Earnings Management and Earnings Quality: Theory and Evidence

Preliminary and Incomplete

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1 Introduction

Earnings management and earnings quality are central to both theoretical and empirical research in accounting. We study a dynamic model of earnings management in which every period a forward looking manager observes the firm's earnings and then issues a report to the market. When issuing a report, the manager is not confined to truthful reporting, however, manipulating the report, which affects both current and future stock prices, is costly to the manager. We study a setting in which the manager's tenure with the firm is finite as well as the steady-state of an infinite horizon setting. In our model, investors are not perfectly informed about the manager's incentives, resulting in an equilibrium which is characterized by ex-post information asymmetry between the manager and the capital market. This feature of the model allows us to break down investors' uncertainty about firm value into two distinct components: (i) economic uncertainty about firm value which is shared by the manager; and (ii) information asymmetry between the manager and investors due to earnings management (reporting quality). We believe that distinguishing between these two components of investors' uncertainty, fundamental uncertainty and reporting quality, is key to understanding the notion of earnings quality and interpreting existing measures of earnings quality. In addition to studying these two components of investors' uncertainty analytically, we structurally estimate our model to empirically parse out the portion of investors' uncertainty due to earnings management (reporting quality) from the one resulting from economic earnings process. This allows us to address existing concerns about archival studies on earnings quality such as the concerns raised by Dechow et al. (2010) and Dichev et al. (2013) that "archival research cannot satisfactorily parse out the portion of managed earnings from the one resulting from fundamental earnings process" (Dichev et al. (2013) p. 1).

The extant theoretical literature on earnings management focuses on two modeling approaches. The first approach studies single period settings in which there are no intertemporal consideration (e.g., Stein (1989), Fischer and Verrecchia (2000) and Guttman et al. (2006)). The second approach studies two period settings and assumes that once the manager determines the bias in his report of the first period, he has no discretion left when issuing the second period's report. In particular, it assumes that the first period's bias fully reverses in the second period (e.g., Sankar and Subramanyam (2001), Kirschenheiter and Melumad

(2002), Ewert and Wagenhofer (2011)).[1] We extend this strand of the literature by studying, what we believe to be, a more realistic setting. In particular, we study a multi-period setting which can accommodate any mechanical reversal of discretionary accruals in subsequent periods. Specifically, we model the manager's cost of biasing the report in a given period as a function of the cumulative bias in reported earnings over the years. In other words, the manager's cost of biasing his report is increasing in the magnitude of the bias in reported equity (Zakolyukina (2012)).

As mentioned earlier, we assume that investors are imperfectly informed about the manager's incentives. In particular, we assume that in each period the manager's cost of manipulating reported firm's equity is not only a function of the magnitude of the bias, but also depends on an additional factor that the manager observes privately and which we refer to as "reporting noise." This factor reflects idiosyncratic circumstances in each period that affect the manager's misreporting costs (Dye and Sridhar (2008)).[2] Finally, as common in this literature, we assume that the manager's incentives are tied to the firm's stock price. When choosing the optimal report in a given period, the manager considers the trade-off between manipulation costs and stock prices. Given the multi period setting, when choosing the report, the manager considers not only the current manipulation costs and stock prices, but also the effect of the current report on the expected future prices and expected future manipulation costs.

The first setting that we study, in which the manager has a finite horizon, generates predictions about the effect of the manager's tenure with the firm on time-series properties of managerial reports and book-to-market ratios. On the one hand, as the manager's tenure with the firm lengthens, investors' uncertainty increases since the manager's report does not fully reveal his private information. Higher uncertainty leads to investors relying more on the manager's report in pricing the firm, and, consequently, to stronger incentives for the manager to bias his report. On the other hand, as the remaining tenure of the manager with the firm shortens, the manager's incentive to bias his report may weaken if the serial correlation of earnings is low. Therefore, the manager's reporting strategy and the resulting

[1]The above mentioned literature focuses on the valuation role of financial reporting in which the manager's compensation is assumed exogenously. A different stream of literature takes a principal-agent approach and focuses on the stewardship role of financial reporting in which the manager's compensation is endogenously set by the principal (see for example, Beyer et al. (2012)).

[2]For a related assumption, see Fischer and Verrecchia (2000).

book-to-market ratios change over time.

The time-series properties of the reports and the book-to-market ratios depend not only on the manager's tenure with the firm, but also on the parameters of our model, namely the underlying process of the firm's economic earnings and the stochastic process of biasing costs. We allow for serial correlation of economic earnings in order to make the model more flexible. In particular, the model can accommodate the fact that different firms/industries have different characteristics and that their time-series properties of managerial reports and book-to-market ratios may qualitatively vary. For example, we find that when economic earnings are not highly serially correlated the bias in reported equity follows an inverse U-shape over the manager's tenure with the firm. That is, the discretionary accruals in reported earnings are initially positive and turn negative towards the end of the manager's tenure with the firm. The reason is that investors' uncertainty about the firm value grows over time, which leads to stronger market reactions to reports. Thus, the manager builds up a larger bias over time, up to the point where the close prospect of retirement means that the manager will no longer fully internalize the long term benefits of his manipulations. In response to this effect, the manager reverses some of the bias accumulated throughout his tenure.

If earnings are highly serially correlated the opposite, U-shape pattern, emerges. That is, the bias is high at the start, but decreases over time until the manager approaches retirement which is when he begins increasing the bias again. This increase in the bias can be explained as follows. Under high serial correlation the firm's price relies more heavily on investors' beliefs about the earnings, rather than the firm's equity. In other words, under high serial correlation the information comes from the firm's growth as opposed to its size. Reporting high growth today comes at the expense of the growth the manager will be able to report tomorrow, other things equal. However, when the manager is close to retirement he does not fully internalize this effect. As a result, the manager optimally increases the bias as he approaches retirement.

If the manager's tenure with the firm is long enough, his bias in cumulative reported earnings is very stable, except for his very early and very late periods with the firm. In order to obtain a better understanding of this intermediate, more stable period of the manager with the firm and to be able to provide more concrete predictions, we study the steady state of a setting in which the manager has an infinite horizon with the firm. In particular, we derive the steady state level of the expected bias in reported equity, the level of investors' uncertainty with respect to the firm's value, and the information asymmetry between the manager and investors resulting from the quality of financial reporting. As expected, a higher volatility in the earnings process increases the overall uncertainty of investors' and the information asymmetry they suffer. Similarly, higher volatility in the accounting system (i.e., misreporting costs) increases both the overall uncertainty and the information asymmetry investors must bear. Thus, the uncertainty investors bear in steady state is caused both by the firm's fundamentals as well as by the properties of the accounting system. The empirical challenge is then to separate the two effects.

We use the steady state model to structurally estimate unobservable parameters such as the distribution of true, unmanaged earnings and the amount of noise earnings management adds to financial reports (reporting noise). Based on the joint distribution of book value of equity and market value of equity, we estimate the following parameters of the model: the persistence of true earnings, the earnings response coefficient, the sensitivity of investors' beliefs about current period true earnings to reported earnings, the variability of earnings shocks, the uncertainty added by earnings manipulation, and the variance of economic earnings. The analysis is performed at the industry level and consequently the parameters estimated from the model are assumed to be constant across firms of the same industry and constant over time. Based on annual data, we find that the persistence of the shock to true earnings is close to 0.9 and the earnings response coefficient is around 1.5. To get a sense of the relative magnitudes of variances, we compute the ratio of the variance of noise introduced by earnings management per period, to the variance of economic earnings innovation per period, and find that, on average, it is around half, suggesting that the noise added by manipulation explains about a third of the uncertainty concerning firms' values. We also perform analysis based on quarterly data. However, the estimates of earnings persistence exceed one and hence are inconsistent with a stable equilibrium of the model.

The paper proceeds as follows. Section 2 lays out the setting of our model. In Section 3 we derive and analyze the equilibrium. We start with the equilibrium of the finite horizon setting and then derive and analyze the steady-state equilibrium of the setting in which the manager's horizon with the firm is infinite. In Section 4 we present the structural estimation of our model and its empirical analysis.

2 Setting

Consider a multi period setting in which a firm generates earnings ε_t in every period $t \in \{1, 2, ..., T\}$. To capture the fact that the earnings of the firm can be correlated over time, we assume that the earnings are determined by the following AR1 process

$$\varepsilon_t = \rho\varepsilon_{t-1} + v_t, \tag{1}$$

where $\rho \in [0, 1]$, $v_t \sim N(0, \sigma_v^2)$ and v_i is independent of v_j for any $i \neq j$. For simplicity (and almost without loss of generality) we assume that $\varepsilon_0 = 0$.

No dividends are ever distributed. Hence, the aggregate earnings of the firm θ_t, which we refer to as the firm's equity at time t, is given by

$$\theta_t = \theta_{t-1} + \varepsilon_t, \tag{2}$$

In every period, the firm's manager privately learns the earnings of the firm and issues a report of the firm's equity, r_t, to the market. The manager can manipulate the report, but he bears a personal costs from doing so. In particular, we assume that the biasing costs are

$$\frac{c\,(r_t - \theta_t - \eta_t)^2}{2},$$

where η_t (aka reporting noise) is the realization of a random variable $\tilde{\eta} \sim N\left(0, \sigma_\eta^2\right)$. The realization of η_t is privately observed by the manager, in each period, and reflects idiosyncratic circumstances that affect the manager's misreporting costs (see, Dye and Sridhar (2008)).[3]

We do not impose the restriction that the bias in reported earnings must reverse at some arbitrary point in time. Instead, the model can accommodate any mechanical reversal of discretionary accruals in subsequent periods.[4] Yet, since the cost of misreporting is convex in

[3]Under Section 404 of the SOX Act, management is required to produce an "internal control report" as part of each annual Exchange Act report. See 15 U.S.C. § 7262. The report must affirm "the responsibility of management for establishing and maintaining an adequate internal control structure and procedures for financial reporting." 15 U.S.C. § 7262(a). The report must also "contain an assessment, as of the end of the most recent fiscal year of the Company, of the effectiveness of the internal control structure and procedures of the issuer for financial reporting."

[4]The full reversal feature of the bias at the end of the manager's tenure can easily be incorporated into this model by assuming that the parameter c depends on t and letting $\lim_{t\to T} c_t = \infty$.

the cumulative bias, $r_t - \theta_t$, a manager who engaged in more aggressive earnings misreporting in the past will face stronger incentives to reverse such manipulations in the future. Observe that the manager's misreporting cost depends on the difference between reported equity r_t and the (true) value of equity θ_t, despite the fact that the firm's equity is only a fraction of the firm's value. This feature captures the notion that the manager is penalized only when he manipulates the accounting reports, which are (often) not meant to be forward looking.

In addition to the biasing costs, the manager's incentives are also tied to the firm's stock price, which we denote by p_t. In particular, the manager's payoff at each period t is assumed to be

$$p_t - \frac{c}{2}\left(r_t - \theta_t - \eta_t\right)^2$$

In each period, the manager maximizes a weighted average of his future payoffs, as in Acharya et al. (2011). In particular, the manager chooses r_t to maximize

$$U_t = \mathbb{E}\left[\sum_{k=t}^{T} \delta^{k-t}\left(p_k - \frac{c}{2}\left(r_k - \theta_k - \eta_k\right)^2\right) \middle| \theta_t, \varepsilon_t, \eta_t\right], \tag{3}$$

where $\delta \in [0,1)$ is the manager's discount factor. First, we analyze the setting in which the manager has a finite horizon with the firm. Then, we analyze the steady state model, in which the manager has been with the firm –and will remain with the firm– for sufficiently long. We structurally estimate the steady state model in Section 4

The firm's price in every period t is set by risk neutral investors based on all the publicly available information, which consists solely of the history of reports. The stock price in period t is set as investors' expectation of the firm's equity value plus the present value of future earnings. For simplicity, and without loss of generality, we assume that the interest rate used by investors in pricing future earnings is zero. The firm's price at time t as a function of prior reports is

$$p_t = \mathbb{E}_t\left[\theta_t + \sum_{k=t+1}^{\infty} \varepsilon_k\right] \tag{4}$$

which given the AR1 structure of earnings yields

$$p_t = \mathbb{E}_t\left[z_t\right],$$

where $\mathbb{E}_t$ is the investors' expectation given the history of reports, $h_t = \{r_1, r_2, ..., r_t\}$, and

$$z_t \equiv \theta_t + \frac{\rho}{1-\rho}\varepsilon_t,$$

is the firm's true economic value in period t. We can think of the stock price as being the sum of two components: an estimate of the firm's assets in place θ_t and an estimate of the firm's future earnings $\frac{\rho}{1-\rho}\varepsilon_t$. As we demonstrate below, investors will remain uncertain about both components of firm value all throughout the manager's tenure.

All the model's parameters, distributions, and the manager's payoff structure are common knowledge.

3 Equilibrium

We study a linear equilibrium in which the manager's report in each period t, r_t, is linear in both the firm's equity, θ_t, and in the cost parameter, η_t. Given that each period's report will be normally distributed, the stock price in period t takes the following linear form

$$p_t = \mu_0 + \sum_{j=1}^{t} \alpha_j^t \left(r_j - b_j\right), \tag{5}$$

for some equilibrium parameters $\left\{\alpha_j^t, b_j\right\}$. Note that α_j^t is the sensitivity of the price at time t to the report issued at time j.

