frac{b\psi}{p\left[q_0 - b\right]\delta} \tag{A.25}$$

$$W_2^n(R) = \frac{b\psi}{p\left[q_1^h(E) - b\right]\delta} \tag{A.26}$$

$$q_1^h(E) = \theta_1^h + \left[1 - \theta_1^h\right]q \tag{A.27}$$

$$q_1^l(E) = q \tag{A.28}$$

Thus,

$$\begin{aligned}\overline{W}_{f1}^h &= \frac{\psi}{p\left[q_0 - b\right]} - \frac{b\psi}{p\left[q_1^h(E) - b\right]} + \frac{b\psi}{p\left[q_1^l(E) - b\right]} \\ &= \frac{\psi}{p\left[q_0 - b\right]} + \frac{b\psi\left[q_1^h - q\right]}{p\left[q_1^h - q\right]\left[q - b\right]}\end{aligned} \tag{A.29}$$

$$\begin{aligned}\overline{W}_{f1}^n &= \frac{b\psi}{p\left[q_0 - b\right]} - \frac{b\psi}{p\left[q_0 - b\right]} + \frac{b\psi}{p\left[q - b\right]} \\ &= \frac{b\psi}{p\left[q - b\right]}\end{aligned} \tag{A.30}$$

Thus, expected first-period contracting costs with C are:

$$\begin{aligned}\mathbb{E}\left[W_C^{nf}\right] &= pq_0\overline{W}_{f1}^h + [1-p]\overline{W}_{f1}^n \\ &= pq_0\left\{\frac{\psi}{p\left[q_0-b\right]} + \frac{b\psi\left[q_1^h-b\right]}{p\left[q_1^h-b\right]\left[q-b\right]} + \frac{[1-p]b\psi}{p[q-b]}\right\}\end{aligned} \tag{A.31}$$

Expected first-period contracting costs with E (with firing after $y_1 = 0$) are:

$$\mathbb{E}\left[W_E^{nf}\right] = pq_0W_1^h + [1-p]W_1^n \tag{A.32}$$

where

$$W_1^h = \frac{\psi}{p\left[q_0-b\right]} - \delta W_2^n(R) \tag{A.33}$$

$$W_1^n = \frac{b\psi}{p\left[q_0-b\right]} - \delta W_2^n(n) \tag{A.34}$$

Using (A.31) and (A.32), we see that

$$\begin{aligned}\mathbb{E}\left[W_C^{nf}\right] - \mathbb{E}\left[W_E^f\right] &= pq_0\delta W_2^n(0) + [1-p]\delta W_2^n(0) \\ &= \frac{pq_0b\psi}{p\left[q-b\right]} + \frac{[1-p]b\psi}{p\left[q-b\right]} \\ &= \frac{b\psi\left\{pq_0+[1-p]\right\}}{p\left[q-b\right]}\end{aligned} \tag{A.35}$$

For C to dominate E we need:

$$\mathbb{E}\left[W_C^{nf}\right] - \mathbb{E}\left[W_E^f\right] > \triangle\hat{V}_C^E \tag{A.36}$$

which is guaranteed by (R.2).

Next it will be shown that E dominates I. Assume that the manager will be fired following $y_1 = 0$; we will verify later that the proof also holds when the manager is never fired with I. In this case, we saw earlier that the expected wage contracting costs is given by (A.9). Since $\mathbb{E}\left[W_E^f\right]$ is the same as $\mathbb{E}\left[W_C^f\right]$ examined in the previous case, we have already proven that

$$\mathbb{E}\left[W_I^f\right] - \mathbb{E}\left[W_E^f\right] > 0 \tag{A.37}$$

For E to dominate I, we need

$$\mathbb{E}\left[W_I^f\right] - \mathbb{E}\left[W_E^f\right] > pq_0\left[r\overline{R}_I^h - R\right] \tag{A.38}$$

which is guaranteed by (R.2).

Thus, the firm chooses E in both periods.

(3) $K > K^*(E)$:

Now the manager is not fired after $y_1 = 0$ with any project. We can no longer rely on comparing only the first-period contracting costs, and must compare expected contracting costs over two periods. With no firing, the expected two-period contracting costs over two periods with I are:

$$\begin{aligned}\mathbb{E}\left[\overline{W}_I^{nf}\right] =& prq_0\left[\underline{W}_{f1}^h + p\underline{q}_1^h(I)W_2^h(R) + [1-p]W_2^n(R)\right]\\ &+ p\left[1-rq_0\right]\left[p\underline{q}_1^l(I)W_2^h(0) + [1-p]W_2^n(0)\right]\\ &+ [1-p]\left[\underline{W}_{f1}^n + pq_0\underline{W}_2^h(n) + [1-p]\underline{W}_2^n(n)\right]\end{aligned} \tag{A.39}$$