We first analyze a setting in which the manager stays with the firm for a finite horizon and then we proceed to the infinite horizon steady state analysis.

3.1 Manager with Finite Horizon

In this part, we assume that the manager's tenure with the firm is finite, i.e., $T < \infty$. When deciding about the bias in the report of the firm's equity, the manager takes into account the effect of his current report on the current biasing cost and on the whole trajectory of future prices. As time goes by, both the information environment (investors' uncertainty about the firm's value) and the remaining tenure of the manager with the firm change, which leads to a reporting strategy that changes over time. We first briefly present the equilibrium reporting strategy of the manager and the market pricing function and then discus the time-series properties of managerial reporting and book-to-market ratios.

Reporting Strategy and the Market Pricing

We use backward induction to solve for the equilibrium. At the last period of the manager with the firm, $t = T$, the manager's optimization problem (3) yields

$$r_T = \theta_T + \eta_T + \frac{\alpha_T^T}{c},$$

and the manager's utility at $t = T$ is given by

$$U_T = \mu_0 + \sum_{j=1}^{T} \alpha_j^T (r_j - b_j) - \frac{c}{2} \left(\alpha_T^T\right)^2.$$

Going backwards one period, at $t = T - 1$ the manager maximizes

$$U_{T-1} = p_{T-1} - \frac{c}{2} (r_{T-1} - \theta_{T-1} - \eta_{T-1})^2 + \delta \mathbb{E} (U_T),$$

which implies an optimal report of

$$r_{T-1} = \theta_{T-1} + \eta_{T-1} + \frac{\alpha_{T-1}^{T-1} + \delta \alpha_{T-1}^T}{c}.$$

By induction, one can see that for any $t \leq T$ the manager's reporting strategy is given by

$$r_t = \theta_t + \eta_t + A_t^T, \tag{6}$$

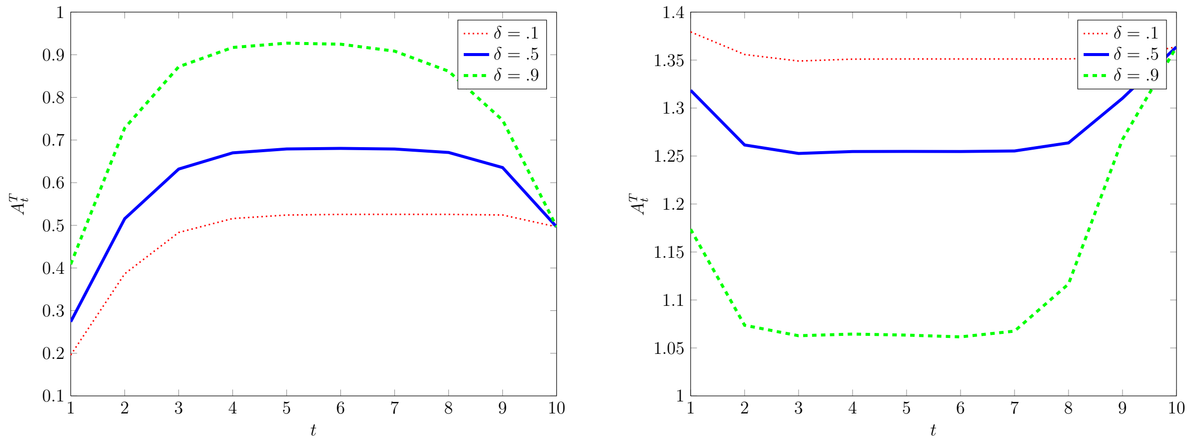
where the (cumulative) bias in reported equity, A_t^T, is

$$A_t^T = \frac{\sum_{k=0}^{T-t} \delta^k \alpha_t^{t+k}}{c}.$$

Thus far, we have not used the properties of the distribution of $\{\theta_t, \eta_t\}$ in the derivation of the reporting strategy, so the structure of such strategy is independent of the details of the distribution of $\{\theta_t, \eta_t\}$.[5]

Reported equity is distorted by two factors. First, a random factor with mean zero, η_t, reflecting the manager's earnings management incentives in period t. The presence of this component results in investors being able to perfectly infer neither the actual value of the firm's equity, θ_t, nor the current period's earnings, ε_t. Second, a deterministic component, A_t^T, that investors perfectly anticipate in equilibrium, that depends both on the manager's tenure with the firm t and his horizon $T - t$.

[5] Of course, the value of A_t^T is affected by the distribution of $\{\theta_t, \eta_t\}$.



(a) $(\sigma_v = 1, \sigma_\eta^2 = 10, \rho = .5)$ (b) $(\sigma_v = 3, \sigma_\eta = 2, \rho = .5)$

Figure 1: The evolution of bias over the manager's tenure. When σ_v and ρ are relatively large (small), the evolution of bias exhibits a U-shape (Hill-shape) pattern. A high serial correlation of earnings means that the difference in consecutive reports, $r_t - r_{t-1}$, as opposed to the level, r_t, contains the most (value) relevant information. This implies that high reports today will have a negative impact on prices tomorrow, other things equal. When the manager approaches retirement he does not fully internalize this effect, which explains why he tends to increase A_t by the end of his tenure. The sign of the effect of δ on A_t^T also depends on $\{\rho, \sigma_v\}$. Under high earnings correlation, a higher bias today leads to a decrease in stock prices tomorrow, ceteris paribus. A more patient manager has then weaker incentives to increase the bias today, since he internalizes more fully the adverse effect of increasing the bias today on future prices.

A_t^T can be interpreted as the expected cumulative discretionary accruals the manager reports in period t. The bias A_t^T is the result of the manager trading-off the benefits from influencing the subsequent stock prices against his current biasing costs. As expected, the higher the sensitivity of subsequent prices to the current report r_t, the larger is the bias A_t^T.[6]

The effect of the manager's discount factor, δ, on A_t^T is ambiguous. In fact, a more patient manager may bias the equity more aggressively. This contrasts with the intuition that misreporting is a phenomenon driven by managerial myopia. In this model, the more the manager cares about the future, the stronger may be his incentives to increase the bias because such increase affects not only current prices but also subsequent ones. Figure 1 shows how A_t^T depends on δ.

Figure 1 reveals the lack of generality of the initial and terminal effects. Indeed, the

[6]In Appendix A page 28 we demonstrate how the coefficients α_j^k in (5) and the conditional variances of both θ_t and ε_t can be computed, given the equilibrium process of reports.

bias may be increasing or decreasing both at the beginning and at the end of the manager's tenure. For example, the manager may start his tenure by injecting a relatively high bias in equity and then partially reduce it over time till a few periods prior to retirement at which time he again increases the bias in equity. Alternatively, the manager may start with a relatively low bias, increase it at the beginning of his tenure, and then reduce it prior to retirement.[7]

Before concluding this section, we consider the evolution of the stock price. Given risk neutral pricing, the price function is a martingale (i.e., the best predictor of p_{t+1} is p_t, in period t) and can thus be expressed as

$$p_t = p_{t-1} + \beta_t \left(r_t - \mathbb{E}_{t-1} \left(r_t \right) \right). \tag{7}$$

where

$$\beta_t = \frac{Cov_{t-1} \left(r_t, z_t \right)}{Var_{t-1} \left(r_t \right)}.$$

(The subscript in Cov_t and Var_t indicates that the variance/covariance is conditioned on the history of reports at time t, i.e., $\{r_1, r_2, ..., r_t\}$). The parameter $\beta_t > 0$ is reminiscent of the earnings response coefficient (ERC) and it changes over the manager's tenure. This confirms that the ERC β_t is not constant in an environment characterized by uncertainty and learning.[8] To conclude this section, Figure 2 simulates the dynamics of the firm's equity θ_t, the book value of equity r_t, the stock price p_t, and the firm's economic value z_t.

3.1.1 Earnings Quality

The analysis of Demski (1973) implies that a general definition of earnings quality is (to put it mildly) elusive. Yet, a higher informativeness is arguably a desirable property of earnings reports, at least one that concerns standard setters and researchers.[9] Here, we adopt a definition of earnings quality that emphasizes informativeness. Formally, we define earnings

[7]The lack of generality of the initial and terminal effects speaks perhaps to the danger of drawing conclusions based on two period models.

[8]This may explain perhaps why, empirically, the ERC has proven unstable (see e.g., Collins and Kothari (1989)). Similarly, note that the book-to-market ratio $\frac{r_t}{p_t}$ evolves throughout the manager's tenure.

[9]In fact, Blackwell (1953)'s informativeness is the only generally desirable property of accounting systems in the setting of Demski (1973).

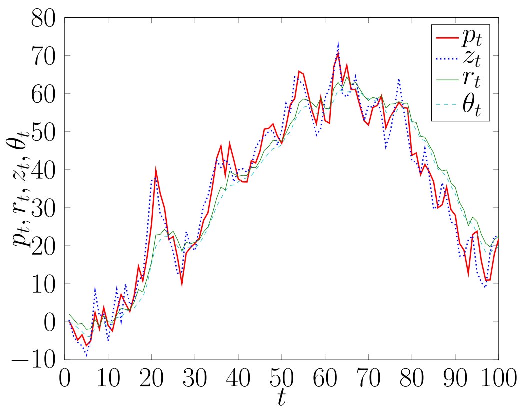


Figure 2: The dynamics of market and book values ($\sigma_v = 1, \sigma_\eta = 1, \rho = .8$). The price p_t tracks closely the movements in the firm's economic value z_t, whereas the book value r_t tracks closely the movements in the firm's equity θ_t. The price p_t is more volatile than reported equity r_t because the latter does not incorporate the implications of the shocks v_t for the firm's future earnings.

quality as[10]

$$EQ_t \equiv -Var_t\left(z_t\right).$$

$Var_t\left(z_t\right)$ measures the information asymmetry, between the manager and investors, that built up over time by the manager's misreporting, given that the manager's misreporting incentives are not fully known. Absent misreporting, $Var_t\left(z_t\right)$ would be zero for all t, and the stock price would be equal to z_t.

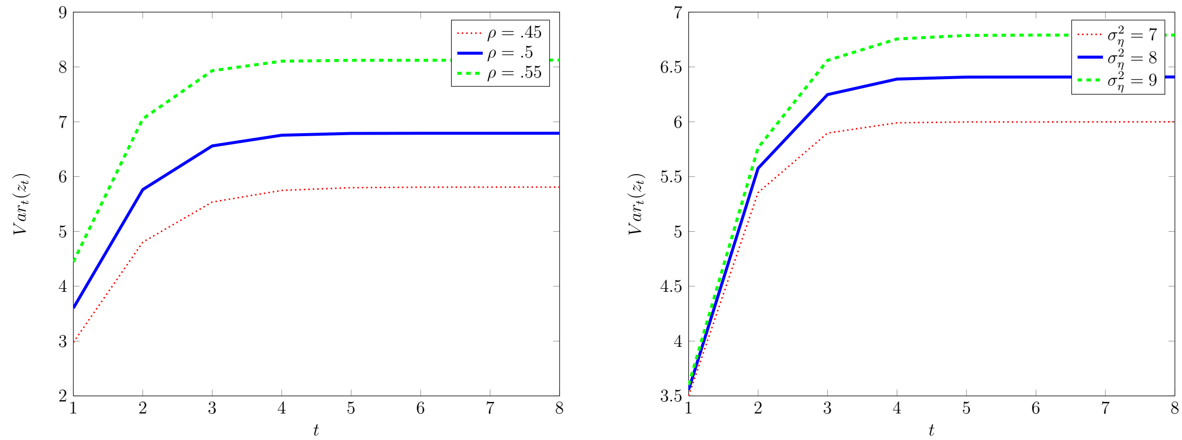
EQ_t captures the informativeness of reports (relative to the manager's information set) from the extent of uncertainty investors are left with after observing the history of reports. Of course, the amount of uncertainty investors are left with critically depends on the amount of uncertainty they faced prior to the reports having being released. In that sense, EQ_t is a well defined measure of informativeness only if the researcher is able to identify investors' prior uncertainty. Formally, this implies that we must control for $\{\sigma_v, \rho\}$ if EQ_t is to provide a meaningful measure of earnings quality.[11] Using equation (7), we can express earnings

[10] In this model an equivalent definition of earnings quality is

$$V_{t-1}\left(z_t\right) - V_t\left(z_t\right),$$

namely the decrease in investors' uncertainty induced by the manager's report.

[11] On the other hand, it is not entirely clear that one should control for σ_v and ρ. After all, we are using the manager's information set as the benchmark, and in each period the manager knows z_t regardless of the values of σ_v and ρ.



(a) $(\sigma_v = 1, \sigma_\eta = 1)$ (b) $(\sigma_v = 1, \rho = .5)$

Figure 3: The effect of (ρ,σ_η) on the evolution of information asymmetry. Information asymmetry increases over time, but it is bounded. Both the serial correlation of earnings ρ and the volatility of reporting noise σ_η increase the level of information asymmetry, at any point in time.

quality as

$$EQ_t = -\left[Var_{t-1}(z_t) - \beta_t^2 Var_{t-1}(r_t)\right]. \tag{8}$$

EQ_t will evolve deterministically throughout the manager's tenure until the steady state is reached. Figure 3 shows that EQ_t decreases over time until it converges to its steady state level. Conversely, the asymmetry of information grows over time, at a decreasing pace, ultimately settling at the steady state level.

Discussion

The empirical accounting literature has studied extensively the bias (manipulation/accruals) of financial reporting. In particular, the time-series of earnings announcements and the book-to-market ratios have been used for both valuation purposes and for various measures of "accounting quality." Yet, to the best of our knowledge, the theoretical literature lacks a dynamic theory of reporting bias to provide insights and guidance to the empirical research on the topic. As such, we do not have a good understanding of how managerial manipulation and book-to-market ratios should evolve over time or whether firms' earnings-response-coefficient (ERC) should be stable over time, as often assumed by empiricists (e.g., Collins and Kothari (1989)) or should change over time.

While our model makes some strong assumptions, mainly in the form of the manager's manipulation cost and his exogenous price dependent incentive, it still has a lot of degrees of freedom that can give rise to different predictions of the time-series properties of managerial reports and firm's book-to-market ratios. In particular, the degree of earnings serial correlation, ρ, affects the predicted properties of the time series of managerial reports and book-to-market ratios. As such, our model can predict different time-series properties for firms and industries with different characteristics.

Consider a firm whose earnings are not correlated over time, or the correlation is sufficiently low. The time series of the bias in reported equity for such a firm takes an inverse U-shape. That is, the bias in reported equity is initially increasing and then starts to decrease until the manager leaves the firm at time T. Equivalently, reported earnings are initially inflated and the book-to-market ratio is increasing, where later on earnings are deflated and the book-to-market ratio is decreasing. Before we provide our intuition for the inverse U-shape of the bias in reported equity, we note two characteristics of the equilibrium: (i) when earnings are uncorrelated over time the stock price at any period equals investors' beliefs about the firm's equity, θ_t. This implies that all future prices are increasing in the current report, and (ii) the variance of investors' beliefs about θ_t is increasing over time and is bounded from above. When determining the bias, the manager considers the trade-off between his biasing cost and the effect of the current report on current and future stock prices. The effect of the current report on prices depends on both the sensitivity of prices to the current report as well as the remaining tenure of the manager with the firm. In early periods, investors' uncertainty about θ_t increases relatively fast, which makes the stock price more responsive to the manager's report. This in turn, results in a relatively large increase in the manager's incentive to bias reported equity. In later periods, the increase in investors' uncertainty decays, which results in a relatively small increase in manager's incentive to bias his report. The effect of time passage on the manager's remaining tenure with the firm, is in the opposite direction. That is, in early periods the shortening of the manager's horizon by one period has a relatively small impact on the manager's incentive to bias his report (due to the manager discounting future consumption) while in later periods this effect is more substantial. Since in early periods the effect that increase the incentive to bias reported equity dominates, and in later stages the effect that decreases the incentive to bias dominates, the bias in reported equity obtains an inverse U-shape in expectation.

If the firm's earnings experience a relatively high positive serial correlation, then the pricing of the firm becomes very sensitive to investors' beliefs about the current level of earnings, and less sensitive to investors' beliefs about the level of equity, θ_t. This complicates the analysis of the time series properties of the firm's report and book-to-market ratios, as the effect of the current report on future prices is no longer always positive. This is intuitive because for given investors' beliefs about θ_t at time t, the lower the beliefs about θ_{t-1} at time t the higher is the stock price at time t. Therefore, the price at time t may be decreasing in the report at time $t-1$ or the report in earlier periods. Numerical analysis demonstrates that for sufficiently high serial correlation of earnings, the shape of the deterministic component of the bias in reported equity, A_t^T, does not follow an inverse U-shape.

In the discussion above, we assumed that the manager's stock base incentive is constant over time. However, this need not be constant over time, or even does not need to have a constant mean over time. While endogenizing the manager's stock based incentives is beyond the scope of our model, one can easily see that the time-series properties of such incentives will affect the manager's reports and the firm's book-to-market ratios. For example, if price related incentives are increasing (decreasing) sufficiently fast over time the bias in the reported earnings will increase (decrease) over time (see Zakolyukina (2012)).

3.2 Steady State

We now turn to the analysis of the infinite horizon case, i.e., $T = \infty$. Furthermore, we focus on the steady state, which is attained once the manager has stayed with the firm sufficiently long. The infinite horizon setting is somewhat unrealistic but particularly amenable for empirical analysis because neither the horizon of the manager nor the time he has already been with the firm play a role in the analysis. The main advantage of studying the steady state is that it achieves tractability and serves as a basis for a structural estimation of our model, which we discuss in Section 4.

In the steady state, the learning process is stationary so that both the price function, and the deterministic component of the bias in reported equity, A_t^T, are constant over time. Denoting the steady state bias in reported equity by A_∞ the manager's report at any period t can be written as

$$r_t = \theta_t + \eta_t + A_\infty. \tag{9}$$

In general, investors use past reports to update their beliefs about both the current equity, θ_t, and about the current period's earning, ε_t. In the steady state, this updating process is stable. For example, the stock price becomes

$$p_t = p_{t-1} + \beta(r_t - \mathbb{E}_{t-1}(r_t)) \tag{10}$$

where β is a constant parameter.

While the price change, $p_t - p_{t-1}$, in equation (10) might seem straight forward, as it is a linear function of the report surprise, $r_t - \mathbb{E}_{t-1}(r_t)$, the expected report, $\mathbb{E}_{t-1}(r_t)$, is in fact a function of the entire history of reports. Some algebra reveals, however, that the price dynamics can be expressed in a compact manner as a function of book and market values of equity in the prior two periods. Denoting by γ the slope of $\hat{\varepsilon}_t \equiv \mathbb{E}_t \varepsilon_t$ with respect to r_t (i.e., $\gamma = \frac{\partial \hat{\varepsilon}_t}{\partial r_t}$), we obtain the following result.

Lemma 1 *In the steady state, the price change is characterized by the difference equation*

$$p_{t+2} = p_{t+1} + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)(p_{t+1} - p_t) + \beta\left(r_{t+2} - p_{t+1}\right) - \rho\beta\left(r_{t+1} - p_t\right). \tag{11}$$

along with the boundary conditions

$$\begin{aligned} p_1 &= \beta r_1 \\ p_2 &= (1-\beta)\,\beta + \frac{\beta\gamma\rho^2}{1-\rho} r_1 + \beta r_2. \end{aligned}$$

In the steady state, the price-change in consecutive periods depends on three variables: the price change in the previous period (which has a momentum flavor to it) and the extent to which the report "beat" the price in the last two periods (in practice these two variables tend to be negative when market values are greater than book values).[12] In section 4, we shall exploit the price function (11) to estimate the equilibrium parameters β, γ as well as the serial correlation of earnings, ρ.

The parameters β and γ are formed in equilibrium based on investors' perceptions of how informative the manager's reports are. Next, we derive the equilibrium values of β and γ.

[12]The structure of the price function is clearly affected by our assumptions about the distribution of $\{\eta_t, \theta_t, \varepsilon_t\}$.

Obtaining γ, β and A_∞ In steady state, the speed of learning is constant so the variance and correlation structure of investors' beliefs is constant, too. As mentioned above, this has two main implications. First, the price coefficients become constant. Second, the steady state bias in reported equity also becomes constant (the latter, in turn, means that earnings reports will contain no discretionary accruals, on average.)

First, we consider the price coefficients β and γ. Recall that these coefficients are given by

$$\begin{aligned}\beta &= \frac{Cov_{t-1}\left(r_t, z_t\right)}{Var_{t-1}\left(r_t\right)},\\ \gamma &= \frac{Cov_{t-1}\left(r_t, \varepsilon_t\right)}{Var_{t-1}\left(r_t\right)}.\end{aligned}$$

The above variances and covariances depend on the covariance matrix,

$$\Sigma = \begin{pmatrix} Var_t(\theta_t) & Cov_t(\theta_t, \varepsilon_t) \\ Cov_t(\theta_t, \varepsilon_t) & Var_t(\varepsilon_t) \end{pmatrix},$$

which describes investors' beliefs about the firm's fundamentals, namely its equity θ_t and earnings ε_t. By definition of steady state, the covariance-matrix Σ is independent of t. Letting $\psi_1 \equiv Var_t(\theta_t)$, $\psi_2 \equiv Cov_t(\theta_t, \varepsilon_t)$ and $\psi_3 \equiv Var_t(\varepsilon_t)$ and noting that

$$\begin{aligned} &\begin{pmatrix} Var_{t-1}(\theta_t) & Cov_{t-1}\left(\theta_t, \varepsilon_t\right) & Cov_{t-1}(r_t, \theta_t) \\ Cov_{t-1}\left(\theta_t, \varepsilon_t\right) & Var_{t-1}(\varepsilon_t) & Cov_{t-1}\left(r_t, \varepsilon_t\right) \\ Cov_{t-1}(r_t, \theta_t) & Cov_{t-1}\left(r_t, \varepsilon_t\right) & Var_{t-1}(r_t) \end{pmatrix} \\ = &\begin{pmatrix} \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \\ \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \\ \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2 \end{pmatrix} \end{aligned} \tag{12}$$

we can solve for ψ by observing that investors will update the covariance matrix Σ using the following rule (see e.g., DeGroot, 1970, pp. 175-176):

$$\begin{aligned}\Sigma &= \begin{bmatrix} Var_{t-1}(\theta_t) & Cov_{t-1}\left(\theta_t, \varepsilon_t\right) \\ Cov_{t-1}\left(\theta_t, \varepsilon_t\right) & Var_{t-1}(\varepsilon_t) \end{bmatrix} \\ &\quad - \frac{\begin{bmatrix} Cov_{t-1}\left(\theta_t, r_t\right) \\ Cov_{t-1}\left(\varepsilon_t, r_t\right) \end{bmatrix} \begin{bmatrix} Cov_{t-1}\left(r_t, \theta_t\right) & Cov_{t-1}\left(r_t, \varepsilon_t\right) \end{bmatrix}}{Var_{t-1}\left(r_t\right)}.\end{aligned}$$

where the first term in the right hand side is the prior covariance matrix, and the second term captures the effect of the report r_t in reducing investors' uncertainty. Notice that the presence of serial correlation in earnings means that the firm's equity will be correlated with its earnings. In turn, this means that investors will have to update their residual variance of equity and that of earnings simultaneously, so as to account for such correlation.

The above updating rule, along with equation (12), yields the following steady state conditions:

$$\begin{aligned}
\psi_1 &= \sigma_\eta^2 \frac{\psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} && (13)\\
\psi_2 &= \sigma_\eta^2 \frac{\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} && (14)\\
\psi_3 &= \frac{\psi_1\rho^2\psi_3 + \rho^2\psi_3\sigma_\eta^2 + \psi_1\sigma_v^2 + \sigma_v^2\sigma_\eta^2 - \rho^2\psi_2^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2}. && (15)
\end{aligned}$$

Solving this system of equations gives ψ as a function of the primitive parameters, $\{\sigma_v, \rho, \sigma_\eta\}$. Naturally, when $\sigma_\eta = 0$ investors's beliefs are degenerate, hence $\psi = \mathbf{0}.$ Conversely, when $\sigma_\eta \to \infty$ accounting becomes uninformative and, over time, the asymmetry of information explodes, i.e., $\psi \to \infty$. More generally, when $\sigma_\eta > 0$ the value of ψ is bounded but positive, meaning that the asymmetry of information regarding the firm value never disappears in this model. In other words, the manager will always know more than investors about the value of the firm's equity and the present value of the future earnings.