where

$$\underline{W}_{f1}^h = \frac{\psi}{p\left[rq_0 - b\right]} - \delta W_2^n(R) + \delta W_2^n(0) \tag{A.40}$$

$$\underline{W}_{f1}^n = \frac{b\psi}{p\left[rq_0 - b\right]} - \delta W_2^n(R) + \delta W_2^n(0) \tag{A.41}$$

$$W_2^h(R) = \frac{\psi}{p\delta\left[q_1^h - b\right]} \tag{A.42}$$

$$W_2^h(0) = \frac{\psi}{p\delta\left[q_1^l(I) - b\right]} \tag{A.43}$$

$$W_2^n(R) = \frac{b\psi}{p\delta\left[q_1^h - b\right]} \tag{A.44}$$

$\theta_1^l = \Pr\left(\tau = T \mid y_1 = 0\right) = 0$, so

$$q_1^l = q \tag{A.45}$$

$$W_2^n(0) = \frac{b\psi}{p\delta\left[q_1^l - b\right]} \tag{A.46}$$

Making the appropriate substitutions, to prove that $\mathbb{E}\left[\overline{W}_I^{nf}\right] < \mathbb{E}\left[\overline{W}_C^{nf}\right]$ (expected contracting costs over two periods with C and no firing), we need to show that

$$
\begin{aligned}
prq_0 & \left[\frac{\psi}{p\left[rq_0-b\right]}-\frac{b\psi}{p\left[q_1^h-b\right]}+\frac{b\psi}{p\left[q-b\right]}+pq_1^hW_2^h(R)+[1-p]W_2^n(R)\right] \\
& \quad +p\left[1-rq_0\right]\left[pq\frac{\psi}{p[q-b]}+\frac{[1-p]b\psi}{p[q-b]}\right] \\
< pq_0 & \left[\frac{\psi}{p\left[q_0-b\right]}-\frac{b\psi}{p\left[q_1^h-b\right]}+\frac{b\psi}{p\left[q-b\right]}+pq_1^hW_2^h(R)+[1-p]W_2^n(R)\right] \\
& \quad +p\left[1-q_0\right]\left[\frac{pq\psi}{p[q-b]}+\frac{[1-p]b\psi}{p[q-b]}\right]
\end{aligned} \tag{A.47}
$$

With simplification, this inequality becomes:

$$
\begin{aligned}
r & \left[\frac{\psi}{p\left[rq_0-b\right]}-\frac{\psi\left[p\underline{q}_1^l+[1-p]b\right]}{p\left[\underline{q}_1^l-b\right]}+pq_1^hW_2^h(R)+[1-p]W_2^h(R)\right] \\
& \quad -\left[\frac{\psi}{p\left[q_0-b\right]}-\frac{\psi[pq+[1-p]b]}{p[q-b]}\right] \\
& < q_0\left[pq_1^hW_2^h(R)+[1-p]W_2^h(R)\right]+\frac{\psi\left[pq+[1-p]b\right]}{p\left[q-b\right]} \\
& \quad -\frac{\psi\left[p\underline{q}_1^l+[1-p]b\right]}{p\left[\underline{q}_1^l-b\right]}
\end{aligned} \tag{A.48}
$$

which holds given (R.3). Thus, expected contracting costs are lower with I than with C, and by implication I also has lower contracting costs than E. Given the positive option value of I relative to E and C, it follows that I is preferred to both E and C.

Thus far we have assumed that the manager is fired with E following $y_1 = 0$. It will now be shown that if $K^*(I) \in (K^*(C), K^*(E))$ so that the manager is fired after $y_1 = 0$ with E but not with I in this range, then E still is the preferred choice when $K \in (K^*(C), K^*(E))$. Note first that not firing the manager following $y_1 = 0$ leads to higher second-period contracting costs for I relative to E. So we can once again focus only on first-period wage contracting costs. To show that expected wage contracting costs are higher with I than with E, we need to show that

$$\begin{aligned} &pq_0 r\left\{\frac{\psi}{p\left[rq_0-b\right]}-\frac{\psi}{p\left[q_1^h-b\right]}+\frac{b\psi}{p\left[\underline{q}_1^l(I)-b\right]}\right\} \\ &+[1-p]\left\{\frac{b\psi}{p\left[rq_0-b\right]}-\frac{b\psi}{\left[rq_0-b\right]}+\frac{b\psi}{p\left[\underline{q}_1^l(I)-b\right]}\right\} \\ >\ &pq_0\left\{\frac{\psi}{p\left[q_0-b\right]}-\delta W_2^n(R)\right\}+[1-p]\left\{\frac{b\psi}{p\left[q_0-b\right]}-\delta W_2^n(n)\right\} \end{aligned} \tag{A.49}$$

This inequality holds since

$$\frac{pq_0 r\psi}{p\left[rq_0-b\right]}>\frac{pq_0\psi}{p\left[q_0-b\right]} \tag{A.50}$$

This completes the proof. ■