Using the solution of ψ we can obtain $\{\beta, \gamma\}$ from the equations:

$$\begin{aligned}
\beta &= \frac{Cov_{t-1}(r_t, z_t)}{Var_{t-1}(r_t)} = \frac{\psi_1 + \frac{2\rho-\rho^2}{1-\rho}\psi_2 + \frac{\rho^2}{1-\rho}\psi_3 + \frac{\sigma_v^2}{1-\rho}}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} && (16)\\
\gamma &= \frac{Cov_{t-1}(r_t, \varepsilon_t)}{Var_{t-1}(r_t)} = \frac{\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} && (17)
\end{aligned}$$

This completes the derivation of the price function, in steady state. It remains to derive the steady state bias A_∞ to complete the characterization of the manager's reporting strategy. This bias is defined as the limit

$$A_\infty \equiv \lim_{t\to\infty} c^{-1} \sum_{k=0}^{\infty} \delta^k \alpha_t^{t+k}. \tag{18}$$

Since in steady state the price coefficients do not depend on the time variable t, in the sequel we drop t and simply write

$$\begin{aligned}
A_\infty &= c^{-1} \sum_{k=1}^{\infty} \delta^{k-1} \alpha_k \\
\text{where } \alpha_k &= \frac{\partial p_{t+k}}{\partial r_t}.
\end{aligned}$$

The sequence of price coefficients $\{\alpha_k\}_{k=1}^{\infty}$ fully summarizes the effect of marginally increasing the current report r_t on current and subsequent prices $\{p_t, p_{t+1}, ..., p_\infty\}$. Our ability to characterize A_∞ hinges upon being able to characterize the sequence $\{\alpha_k\}_{k=1}^{\infty}$. Fortunately, the analysis of the price function (11), along with the equilibrium reporting strategy, readily show that the sequence of price coefficients is determined by the second-order difference equation:

$$\alpha_{t+2} = \left(1 - \beta + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\right)\alpha_{t+1} - \left(\rho\left(1 + \gamma\frac{\rho}{1-\rho}\right) - \rho\beta\right)\alpha_t \tag{19}$$

with the boundary conditions:

$$\begin{aligned}\alpha_1 &= \beta \\ \alpha_2 &= (1-\beta)\beta + \frac{\beta\gamma\rho^2}{1-\rho}\end{aligned}$$

The solution of the difference equation (19) is

$$\alpha_t = \phi_1 m_1^t + \phi_2 m_2^t, \tag{20}$$

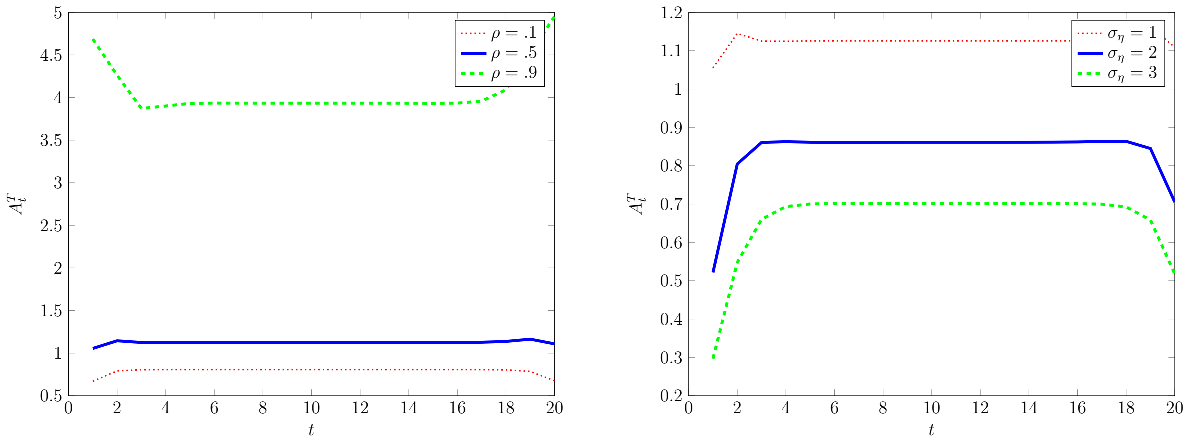
where the ϕ's and m's are constants that depend on $\{\beta, \gamma, \rho\}$. In deriving this sequence, we have relied on the simple structure of the manager's reporting strategy, notably the fact that given $\{\theta_t, \eta_t\}$ the reporting strategy r_t is independent of the history of reports.

Substituting equation (20) into the bias equation (18) yields

$$A_\infty = \frac{\frac{\phi_1}{1-\delta m_1} + \frac{\phi_2}{1-\delta m_2}}{c}.$$

Ideally, as researchers, we would like to empirically learn the value of A_∞, both for valuation purposes, and (perhaps more interestingly) to quantify the intensity of misreporting. Unfortunately, the value of A_∞ can only be identified up to the scale factor c, because the value of c is not identified by the time-series of reports and prices. This is, however, not an obstacle for empirically identifying earnings quality, as we demonstrate in Section 4.

Characterizing the Steady State Given the manager's incentives to inflate investors' perceptions of firm value, the bias A_∞ is positive in this model. The more interesting question though is under which circumstances the manager is expected to manipulate the reports more aggressively. To address this question, we provide comparative statics for the bias A_∞.



(a) $(\sigma_v = 2, \sigma_\eta = 2, \delta = .5)$

(b) $(\sigma_v = 2, \rho = .5, \delta = .5)$

Figure 4: Panel (a): The effect of ρ on the bias, A_t^T. Panel (b): The effect of σ_η on the bias, A_t^T. Higher reporting noise, σ_η, results in a smaller bias. This is because the market response to the manager's report is weaker when investors anticipate noisier reports, hence the manager's incentive to incur the cost of misreporting is also weaker.

Corollary 1 *The bias A_∞ increases in σ_v and ρ, but decreases in σ_η and c. The effect of δ on A_∞ is ambiguous.*

Proof. To be completed. ■

These comparative statics are intuitive. Misreporting is a phenomenon that develops when valuation uncertainty is higher, since the market response to reports is stronger in those circumstances. In this model, uncertainty increases in the volatility of earnings σ_v; the managers' misreporting incentives are thus directly affected by σ_v. Prices also tend to be more sensitive to reports when the serial correlation of earnings ρ is higher, because the serial correlation of earnings amplify the uncertainty of investors. This explains why A_∞ increases in ρ.

Intuitively, a higher c reduces the manager's biasing incentives (almost, in a a mechanical fashion) thus leading to a lower bias A_∞. At first sight, this would seem as a desirable effect, however, Lemma 2 and Corollary 2 establish that earnings quality is independent of of c.

Lemma 2 *In steady state, earnings quality equals*

$$
\begin{aligned}
EQ_\infty &= -\Bigg[\psi_1 + 2\left(\frac{\rho}{1-\rho}\right)\psi_2 + \left(\frac{\rho}{1-\rho}\right)^2\psi_3 + \frac{\sigma_v^2}{(1-\rho)^2} \\
&\quad - \frac{\left(\psi_1 + \frac{2\rho-\rho^2}{1-\rho}\psi_2 + \frac{\rho^2}{1-\rho}\psi_3 + \frac{\sigma_v^2}{1-\rho}\right)^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2}\Bigg]
\end{aligned}
$$

Corollary 2 *EQ_∞ decreases in σ_v, ρ, and σ_η. EQ_∞ is independent of the manager's discount factor δ and the cost of misreporting c.*

Proof. To be completed. ■

The analysis reveals that the bias A_∞ is large precisely when earnings quality is high, which contrasts with the intuition behind some existing measures of earnings quality, in particular those relying on the levels of discretionary accruals (see Dechow et al. (2010)). The reason is that higher earnings quality leads to stronger price reactions, which in turn boosts the manager's misreporting incentives. This suggests that existing measures of earnings quality based on the level of discretionary accruals could be misleading, under the assumptions of this model.

On the other hand, one may wonder whether EQ_t is consistent with other routinely used measures of earnings quality, such as for example, *smoothness* ($Var\left(\Delta r_t\right)$) or *persistence* ($Corr\left(\Delta r_t, \Delta r_{t-1}\right)$). It is not difficult to see that in this model a higher reporting noise σ_η will result in a lower persistence of reported earnings as well as in more volatile earnings.

$$
Var\left(\Delta r_t\right) = Var\left(\varepsilon_t\right) + 2\sigma_\eta^2,
$$

$$
Cov\left(\Delta r_t, \Delta r_{t-1}\right) = Cov(\varepsilon_t, \varepsilon_{t-1}) - \sigma_\eta^2
$$

In that sense, persistence and smoothness seem as appropriate measures of earnings quality, in this setting. But, again, these measures only are meaningful when one has a reliable method for identifying σ_v and ρ.[13]

[13] Gerakos and Kovrijnykh (2013) study the effect of earnings management on the time series of reported earnings. They argue that earnings management has the following implication for the time-series of earnings: the residual from a regression of earnings (Δr_t) on the first order lag of earnings (Δr_{t-1}) should exhibit negative second order autocorrelation. We consider this prediction in our setting. From equation (9), one can express

$$
\Delta r_t = \rho \Delta r_{t-1} + \eta_t - (1+\rho)\eta_{t-1} + \rho\eta_{t-1}
$$

4 Empirical Analysis

Predire n'est pas expliquer, René Thom

This paper's goal is to perform an empirical analysis of misreporting that satisfies three properties:

1. all the assumptions of the underlying theory are explicitly outlined,
2. the theory and empirical methods are mutually consistent, and
3. the properties of the theory are analytically clear and can be understood generally (not just numerically).

We believe these properties help interpret in a clear manner the data we observe in the real world. Clarity has a price: since the model is stylized, the empirical method will inherit its simplicity and perhaps naivete.[14] In other words, by emphasizing clarity we will likely loose predictive power.[15,16]

We acknowledge that in order to achieve the third property outlined above, we have made strong assumptions, most significantly the quadratic misreporting cost. We do not know what the full implications of dropping this assumption will be for the empirical analysis. We hope that future work will assess the impact of this and other assumptions.

4.1 Estimation Method and Identification

The objective of this section is to use the structural model of the steady state in Section 3.2 to estimate the persistence of true earnings ρ, the earnings response coefficient β, the

Hence, Gerakos and Kovrijnykh (2013)'s residual is here

$$n_t = \eta_t - (1+\rho)\eta_{t-1} + \rho\eta_{t-2}$$

From this equation we see that

$$Cov(n_t, n_t - 2) = \rho\sigma_v^2 > 0$$

which contrasts with the theoretical prediction of Gerakos and Kovrijnykh (2013).

[14]This is both a virtue – to the extent that a good theory should be as simple as possible – and a drawback – to the extent the model may overlook other important effects.

[15]Clarity has priority in science, since there cannot be a meaningful prediction in the absence of a clear theory. For Aristotle, scientific knowledge is "knowledge of causes."

[16]Needless to say, we do not view our approach as the only valid approach, not even the most desirable one, but merely as a complement to alternative existing methods, in line with Aristotle's teachings: "It is evidently equally foolish to accept probable reasoning from a mathematician and to demand from a rhetorician demonstrative proofs." Aristotle Nicomachean Ethics, 1094b, 25-7

sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, the variability of earnings shocks σ_v^2, and the uncertainty added by earnings manipulation σ_η^2 based on the joint distribution of book value of equity (or reported earnings, which is informationally equivalent) and market value of equity. Since our model suggests a deterministic relation between reported earnings histories and prices, we add the statistical assumption that the price is affected by unobserved random variables, such as liquidity trades, that are proportional to the price at the beginning of the period.

$$p_t = \left(1+\tilde{\zeta}\right) p_{t-1} + \rho \left(1+\gamma\frac{\rho}{1-\rho}\right)(p_{t-1}-p_{t-2}) + \beta\left(r_t - p_{t-1}\right) - \rho\beta(r_{t-1}-p_{t-2})$$

We assume that $\tilde{\zeta}$ is normally distributed with mean 0. Since the manager is risk-neutral and investors do not have any private information, a zero-mean random shock to price will not alter the manager's reporting strategy and will not affect the information contained in a firm's history of prices and reported earnings. We further assume that $\tilde{\zeta}$ is distributed independently across time and of the entire history of earnings innovations, shocks to the manipulation costs and past price distortions $\tilde{\zeta}$. Since our analysis is performed at the industry level we also require that the price distortions are independent across firms in the same industry as well as the parameters being the same for all firms within each industry. These are clearly strong (and possibly unrealistic) assumptions to make but relaxing them is beyond the current scope of the paper.

At first, it might appear that the most natural method to estimate this model is by taking the following two-step approach:

1. Estimate $\{\beta, \gamma, \rho\}$ from the price equation (11) via OLS.
2. Based on the estimates, $\left\{\hat{\beta}, \hat{\gamma}, \hat{\rho}\right\}$, solve the five steady state equations given in (13) – (15) and (16) – (17) to obtain estimates for $\left\{\sigma_v^2, \sigma_\eta^2, \psi_1, \psi_2, \psi_3\right\}$.

Unfortunately, this system of equations does not identify the values of $\{\sigma_v, \sigma_\eta\}$ because both β and γ depend only on the ratio σ_v^2/σ_η^2. In other words, observing $\{\beta, \gamma, \rho\}$ does not contain information about the levels of σ_v and σ_η. Hence, in order to estimate $\{\sigma_v, \sigma_\eta\}$ we need information in addition to that contained in the price function in (11). Next, we describe a method that identifies σ_v and σ_η using additional moment conditions.

Recall that in steady state the stochastic process of a firm's true economic value of equity θ_t, true economic earnings ε_t and reported book value of equity r_t is given by

$$\begin{aligned}
\theta_t &= \theta_{t-1} + \varepsilon_t \\
\varepsilon_t &= \rho\varepsilon_{t-1} + \tilde{\upsilon}_t \\
r_t &= \theta_t + \tilde{\eta}_t + A_\infty
\end{aligned}$$

where $\tilde{\upsilon}_t$ and $\tilde{\eta}_t$ are independent normally distributed shocks with mean 0.[17] Since the main focus of the analysis are the persistence and variance of true economic earnings compared to the uncertainty induced by earnings manipulation, we use the variances of two measures of changes in book value of equity as part of our moment conditions. In particular, the moment conditions are based on the first-order difference in book value $\Delta r_t \equiv r_t - r_{t-1}$ as well as a first- and a second-order difference adjusted for the persistence of shocks to true equity, ρ.

$$\begin{aligned}
\Delta r_t - \rho\Delta r_{t-1} &= \upsilon_t + \eta_t - (1+\rho)\,\eta_{t-1} + \rho\eta_{t-2} \\
\Delta^2 r_t - \rho\Delta^2 r_{t-1} &= \upsilon_t - \upsilon_{t-1} + \eta_t - (2+\rho)\,\eta_{t-1} + (1+2\rho)\,\eta_{t-2} - \rho\eta_{t-3}
\end{aligned}$$

where $\Delta^2 r_t$ denotes the change in reported earnings or, equivalently, the second order difference in the book value of equity, $\Delta^2 r_t \equiv \Delta r_t - \Delta r_{t-1}$. To summarize, we use the following 8-dimensional vector of moments to estimate the model parameters:

$$\begin{aligned}
E\left[\Delta r_t - \rho\Delta r_{t-1} - \mu_1\right] &= 0 \\
E\left[(\Delta r_t - \rho\Delta r_{t-1} - \mu_1)^2 - \left(\sigma_\upsilon^2 + 2\left(1+\rho+\rho^2\right)\sigma_\eta^2\right)\right] &= 0 \\
E\left[\Delta^2 r_t - \rho\Delta^2 r_{t-1} - \mu_2\right] &= 0 \\
E\left[\left(\Delta^2 r_t - \rho\Delta^2 r_{t-1} - \mu_2\right)^2 - \left(2\sigma_\upsilon^2 + 2\left(3+4\rho+3\rho^2\right)\sigma_\eta^2\right)\right] &= 0 \\
E\left[x\zeta\right] &= 0
\end{aligned}$$

where $x' = \left[1, 1 - \frac{p_{t-2}}{p_{t-1}}, \frac{r_t}{p_{t-1}} - 1, \frac{r_{t-1}}{p_{t-1}} - \frac{p_{t-2}}{p_{t-1}}\right]$. Since we can express the moments in closed form, we use the Generalized Method of Moments (GMM).

4.2 Data

The model requires data on market value and book value of equity. Since we expect the model parameters to vary across industry due to different technologies, investment oppor-

[17] We realize that normal distributions are unrealistic given the value restrictions on the book value and market value of equity. We maintain the assumption for tractability reasons.

tunities and competitive situations, we also require the SIC code to classify firms into an industry following the Fama-French 12 industry classification. Data on stock prices, number of shares outstanding, common stockholders' equity and SIC code are obtained from Compustat for the period from 1990 to 2010. We delete observations with negative market value or book value of equity and truncate all variables at the 1% level. Moreover, we seasonally adjust reported equity values by subtracting the mean calculated for each fiscal quarter and each firm.

		quarterly data	annual data
N		382,199	65,447
Number of firms		14,398	9,421
Book value of equity	mean	950.6	1,603.3
	std. dev.	4,510.7	6,772.7
Market value of equity	mean	2,490.0	4,029.0
	std. dev.	12,639.6	16,996.1

Table 1: Descriptive statistics for quarterly and annual data.

4.3 Findings

We first estimate the returns regression to verify that b_1 and b_2 are positive and b_3 is negative.

$$\frac{p_t}{p_{t-1}} = b_0 + b_1\left(1 - \frac{p_{t-2}}{p_{t-1}}\right) + b_2\left(\frac{r_t}{p_{t-1}} - 1\right) + b_3\left(\frac{r_{t-1}}{p_{t-1}} - \frac{p_{t-2}}{p_{t-1}}\right) + \tilde{\zeta} \tag{21}$$

The results for quarterly and annual data on an industry level are summarized in Table 8 and Table 9, respectively. All detailed tables are included in Appendix B. All coefficients have the expected sign and are significant at the one percent level where standard errors are clustered by fiscal year and firm. However, the coefficients are larger in magnitude for the annual data than for the quarterly data and the R-squared is generally higher for annual data.

Even though the persistence of true earnings ρ, the earnings response coefficient β and the sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ can be calculated based on the estimates obtained in Table 8 and Table 9, we use the Generalized Method of Moments (GMM) to estimate all five model parameters and the corresponding standard errors.

Table 2 provides the mean and median of the estimates we obtain based on both the quarterly and annual data for the twelve Fama-French industries. Detailed estimates including standard deviation and confidence intervals are summarized for each industry in Tables 10 and 11 in the Appendix. For quarterly data, the persistence of earnings shocks, ρ, is estimated to be 1.1 on average across industries. Since the model restricts ρ to be in $[0, 1]$ the model is inconsistent with the quarterly data. For this reason, in the following, we discuss the results based on annual data.

The persistence of earnings shocks, ρ is close to 0.8 for all industries based on annual data suggesting that annual economic earnings innovations are highly persistent. We obtain the lowest estimate for the persistence parameter for the Telecom (Industry 7) with $\hat{\rho} = 0.740$ and the highest estimate for Consumer Non-Durables (Industry 1) with $\hat{\rho} = 0.907$. γ measures the sensitivity of investors' beliefs about current period economic earnings to reported earnings (or equity). Estimates of γ are generally close to 0.05. The earnings response coefficient, β, is estimated to be about 1.5. σ_v^2 and σ_η^2 measure the variability of shocks to economic earnings and the uncertainty added by earnings manipulation, respectively. Estimates of the variances for v and η are positive and significantly different from zero. The variance of true earnings shocks is estimated to be 0.013. Estimates of the uncertainty added by earnings manipulation yield 0.005.

Parameter	ρ	γ	β	σ_v^2	σ_η^2
Quarterly data					
Mean	1.088	.045	.444	.003	.002
Median	1.095	.048	.447	.003	.002
Annual data					
Mean	.861	.077	1.615	.013	.005
Median	.876	.040	1.443	.013	.005

Table 2: Estimates for model parameters based on quarterly and annual data using GMM. Estimates are averaged across the twelve industries.

To get a sense of the relative magnitudes, we compute the ratio of the variance of noise introduced by earnings management per period, σ_η^2, to the variance of earnings innovation per period, σ_v^2. Table 3 provides the mean and median ratio across the Fama-French industries.

While the mean and median are 53% and 37%, respectively, for annual data, the estimates for individual industries vary widely. We obtain the lowest estimate for Consumer Durables (Industry 2) with 16.2% while the highest estimate is 121.3% for Utilities (Industry 8). Details are provided in Table 13.

σ_η^2/σ_v^2	quarterly data	annual data
Mean	139%	53%
Median	65%	37%

Table 3: Mean and median ratio of estimates across industries of the variance of the noise added by earnings manipulation, σ_η^2, to the variance of the earnings innovation, σ_v^2, for both quarterly and annual data.

5 Appendix

Derivation of price coefficients. Define the variance-covariance matrix

$$\Sigma_{rr}(t) \equiv Var(\mathbf{r}_t)$$

and the covariance vector

$$\Sigma_{rz}(t) \equiv Cov(\mathbf{r}_t, z_t).$$

The price coefficients $\alpha_t = \left\{\alpha_j^t\right\}_{j=1}^{j=t}$ are thus given by

$$\left[\Sigma_{rr}(t)' \Sigma_{rr}(t)\right]^{-1} \Sigma_{rz}(t)$$

Below we compute the elements of $\Sigma_{rr}(t)$ and $\Sigma_{rz}(t)$. First note that the value of θ_t can be written as

$$\theta_t = \sum_{k=0}^{t-1} \frac{v_{k+1}\left(1-\rho^{t-k}\right)}{1-\rho}$$

Similarly,

$$\varepsilon_t = \sum_{k=0}^{t-1} \rho^{t-1-k} v_{k+1}$$

Therefore, using

$$z_t = \theta_t + \frac{\rho}{1-\rho}\varepsilon_t,$$

we get that

$$\begin{aligned}
z_t &= \sum_{k=0}^{t-1} v_{k+1}\frac{1-\rho^{t-k}}{1-\rho} + \frac{\rho}{1-\rho}\sum_{k=0}^{t-1}\rho^{t-1-k}v_{k+1} \\
&= \sum_{k=0}^{t-1} v_{k+1}\left(\frac{1-\rho^{t-k}}{1-\rho} + \frac{\rho}{1-\rho}\rho^{t-1-k}\right) \\
&= \sum_{k=0}^{t-1} \frac{v_{k+1}}{1-\rho}
\end{aligned}$$

Ignoring the deterministic part of the bias A_t^T, the report is given by

$$r_t = \sum_{k=0}^{t-1} \frac{v_{k+1}}{1-\rho}\left(1-\rho^{t-k}\right) + \eta_t$$

The variance covariance matrix of $\mathbf{r}_t$ is then characterized by

$$Var(r_t) = \sigma_v^2 \sum_{k=0}^{t-1}\left(\frac{\left(1-\rho^{t-k}\right)}{1-\rho}\right)^2 + \sigma_\eta^2$$

$$Cov\left(r_t, r_j\right) = \sigma_v^2 \sum_{k=0}^{j-1} \frac{\left(1-\rho^{t-k}\right)\left(1-\rho^{j-k}\right)}{\left(1-\rho\right)^2} \text{ for } j < t$$

Similarly the elements of $\Sigma_{rz}\left(t\right)$ are given by

$$\begin{aligned}
Cov\left(z_t, r_j\right) &= Cov(\sum_{k=0}^{t-1} \frac{v_{k+1}}{1-\rho}, \sum_{k=0}^{j-1} \frac{v_{k+1}\left(1-\rho^{j-k}\right)}{1-\rho}) \\
&= Cov(\sum_{k=0}^{t-j-1} \frac{v_{k+1}}{1-\rho}, \sum_{k=0}^{j-1} \frac{v_{k+1}\left(1-\rho^{j-k}\right)}{1-\rho}) \\
&= \sum_{k=0}^{j-1} \frac{\sigma_v^2\left(1-\rho^{j-k}\right)}{\left(1-\rho\right)^2} \\
&= \sigma_v^2 \frac{-j + j\rho + \rho - \rho^{j+1}}{\left(-1+\rho\right)^3}.
\end{aligned}$$

■

Proof of Lemma 1. In the steady state, the price function can be written as

$$\begin{aligned}
p_t &= p_{t-1} + \beta(r_t - \mathbb{E}_{t-1}(r_t)) \\
&= p_{t-1} + \beta(r_t - \mathbb{E}_{t-1}[\theta_t + \eta_t]) + \text{constant} \\
&= p_{t-1} + \beta(r_t - \mathbb{E}_{t-1}[\theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \eta_t]) \\
&= p_{t-1} + \beta(r_t - \mathbb{E}_{t-1}[\theta_{t-1} + \rho\varepsilon_{t-1}]) \\
&= p_{t-1} + \beta(r_t - p_{t-1} - \mathbb{E}_{t-1}[\rho\varepsilon_{t-1} - \frac{\rho}{1-\rho}\varepsilon_{t-1}]) \\
&= p_{t-1} + \beta(r_t - p_{t-1} + \mathbb{E}_{t-1}[\frac{\rho^2}{1-\rho}\varepsilon_{t-1}]) \\
&= p_{t-1} + \beta(r_t - p_{t-1} + \frac{\rho^2}{1-\rho}\hat{\varepsilon}_{t-1})
\end{aligned}$$

where $\hat{\varepsilon}_t = \mathbb{E}_t \varepsilon_t$. Now,

$$\begin{aligned}
\hat{\varepsilon}_{t-1} &= \mathbb{E}_{t-2}\varepsilon_{t-1} + \gamma(r_{t-1} - \mathbb{E}_{t-2}r_{t-1}) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - \mathbb{E}_{t-2}r_{t-1}) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - \mathbb{E}_{t-2}[\theta_{t-1}]) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - \mathbb{E}_{t-2}[\theta_{t-2} + \varepsilon_{t-1}]) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - \mathbb{E}_{t-2}[\theta_{t-2} + \rho\varepsilon_{t-2}]) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - p_{t-2} - \mathbb{E}_{t-2}\left[-\frac{\rho}{1-\rho}\varepsilon_{t-2} + \rho\varepsilon_{t-2}\right]) \\
&= \rho\hat{\varepsilon}_{t-2} + \gamma(r_{t-1} - p_{t-2} + \frac{\rho^2}{1-\rho}\hat{\varepsilon}_{t-2}) \\
&= \gamma(r_{t-1} - p_{t-2}) + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\hat{\varepsilon}_{t-2}
\end{aligned}$$

Plugging $\hat{\varepsilon}_{t-1}$ back to the price p_t we get

$$\begin{aligned}
p_t &= p_{t-1} + \beta(r_t - p_{t-1} + \frac{\rho^2}{1-\rho}[\gamma(r_{t-1} - p_{t-2}) + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\hat{\varepsilon}_{t-2}]) \\
&= p_{t-1} + \beta(r_t - p_{t-1} + \frac{\rho^2}{1-\rho}[\gamma(r_{t-1} - p_{t-2}) + \left(\rho + \gamma\frac{\rho^2}{1-\rho}\right)[\gamma(r_{t-2} - p_{t-3}) + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\hat{\varepsilon}_{t-3}]]) \\
&= p_{t-1} + \beta(r_t - p_{t-1} + \frac{\rho^2}{1-\rho}[\gamma(r_{t-1} - p_{t-2}) + \\
&\quad \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)[\gamma(r_{t-2} - p_{t-3}) + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\left[[\gamma(r_{t-3} - p_{t-4}) + \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\hat{\varepsilon}_{t-4}]\right]]])
\end{aligned}$$

Following the above expansion one can see that in the steady state the price evolves as

$$p_t = p_{t-1} + \beta\left(r_t - p_{t-1}\right) + \frac{\beta\gamma\rho^2}{1-\rho}\sum_{k=1}^{\infty}\left(\rho + \gamma\frac{\rho^2}{1-\rho}\right)^{k-1}\left(r_{t-k} - p_{t-k-1}\right)$$

This implies that

$$p_{t-1} = p_{t-2} + \beta\left(r_{t-1} - p_{t-2}\right) + \frac{\beta\gamma\rho^2}{1-\rho}\sum_{k=1}^{\infty}\left(\rho + \gamma\frac{\rho^2}{1-\rho}\right)^{k-1}\left(r_{t-1-k} - p_{t-1-k-1}\right)$$

or, equivalently, that

$$\frac{\beta\gamma\rho^2}{1-\rho}\sum_{k=1}^{\infty}\left(\rho + \gamma\frac{\rho^2}{1-\rho}\right)^{k-1}\left(r_{t-1-k} - p_{t-2-k}\right) = p_{t-1} - p_{t-2} - \beta\left(r_{t-1} - p_{t-2}\right).$$

Taken together these results imply that

$$p_t = p_{t-1} + \beta\left(r_t - p_{t-1}\right) + \left(\rho + \gamma\frac{\rho^2}{1-\rho}\right)\left(p_{t-1} - p_{t-2}\right) - \rho\beta(r_{t-1} - p_{t-2})$$

■

Proof of Equation 12. Since in the steady state the variance covariance matrix

$$\Sigma = \begin{pmatrix} Var_t(\theta_t) & Cov(\theta_t, \varepsilon_t) \\ Cov(\theta_t, \varepsilon_t) & Var_t(\varepsilon_t) \end{pmatrix} \equiv \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{pmatrix}$$

is independent of t, the following results follow:

$$\begin{aligned}
Var_{t-1}(\theta_t) &= Var_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t) = \psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 \\
Var_{t-1}(\varepsilon_t) &= Var_{t-1}(\rho\varepsilon_{t-1} + v_t) = \rho^2\psi_3 + \sigma_v^2 \\
Var_{t-1}(r_t) &= Var_{t-1}(\theta_t + \eta_t) = \psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2 \\
Cov_{t-1}(\theta_t, \varepsilon_t) &= Cov_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t, \rho\varepsilon_{t-1} + v_t) = \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \\
Cov_{t-1}(r_t, \theta_t) &= Cov_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t, \theta_{t-1} + \rho\varepsilon_{t-1} + v_t) = \psi_1 + \rho^2\psi_3 + \sigma_v^2 + 2\rho\psi_2 \\
Cov_{t-1}(r_t, \varepsilon_t) &= Cov_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t, \rho\varepsilon_{t-1} + v_t) = \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2
\end{aligned}$$

■

Proof of Equations 13,14,15. The variance covariance matrix of $\{\varepsilon_t, \theta_t\}$ is updated from period $t-1$ to period t, in the following way:

$$\begin{aligned}
\Sigma &= \begin{pmatrix} Var_{t-1}(\theta_t) & Cov_{t-1}(\theta_t, \varepsilon_t) \\ Cov_{t-1}(\theta_t, \varepsilon_t) & Var_{t-1}(\varepsilon_t) \end{pmatrix} - \frac{\begin{pmatrix} Cov_{t-1}(\theta_t, r_t) \\ Cov_{t-1}(\varepsilon_t, r_t) \end{pmatrix} \begin{pmatrix} Cov_{t-1}(r_t, \theta_t) & Cov_{t-1}(r_t, \varepsilon_t) \end{pmatrix}}{Var_{t-1}(r_t)}. \\
\Sigma &= \begin{pmatrix} \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \\ \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho^2\psi_3 + \sigma_v^2 \end{pmatrix} \\
&\quad - \frac{\begin{pmatrix} \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \end{pmatrix}^T \begin{pmatrix} \psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 & \rho\psi_2 + \rho^2\psi_3 + \sigma_v^2 \end{pmatrix}}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \begin{pmatrix} \sigma_\eta^2 \frac{\psi_1 + 2\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} & \sigma_\eta^2 \frac{\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\ \sigma_\eta^2 \frac{\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} & \frac{\psi_1\rho^2\psi_3 + \rho^2\psi_3\sigma_\eta^2 + \psi_1\sigma_v^2 + \sigma_v^2\sigma_\eta^2 - \rho^2\psi_2^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \end{pmatrix}
\end{aligned}$$

■

Proof of Equations 14 and 17.

$$\begin{aligned}
\beta &= \frac{Cov_{t-1}\left(r_t, \theta_t + \frac{\rho}{1-\rho}\varepsilon_t\right)}{Var_{t-1}\left(r_t\right)} \\
&= \frac{Cov_{t-1}\left(\theta_t + \eta_t, \theta_t + \frac{\rho}{1-\rho}\varepsilon_t\right)}{Var_{t-1}\left(r_t\right)} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \eta_t, \theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \frac{\rho}{1-\rho}(\rho\varepsilon_{t-1} + v_t)\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t, \theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \frac{\rho}{1-\rho}(\rho\varepsilon_{t-1} + v_t)\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t, \theta_{t-1} + \rho\varepsilon_{t-1} + v_t\frac{1}{1-\rho} + \frac{\rho^2}{1-\rho}\varepsilon_{t-1}\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1}, \theta_{t-1} + \rho\varepsilon_{t-1} + \frac{\rho^2}{1-\rho}\varepsilon_{t-1}\right) + \frac{\sigma_v^2}{1-\rho}}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1}, \theta_{t-1} + \frac{\rho}{1-\rho}\varepsilon_{t-1}\right) + \frac{\sigma_v^2}{1-\rho}}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{\psi_1 + \frac{2\rho-\rho^2}{1-\rho}\psi_2 + \frac{\rho^2}{1-\rho}\psi_3 + \frac{\sigma_v^2}{1-\rho}}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\gamma &= \frac{Cov_{t-1}\left(r_t, \varepsilon_t\right)}{Var_{t-1}\left(r_t\right)}. \\
&= \frac{Cov_{t-1}\left(r_t, \varepsilon_t\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \varepsilon_t + \eta, \varepsilon_t\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \eta, \rho\varepsilon_{t-1} + v_t\right)}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{Cov_{t-1}\left(\theta_{t-1} + \rho\varepsilon_{t-1}, \rho\varepsilon_{t-1}\right) + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2} \\
&= \frac{\rho\psi_2 + \rho^2\psi_3 + \sigma_v^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2}
\end{aligned}$$

■

Proof of Equation 19. Consider the marginal effect of r_t on p_t and subsequent prices.

Using the steady state equation for the price,

$$p_{t+2} = p_{t+1} + \beta \left(r_{t+2} - p_{t+1} \right) + \rho \left(1 + \gamma \frac{\rho}{1-\rho} \right) \left(p_{t+1} - p_t \right) - \rho\beta \left(r_{t+1} - p_t \right)$$

one can see that the marginal effect $\frac{\partial}{\partial r_t} p_t = \beta$. Similarly, the marginal effect of r_t on p_{t+1} can be obtained from

$$p_{t+1} = p_t + \beta \left(r_{t+1} - p_t \right) + \rho \left(1 + \gamma \frac{\rho}{1-\rho} \right) \left(p_t - p_{t-1} \right) - \rho\beta \left(r_t - p_{t-1} \right)$$

which gives $\frac{\partial p_{t+1}}{\partial r_t} = (1-\beta)\,\beta + \frac{\beta\gamma\rho^2}{1-\rho}$. In general, the marginal effect follows:

$$\alpha_{t+2} = a\alpha_{t+1} - b\alpha_t \tag{22}$$

where

$$\begin{aligned} a &= (1-\beta) + \rho \left(1 + \gamma \frac{\rho}{1-\rho} \right) \\ b &= \rho \left(1 + \gamma \frac{\rho}{1-\rho} - \beta \right) \end{aligned}$$

Furthermore, the boundary conditions are

$$\begin{aligned} \alpha_1 &= \beta \\ \alpha_2 &= d \end{aligned}$$

where

$$d = (1-\beta)\,\beta + \frac{\beta\gamma\rho^2}{1-\rho}.$$

The solution of the difference equation (22) can be obtained from Mathematica (see "steadystate.nb") giving:

$$\alpha_t = \phi_1 m_1^t + \phi_2 m_2^t$$

where $\phi_1 = \frac{-2b\beta + (a+\sqrt{a^2-4b})(-d+a\beta)}{2b\sqrt{a^2-4b}}$ and $\phi_2 = \frac{2b\beta + (a-\sqrt{a^2-4b})(-d+a\beta)}{2b\sqrt{a^2-4b}}$ and $m_1 = \frac{a-\sqrt{a^2-4b}}{2}$ and $m_2 = \frac{a+\sqrt{a^2-4b}}{2}$. ■

Proof of Lemma 2.

$$\begin{aligned}
Var_t(\theta_t + \frac{\rho}{1-\rho}\varepsilon_t) &= Var_{t-1}(\theta_t + \frac{\rho}{1-\rho}\varepsilon_t) - \beta^2 Var_{t-1}(r_t) \\
&= Var_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \frac{\rho}{1-\rho}\varepsilon_t) - \beta^2 Var_{t-1}(r_t) \\
&= Var_{t-1}(\theta_{t-1} + \rho\varepsilon_{t-1} + v_t + \frac{\rho^2}{1-\rho}\varepsilon_{t-1} + \frac{\rho}{1-\rho}v_t) - \beta^2 Var_{t-1}(r_t) \\
&= Var_{t-1}(\theta_{t-1} + \frac{\rho}{1-\rho}\varepsilon_{t-1} + \frac{1}{1-\rho}v_t) - \beta^2 Var_{t-1}(r_t) \\
&= Var_{t-1}(\theta_{t-1} + \frac{\rho}{1-\rho}\varepsilon_{t-1}) + \frac{\sigma_v^2}{(1-\rho)^2} - \beta^2 Var_{t-1}(r_t) \\
&= \psi_1 + 2\left(\frac{\rho}{1-\rho}\right)\psi_2 + \left(\frac{\rho}{1-\rho}\right)^2\psi_3 + \frac{\sigma_v^2}{(1-\rho)^2} - \frac{\left(\psi_1 + \frac{2\rho-\rho^2}{1-\rho}\psi_2 + \frac{\rho^2}{1-\rho}\psi_3 + \frac{\sigma_v^2}{1-\rho}\right)^2}{\psi_1 + \rho^2\psi_3 + 2\rho\psi_2 + \sigma_v^2 + \sigma_\eta^2}
\end{aligned}$$

■

6 Appendix B

Variable	Mean	Median	Std. Dev.	Min	Max
p_t	1.040	.015	.268	.306	2.796
r_t	-.171	-.005	.6618	-15.068	.901
$p_{t-1} - p_{t-2}$	-.023	.015	.278	-2.238	.640
$r_t - p_{t-1}$	-1.171	-1.004	.662	-16.068	-.099
$r_{t-1} - p_{t-2}$	-1.197	-1.023	.712	-16.005	-.100
Δr_t	.003	.005	.096	-.970	1.076
Δr_{t-1}	.003	.005	.0938	-1.012	1.060
$\Delta^2 r_t$	-.001	-0.001	.133	-1.797	1.997
$\Delta^2 r_{t-1}$	-.003	-0.001	.141	-1.671	1.608

Table 4: Descriptive statistics for quarterly data ($N = 382,389$). All variables are scaled by p_{t-1}.

	p_t	r_t	$p_{t-1}-p_{t-2}$	$r_{t-1}-p_{t-2}$	Δr_t	Δr_{t-1}	$\Delta^2 r_t$	$\Delta^2 r_{t-1}$
p_t		-.0552	.0087	-.0831	.1837	.0809	.0784	.0400
r_t	-.0751		-.0448	.7182	.1015	.1142	.0023	.0018
$p_{t-1}-p_{t-2}$	-.0028	.0074		.5365	.1234	.1744	-.0303	.0751
$r_{t-1}-p_{t-2}$	-.0922	.3779	.9190		.0105	.1832	-.1189	.0475
Δr_t	.1585	.1411	.0968	.0101		.0458	.6278	.0462
Δr_{t-1}	.0515	.1578	.0967	.1484	.0225		-.6195	.6140
$\Delta^2 r_t$	.0783	-.0094	.0017	-.0975	.7078	-.6903		-.4037
$\Delta^2 r_{t-1}$	.0306	.0596	.0142	.0317	.0351	.6316	-.4209	

Table 5: Pearson (below the diagonal) and Spearman (above the diagonal) for quarterly data ($N = 382,389$). All variables are scaled by p_{t-1}.

Variable	Mean	Median	Std. Dev.	Min	Max
p_t	1.167	1.062	.624	.139	7.573
r_t	.677	.581	.456	.025	4.044
$p_{t-1}-p_{t-2}$	-.058	.069	.592	-5.429	.878
r_t-p_{t-1}	-.323	-.419	.455	-.975	3.043
$r_{t-1}-p_{t-2}$	-.416	-.367	.603	-5.195	2.194
Δr_t	.034	.036	.174	-1.280	1.295
Δr_{t-1}	.046	.039	.167	-1.035	1.323
$\Delta^2 r_t$	-.012	-.006	.218	-1.478	1.417
$\Delta^2 r_{t-1}$	-.010	-.005	.233	-1.995	1.508

Table 6: Descriptive statistics for annual data ($N = 65,447$). All variables are scaled by p_{t-1}.

	p_t	r_t	$p_{t-1}-p_{t-2}$	$r_{t-1}-p_{t-2}$	Δr_t	Δr_{t-1}	$\Delta^2 r_t$	$\Delta^2 r_{t-1}$
p_t		.2555	-.0161	.0374	.4905	.0187	.3759	.0243
r_t	.2441		-.2631	.4730	.2613	.0158	.1779	-.04460
$p_{t-1}-p_{t-2}$	-.1064	-.2842		.5873	.1929	.4784	-.1971	.3675
$r_{t-1}-p_{t-2}$	-.0384	.4194	.7109		.1146	.3670	-.1973	.2799
Δr_t	.4094	.1982	.1962	.0533		.2461	.5638	.0795
Δr_{t-1}	-.0279	-.0470	.3544	.2596	.1837		-.5224	.5336
$\Delta^2 r_t$	.3484	.1943	-.1147	-.1562	.6582	-.6191		-.3502
$\Delta^2 r_{t-1}$	.0128	-.0383	.2717	.2175	.0707	.5625	-.3743	

Table 7: Pearson (below the diagonal) and Spearman (above the diagonal) for annual data ($N = 65,447$). All variables are scaled by p_{t-1}.

Table 8: This table displays the results for the regression of returns on prior book values and market values of equity using quarterly data. The Specifically, this table summarizes the regression results for

$$\frac{p_t}{p_{t-1}} = b_0 + b_1 \left(1 - \frac{p_{t-2}}{p_{t-1}}\right) + b_2 \left(\frac{r_t}{p_{t-1}} - 1\right) + b_3 \left(\frac{r_{t-1}}{p_{t-1}} - \frac{p_{t-2}}{p_{t-1}}\right) + \tilde{\zeta}$$

where r_t is the book value of equity and p_t is the market value of equity. The book value of equity is the value of common shareholders' equity at the end of the quarter (fundq:ceqq) and the market value of equity is calculated as the product of the closing price at the end of the quarter (fundq:priccq) and the numbers of common shares outstanding (fundq:cshoq). Standard errors are clustered by fiscal year (fundq:fyearq) and firm (fundq:gvkey2). The regression is pooled for each industry of the Fama-French 12-industry classification.

	(1) ConsNonDur	(2) ConsDur	(3) Manuf	(4) Energy	(5) Chems	(6) BusEq	(7) Telecom	(8) Utilities	(9) Shops	(10) Health	(11) Finance	(12) Other
b_1	0.535**	0.492**	0.500**	0.343**	0.416**	0.539**	0.475**	0.350**	0.498**	0.590**	0.442**	0.414**
	(0.049)	(0.073)	(0.058)	(0.051)	(0.075)	(0.060)	(0.058)	(0.054)	(0.032)	(0.055)	(0.041)	(0.039)
b_2	0.524**	0.480**	0.494**	0.290**	0.408**	0.513**	0.410**	0.331**	0.478**	0.567**	0.415**	0.413**
	(0.036)	(0.064)	(0.048)	(0.034)	(0.065)	(0.048)	(0.043)	(0.062)	(0.028)	(0.050)	(0.033)	(0.027)
b_3	-0.568**	-0.540**	-0.540**	-0.302**	-0.448**	-0.565**	-0.460**	-0.343**	-0.527**	-0.633**	-0.440**	-0.448**
	(0.039)	(0.064)	(0.049)	(0.036)	(0.065)	(0.051)	(0.042)	(0.065)	(0.031)	(0.053)	(0.035)	(0.028)
b_0	0.976**	0.964**	0.981**	1.038**	0.984**	0.988**	0.987**	1.011**	0.973**	0.971**	1.002**	0.994**
	(0.011)	(0.011)	(0.015)	(0.016)	(0.018)	(0.018)	(0.019)	(0.011)	(0.014)	(0.014)	(0.013)	(0.013)
Obs.	19,545	9,024	38,114	15,074	7,944	67,401	9,288	11,229	36,768	37,605	81,731	48,476
R^2	0.042	0.044	0.036	0.030	0.023	0.032	0.042	0.018	0.041	0.036	0.055	0.036

Standard errors in parentheses
** $p<0.01$, * $p<0.05$, + $p<0.1$

Table 9: This table displays the results for the regression of returns on prior book values and market values of equity using annual data. The Specifically, this table summarizes the regression results for

$$\frac{p_t}{p_{t-1}} = b_0 + b_1\left(1 - \frac{p_{t-2}}{p_{t-1}}\right) + b_2\left(\frac{r_t}{p_{t-1}} - 1\right) + b_3\left(\frac{r_{t-1}}{p_{t-1}} - \frac{p_{t-2}}{p_{t-1}}\right) + \tilde{\zeta}$$

where r_t is the book value of equity and p_t is the market value of equity. The book value of equity is the value of common shareholders' equity at the end of the fiscal year (funda:ceqq) and the market value of equity is calculated as the product of the closing price at the end of the quarter (funda:priccq) and the numbers of common shares outstanding (funda:cshoq). Standard errors are clustered by fiscal year (funda:fyearq) and firm (funda:gvkey2). The regression is pooled for each industry of the Fama-French 12-industry classification.

	(1) ConsNonDur	(2) ConsDur	(3) Manuf	(4) Energy	(5) Chems	(6) BusEq	(7) Telecom	(8) Utilities	(9) Shops	(10) Health	(11) Finance	(12) Other
b_1	1.186**	1.170**	1.299**	1.423**	1.162**	1.836**	0.784**	1.051**	1.239**	1.984**	1.151**	1.055**
	(0.080)	(0.104)	(0.082)	(0.198)	(0.171)	(0.160)	(0.173)	(0.105)	(0.070)	(0.139)	(0.095)	(0.074)
b_2	1.391**	1.494**	1.620**	1.876**	1.385**	2.420**	1.172**	1.340**	1.556**	2.478**	1.280**	1.370**
	(0.090)	(0.116)	(0.080)	(0.194)	(0.165)	(0.149)	(0.136)	(0.063)	(0.101)	(0.161)	(0.066)	(0.052)
b_3	-1.262**	-1.333**	-1.425**	-1.540**	-1.246**	-2.043**	-0.867**	-1.176**	-1.328**	-2.168**	-1.137**	-1.167**
	(0.081)	(0.125)	(0.083)	(0.151)	(0.161)	(0.176)	(0.160)	(0.082)	(0.080)	(0.169)	(0.051)	(0.057)
b_0	1.115**	1.109**	1.142**	1.279**	1.134**	1.326**	1.270**	1.109**	1.152**	1.323**	1.099**	1.174**
	(0.031)	(0.039)	(0.038)	(0.062)	(0.043)	(0.058)	(0.072)	(0.032)	(0.043)	(0.053)	(0.042)	(0.044)
Obs.	3,620	1,673	7,048	2,655	1,461	11,199	1,615	2,222	6,192	6,098	13,696	7,968
R^2	0.212	0.242	0.237	0.291	0.165	0.256	0.121	0.217	0.228	0.242	0.282	0.216

Standard errors in parentheses
** p<0.01, * p<0.05, + p<0.1

Table 10: GMM estimates using quarterly data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
Industry 1: Consumer Non-Durables ($N = 19,545$)						
ρ	1.084	.009	117.392	.000	1.066	1.102
γ	.039	.005	7.352	.000	.029	.050
β	.524	.029	18.234	.000	.468	.580
σ_v^2	.003	.000	7.871	.000	.002	.003
σ_η^2	.001	.000	15.498	.000	.001	.002
intercept	.976	.005	201.468	.000	.967	.986
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.001	.000	-3.522	.000	-.002	-.001
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.002	.001	3.017	.003	.001	.003
Industry 2: Consumer Durables ($N = 9,024$)						
ρ	1.124	.018	63.984	.000	1.089	1.158
γ	.062	.011	5.566	.000	.040	.084
β	.480	.042	11.442	.000	.398	.563
σ_v^2	.004	.001	5.730	.000	.002	.005
σ_η^2	.001	.000	11.540	.000	.001	.002
intercept	.964	.007	132.781	.000	.949	.978
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.002	.000	-3.903	.000	-.003	-.001
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.003	.001	3.260	.001	.001	.005
Industry 3: Manufacturing ($N = 38,114$)						
ρ	1.093	.008	135.380	.000	1.078	1.109
γ	.046	.005	9.436	.000	.037	.056
β	.494	.023	21.949	.000	.450	.538
σ_v^2	.003	.000	1.157	.000	.002	.003
σ_η^2	.001	.000	24.007	.000	.001	.002
intercept	.981	.004	262.980	.000	.974	.989
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.001	.000	-5.385	.000	-.002	-.001
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.001	.000	1.785	.074	-.000	.002
Industry 4: Energy ($N = 15,074$)						
ρ	1.041	.007	158.723	.000	1.028	1.054
γ	.027	.005	5.815	.000	.018	.035
β	.290	.024	12.233	.000	.244	.337
σ_v^2	.009	.002	4.020	.000	.004	.013
σ_η^2	.006	.000	12.489	.000	.005	.007
intercept	1.038	.003	353.925	.000	1.032	1.044

$E[\Delta r_t - \rho \Delta r_{t-1}]$	.000	.001	.028	.978	-.002	.002
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.006	.001	4.949	.000	.004	.009

Table 10 (cont.): GMM estimates using quarterly data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
			Industry 5: Chemicals ($N = 7,944$)			
ρ	1.097	.022	49.654	.000	1.054	1.140
γ	.055	.015	3.548	.000	.025	.085
β	.408	.059	6.932	.000	.293	.523
σ_v^2	.001	.000	3.056	.002	.001	.002
σ_η^2	.001	.000	9.965	.000	.001	.001
intercept	.984	.008	13.611	.000	.970	.999
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.001	.000	-1.400	.161	-.001	.000
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.002	.001	3.172	.002	.001	.004
			Industry 6: Business Equipment ($N = 67,401$)			
ρ	1.103	.007	166.178	.000	1.090	1.116
γ	.048	.004	12.049	.000	.040	.056
β	.513	.019	27.395	.000	.476	.549
σ_v^2	.004	.000	13.053	.000	.004	.005
σ_η^2	.002	.000	3.018	.000	.002	.002
intercept	.988	.003	301.006	.000	.982	.995
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.000	.000	-1.041	.298	-.001	.000
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.002	.000	6.018	.000	.002	.003
			Industry 7: Telecom ($N = 9,288$)			
ρ	1.120	.025	44.053	.000	1.070	1.170
γ	.062	.015	4.226	.000	.033	.090
β	.410	.043	9.562	.000	.326	.495
σ_v^2	.001	.002	.311	.756	-.003	.004
σ_η^2	.005	.000	1.986	.000	.004	.006
intercept	.987	.010	103.280	.000	.969	1.006
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.002	.001	-1.949	.051	-.003	.000
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.003	.001	2.095	.036	.000	.006
			Industry 8:Utilities ($N = 11,229$)			
ρ	1.036	.008	133.378	.000	1.021	1.051
γ	.023	.005	4.230	.000	.012	.034
β	.331	.038	8.778	.000	.257	.405
σ_v^2	.001	.000	4.409	.000	.000	.001
σ_η^2	.001	.000	12.088	.000	.000	.001
intercept	1.011	.003	339.475	.000	1.005	1.017
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.000	.000	-1.537	.124	-.001	.000

$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	.000	.000	1.211	.226	-.000	.001

Table 10 (cont.): GMM estimates using quarterly data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
			Industry 9: Wholesale and Retail ($N = 36,768$)			
ρ	1.103	.007	147.268	.000	1.088	1.118
γ	.051	.005	1.854	.000	.042	.061
β	.478	.021	22.973	.000	.437	.518
σ_v^2	.004	.000	9.928	.000	.003	.004
σ_η^2	.002	.000	22.216	.000	.002	.002
intercept	.973	.003	291.777	.000	.967	.980
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.002	.000	-7.785	.000	-.003	-.002
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.002	.001	4.070	.000	.001	.003
			Industry 10: Healthcare ($N = 37,605$)			
ρ	1.116	.009	119.179	.000	1.098	1.135
γ	.049	.006	8.555	.000	.038	.060
β	.567	.028	19.942	.000	.512	.623
σ_v^2	.002	.000	6.523	.000	.001	.003
σ_η^2	.002	.000	26.444	.000	.002	.002
intercept	.971	.005	205.042	.000	.962	.981
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.001	.000	-4.725	.000	-.002	-.001
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.003	.000	6.170	.000	.002	.004
			Industry 11: Finance ($N = 81,731$)			
ρ	1.060	.004	263.695	.000	1.052	1.068
γ	.033	.002	13.184	.000	.028	.038
β	.415	.012	34.928	.000	.392	.439
σ_v^2	.003	.000	11.817	.000	.003	.004
σ_η^2	.002	.000	31.882	.000	.002	.002
intercept	1.002	.002	588.687	.000	.999	1.006
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.001	.000	-7.089	.000	-.002	-.001
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.002	.000	6.586	.000	.002	.003
			Industry 12: Other ($N = 48,476$)			
ρ	1.085	.008	139.667	.000	1.069	1.100
γ	.048	.005	1.074	.000	.039	.058
β	.413	.017	24.400	.000	.380	.446
σ_v^2	.005	.001	9.520	.000	.004	.006
σ_η^2	.003	.000	26.928	.000	.003	.003
intercept	.994	.003	298.286	.000	.988	1.001
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.002	.000	-5.126	.000	-.002	-.001

$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.004	.001	6.413	.000	.003	.005

Table 11: GMM estimates using annual data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
		Industry 1: Consumer Non-Durables ($N = 3,620$)				
ρ	.907	.013	69.653	.000	.881	.932
γ	.032	.008	3.815	.000	.015	.048
β	1.391	.072	19.346	.000	1.250	1.532
σ_v^2	.016	.002	7.672	.000	.012	.020
σ_η^2	.003	.000	8.934	.000	.003	.004
intercept	1.115	.010	108.520	.000	1.095	1.136
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.008	.002	-4.198	.000	-.012	-.004
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.004	.003	-1.586	.113	-.009	.001
		Industry 2: Consumer Durables ($N = 1,673$)				
ρ	.892	.030	29.419	.000	.833	.951
γ	.038	.018	2.113	.035	.003	.073
β	1.494	.103	14.486	.000	1.292	1.696
σ_v^2	.019	.003	5.703	.000	.012	.025
σ_η^2	.003	.001	5.358	.000	.002	.004
intercept	1.109	.022	51.310	.000	1.066	1.151
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.009	.003	-3.213	.001	-.015	-.004
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.001	.004	-.372	.710	-.009	.006
		Industry 3: Manufacturing ($N = 7,048$)				
ρ	.880	.012	72.945	.000	.856	.903
γ	.065	.010	6.268	.000	.045	.086
β	1.620	.059	27.558	.000	1.505	1.735
σ_v^2	.012	.001	8.726	.000	.009	.014
σ_η^2	.004	.000	15.307	.000	.004	.005
intercept	1.142	.009	128.381	.000	1.125	1.160
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.004	.001	-3.062	.002	-.007	-.001
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.003	.002	-1.626	.104	-.007	.001
		Industry 4: Energy ($N = 2,655$)				
ρ	.821	.026	31.003	.000	.769	.873
γ	.160	.035	4.548	.000	.091	.229
β	1.876	.101	18.500	.000	1.677	2.075
σ_v^2	.009	.004	2.608	.009	.002	.016
σ_η^2	.011	.001	1.433	.000	.009	.013
intercept	1.279	.024	52.511	.000	1.231	1.327

$E[\Delta r_t - \rho \Delta r_{t-1}]$	.007	.004	1.952	.051	-.000	.014
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.001	.004	-.154	.878	-.009	.007

Table 11 (cont.): GMM estimates using annual data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
			Industry 5: Chemicals ($N = 1,461$)			
ρ	.900	.030	3.325	.000	.842	.958
γ	.032	.020	1.642	.101	-.006	.071
β	1.385	.166	8.328	.000	1.059	1.711
σ_v^2	.008	.002	3.123	.002	.003	.012
σ_η^2	.003	.000	6.389	.000	.002	.004
intercept	1.134	.024	47.796	.000	1.087	1.180
$E[\Delta r_t - \rho\Delta r_{t-1}]$	.000	.002	.190	.849	-.004	.005
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.008	.004	2.298	.022	.001	.015
			Industry 6: Business Equipment ($N = 11,199$)			
ρ	.844	.011	74.560	.000	.822	.866
γ	.217	.021	1.335	.000	.176	.259
β	2.420	.065	37.002	.000	2.292	2.549
σ_v^2	.013	.001	1.920	.000	.011	.015
σ_η^2	.006	.000	2.908	.000	.005	.006
intercept	1.326	.016	81.214	.000	1.294	1.358
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.002	.001	-2.090	.037	-.004	-.000
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	.001	.002	.397	.691	-.002	.004
			Industry 7: Telecom ($N = 1,615$)			
ρ	.740	.060	12.393	.000	.623	.857
γ	.021	.039	.536	.592	-.056	.098
β	1.172	.117	1.006	.000	.942	1.401
σ_v^2	.014	.004	3.329	.001	.006	.022
σ_η^2	.007	.001	8.916	.000	.006	.009
intercept	1.270	.037	34.332	.000	1.197	1.342
$E[\Delta r_t - \rho\Delta r_{t-1}]$	-.009	.003	-2.618	.009	-.015	-.002
$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	-.007	.005	-1.537	.124	-.016	.002
			Industry 8:Utilities ($N = 2,222$)			
ρ	.878	.023	37.384	.000	.832	.924
γ	.028	.011	2.423	.015	.005	.050
β	1.340	.079	16.860	.000	1.184	1.496
σ_v^2	.002	.001	1.962	.050	.000	.004
σ_η^2	.002	.000	8.126	.000	.002	.003
intercept	1.109	.012	88.983	.000	1.085	1.133
$E[\Delta r_t - \rho\Delta r_{t-1}]$	.003	.002	1.692	.091	-.000	.006

$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	-.003	.002	-1.847	.065	-.006	.000

Table 11 (cont.): GMM estimates using annual data for the following model parameters: persistence of true earnings ρ, earnings response coefficient β, sensitivity of investors' beliefs about current period true earnings to reported earnings (or equity) γ, variability of earnings shocks σ_v^2, and uncertainty added by earnings manipulation σ_η^2.

Parameter	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
			Industry 9: Wholesale and Retail ($N = 6,192$)			
ρ	.854	.013	65.861	.000	.828	.879
γ	.077	.013	6.055	.000	.052	.102
β	1.556	.067	23.280	.000	1.425	1.687
σ_v^2	.018	.002	9.187	.000	.014	.022
σ_η^2	.004	.000	1.708	.000	.003	.004
intercept	1.152	.010	117.929	.000	1.133	1.171
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.009	.002	-5.631	.000	-.013	-.006
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.005	.002	-2.222	.026	-.009	-.001
			Industry 10: Healthcare ($N = 6,098$)			
ρ	.875	.015	59.803	.000	.846	.904
γ	.181	.027	6.662	.000	.128	.234
β	2.478	.102	24.278	.000	2.278	2.678
σ_v^2	.006	.002	3.595	.000	.003	.010
σ_η^2	.007	.000	16.431	.000	.006	.008
intercept	1.323	.025	53.336	.000	1.275	1.372
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.004	.002	-2.745	.006	-.007	-.001
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.001	.002	-.276	.783	-.005	.004
			Industry 11: Finance ($N = 13,696$)			
ρ	.888	.009	103.002	.000	.871	.905
γ	.037	.005	8.118	.000	.028	.046
β	1.280	.029	44.778	.000	1.224	1.336
σ_v^2	.017	.001	12.958	.000	.015	.020
σ_η^2	.005	.000	2.978	.000	.004	.005
intercept	1.099	.004	245.254	.000	1.090	1.108
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.005	.001	-4.839	.000	-.008	-.003
$E[\Delta^2 r_t - \rho \Delta^2 r_{t-1}]$	-.005	.001	-3.799	.000	-.008	-.003
			Industry 12: Other ($N = 7,968$)			
ρ	.852	.012	73.638	.000	.829	.874
γ	.042	.010	4.372	.000	.023	.060
β	1.370	.050	27.527	.000	1.272	1.467
σ_v^2	.020	.002	8.211	.000	.015	.024
σ_η^2	.007	.000	15.290	.000	.006	.008
intercept	1.174	.008	139.163	.000	1.158	1.191
$E[\Delta r_t - \rho \Delta r_{t-1}]$	-.013	.002	-7.332	.000	-.017	-.010

$E[\Delta^2 r_t - \rho\Delta^2 r_{t-1}]$	-.008	.002	-3.429	.001	-.013	-.004

Table 12: Ratio of of the variance of the noise added by earnings manipulation, σ_η^2, to the variance of the earnings innovation, σ_v^2, for quarterly data.

	σ_v^2	σ_η^2	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
Industry 1: Consumer non-Durables	.003	.001	.543	.083	6.533	.000	.380	.705
Industry 2: Consumer Durables	.004	.001	.402	.087	4.618	.000	.231	.572
Industry 3: Manufacturing	.003	.001	.548	.064	8.602	.000	.423	.673
Industry 4: Energy	.009	.006	.716	.210	3.407	.001	.304	1.129
Industry 5: Chemicals	.001	.001	.781	.294	2.659	.008	.205	1.357
Industry 6: Business Equipment	.004	.002	.540	.050	1.877	.000	.443	.638
Industry 7: Telecom	.001	.005	9.585	31.312	.306	.760	-51.785	7.955
Industry 8: Utilities	.001	.001	.714	.188	3.797	.000	.345	1.082
Industry 9: Wholesale and Retail	.004	.002	.537	.065	8.258	.000	.410	.665
Industry 10: Healthcare	.002	.002	1.044	.179	5.822	.000	.692	1.395
Industry 11: Finance	.003	.002	.652	.064	1.187	.000	.527	.777
Industry 12: Other	.005	.003	.639	.077	8.277	.000	.488	.791
Mean			1.392					
Median			.646					

References

Acharya, Viral V., Peter DeMarzo, and Ilan Kremer, "Endogenous Information Flows and the Clustering of Announcements," *American Economic Review*, September 2011, *101* (7), 2955–79.

Beyer, Anne, Ilan Guttman, and Ivan Marinovic, "Optimal Contracts with Performance Manipulation," Working Paper, Stanford University 2012.

Collins, Daniel W. and S.P. Kothari, "An analysis of intertemporal and cross-sectional determinants of earnings response coefficients," *Journal of Accounting and Economics*, 1989, *11* (23), 143 – 181.

Dechow, Patricia, Weili Ge, and Catherine Schrand, "Understanding earnings quality: A review of the proxies, their determinants and their consequences," *Journal of Accounting and Economics*, 2010, *50* (23), 344 – 401.

Demski, Joel S., "The General Impossibility of Normative Accounting Standards," *The Accounting Review*, 1973, *48* (4), pp. 718–723.

Dichev, Ilia D., John Graham, Campbell R. Harvey, and Shiva Rajgopal, "Earnings Quality: Evidence from the Field," *Journal of Accounting and Economics*, 2013, (0), –.

Dye, Ronald A. and Sri S. Sridhar, "A positive theory of flexibility in accounting standards," *Journal of Accounting and Economics*, 2008, *46* (23), 312 – 333.

Ewert, Ralf and Alfred Wagenhofer, "Earnings Quality Metrics and What They Measure," Working Paper, University of Graz 2011.

Fischer, Paul E. and Robert E. Verrecchia, "Reporting Bias," *The Accounting Review*, 2000, *75* (2), pp. 229–245.

Gerakos, Joseph and Andrei Kovrijnykh, "Performance Shocks and Misreporting," Working Paper, University of Chicago 2013.

Guttman, Ilan, Ohad Kadan, and Eugene Kandel, "A Rational Expectations Theory of Kinks in Financial Reporting," *The Accounting Review*, 2006, *81* (4), pp. 811–848.

Kirschenheiter, Michael and Nahum D. Melumad, "Can "Big Bath" and Earnings Smoothing Co-Exist as Equilibrium Financial Reporting Strategies?," *Journal of Accounting Research*, 2002, *40* (3), pp. 761–796.

Sankar, Mandira Roy and K. R. Subramanyam, "Reporting Discretion and Private Information Communication through Earnings," *Journal of Accounting Research*, 2001, *39* (2), pp. 365–386.

Stein, Jeremy C., "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *The Quarterly Journal of Economics*, 1989, *104* (4), 655–669.

Zakolyukina, Anastasia A., "Measuring Intentional Manipulation: A Structural Approach," Working Paper, University of Chicago 2012.

Table 13: Ratio of of the variance of the noise added by earnings manipulation, σ_η^2, to the variance of the earnings innovation, σ_v^2, for annual data.

	σ_v^2	σ_η^2	*Estimate*	*Std. Dev.*	*z-Statistic*	*p-Value*	95% *Conf. Interval*	
Industry 1: Consumer non-Durables	.016	.003	.220	.044	4.995	.000	.134	.306
Industry 2: Consumer Durables	.019	.003	.162	.050	3.275	.001	.065	.260
Industry 3: Manufacturing	.012	.004	.365	.055	6.626	.000	.257	.472
Industry 4: Energy	.009	.011	1.135	.502	2.260	.024	.151	2.120
Industry 5: Chemicals	.008	.003	.367	.154	2.387	.017	.066	.669
Industry 6: Business Equipment	.013	.006	.443	.054	8.236	.000	.337	.548
Industry 7: Telecom	.014	.007	.530	.191	2.780	.005	.156	.904
Industry 8: Utilities	.002	.002	1.213	.712	1.703	.089	-.183	2.610
Industry 9: Wholesale and Retail	.018	.004	.209	.037	5.614	.000	.136	.281
Industry 10: Healthcare	.006	.007	1.077	.338	3.183	.001	.414	1.740
Industry 11: Finance	.017	.005	.282	.030	9.547	.000	.224	.340
Industry 12: Other	.020	.007	.362	.061	5.950	.000	.242	.481
Mean			.530					
Median			.366					