Competition and Transparency in Financial Markets[*]

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December 28, 2010

Abstract

Is competition sufficient to induce transparency in financial markets? We examine this question taking into consideration that competition in financial markets frequently resembles a tournament, where superior relative performance and greater visibility are rewarded with convex payoffs. We show under fairly general conditions (i.e., model variations) that higher competition for this remuneration often makes discretionary disclosure less likely. In the limit when the market is perfectly competitive, transparency is minimized. We show that this effect may decrease efficiency and per capita welfare, particularly when it is optimal to screen all market participants to determine whether they should be allocated a scarce good. Our analysis implies, then, that competition might be unreliable as a driver of transparency and efficiency in financial markets, especially in settings where tournament-style remuneration takes place.

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1   Introduction

Is competition sufficient to induce transparency in financial markets or is regulation needed to achieve this? This question often provokes visceral ideology and hot debate. Many economists believe that tightening competition should drive market participants to give up small advantages, which includes their private information. Others are more skeptical that the Invisible Hand can enforce truthful disclosure and increase market transparency; instead they support interventions aimed at protecting consumers and investors in the market.

An important, but overlooked, aspect of this debate rests on the fact that competition in financial markets frequently resembles a tournament, where superior relative performance is rewarded with convex payoffs.[1] Indeed, in many settings, rankings drive remuneration (i.e., order statistics matter). For example, mutual funds that advertise better past performance or achieve greater visibility experience convex investor flows (e.g., Brown, Harlow, and Starks, 1996; Berk and Green, 2004; Del Guercio and Tkac, 2008).[2] Firms with higher status attract superior human capital, especially when labor is scarce (e.g., Gatewood, Gowan, and Lautenschlager, 1993).[3] CEO's who receive higher public praise are rewarded with positively skewed payoffs (e.g., Gibbs, 1994; Malmendier and Tate, 2009). Firms that win investor relations awards enjoy more analyst coverage, higher abnormal returns, and a lower cost of capital (e.g., Lang and Lundholm, 1996; Agarwal, Liao, Nash, and Taffler, 2010). Such convexity is also likely to exist when other scarce resources are allocated: supplies and supplier credit, venture capital (e.g., Hsu, 2004), and investor attention (e.g., Hendricks and Singhal, 1996).

How, then, does tournament-like competition for such prizes affect each participants's incentives to reveal private information to the public? The answer to this question turns out to be non-obvious. Though it might seem intuitive that higher competition should increase incentives to reveal private information, we find that this is often not the case. Instead, since greater competition makes it harder to access such remuneration, firms have greater incentives to hoard private information.

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[1]As first pointed out by Rosen (1981), convex payoffs mean that small differences in performance at the high end of the spectrum become magnified in larger earnings differences or returns to effort.

[2]Capon, Fitzsimons, and Prince (1996) and Sirri and Tufano (1998) show that past performance is the crucial input in investors' choice of mutual fund. Brown, Harlow, and Starks (1996) are credited as being the first to point out the tournament-nature of mutual fund markets and the effects this has on managerial incentives. See Berk and Green (2004) for a theoretical model of convex performance incentives in mutual fund markets. See Gallaher, Kaniel, and Starks (2005), Gualtieri and Petrella (2005), Del Guercio and Tkac (2008), and Starks and Yates (2008) for the affects of visibility and reputation on mutual fund flows. See Nanda, Wang, and Zheng (2004) and Gaspar, Massa and Matos (2006) for the effect that past returns and visibility have on flows to other offerings in fund families.

[3]See also Chauvin and Guthrie (1994) and Turban and Greening (1996).

In fact, when there is perfect competition (i.e., the number of firms tends to infinity), firms act as though payoffs are not convex whatsoever, even though prizes are awarded are ex post. These effects have important welfare implications, and we show that in some reasonable cases, market efficiency drops as a result of competition.

Based on this, we add an important dimension to the aforementioned debate: policy makers need to carefully consider the type of competition that takes place in markets before deciding whether regulation is necessary. Moreover, competition should *not* be viewed as a panacea to assure information disclosure and self-regulation by participants in financial markets, especially in settings where tournament-style remuneration takes place.

Summarizing our base model makes it easy to appreciate the intuition for this result. We build on the model of Dye (1985), where incomplete disclosure results from investors' uncertainty as to whether or not management possesses relevant information. In our variant, a finite number of firms compete in the market. All firms experience a random shock that changes their fundamental value. Each firm may or may not observe the precise value of their shock. Firms that make an observation simultaneously choose whether to announce it publicly, while firms with no new information have nothing to reveal. The firm with the best announcement gets a fixed prize from the market, which represents the rank-based convex remuneration previously described.

In the symmetric equilibrium of the game, each firm with new information applies a threshold in deciding whether or not to reveal its news. If the observed shock value is above this threshold, the firm announces it and competes for the prize. If the observed shock is lower, however, the firm conceals its information. The presence of uninformed firms lends plausible deniability to informed firms wishing to conceal a bad observation. Rational investors use Bayesian learning to adjust the market price of firms that do not release any news.

Because the probability of winning the prize drops when more firms compete, the benefit of making announcements decreases with competition. Therefore, increasing competition leads to decreased information revelation and lower market transparency. In the limit, when the market is perfectly competitive, transparency is minimized because the individual probability of winning the prize goes to zero.

We explore the welfare consequences of this effect by considering that a market participant optimally offers the prize to improve their ability to screen potential business partners and allocate scarce goods. We show that when it is only necessary to identify the best counterparty in the market, welfare rises with competition, despite the incentives that some firms have to conceal private information. However, when it is efficient to screen all potential business partners, competition

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may adversely affect efficiency and per capita welfare. Once the number of firms increases past a modest threshold, the market becomes more inefficient. In the limit under perfect competition, the effectiveness of giving a prize disappears and per capita welfare is minimized.

We complete our analysis by considering several other prize structures and model variations to evaluate the robustness of our results: progressive reward systems (i.e., prizes are awarded to runners-up), prizes that change in size as a result of competition, prizes awarded based on percentile, sequential disclosure, and product market competition. Perfect competition leads to minimal market transparency in all of these variations except one: when the prize grows exponentially with competition ad infinitum. Given that this is rather unlikely to occur in reality, we view this result with considerable generality.

The most direct evidence that our theory is empirically plausible is Cheng (2010), who shows that as competition rises, firms have an increased tendency to participate in earnings manipulation. Since executives have career concerns and compete in the labor market (i.e., for convex compensation), greater competition leads to more exaggeration of earnings and lower transparency. Other evidence also appears consistent with our results. Deegan and Carroll (1993) show that the propensity of Australian firms to make voluntarily disclosures and compete for reporting excellence awards is positively influenced by industry concentration. Wong (1988) and Deegan and Hallam (1991) find that the extent to which firms augment their financial reports with discretionary items is positively related to industry concentration. Finally, Gallego-Álvarez et al.(2008) perform content analysis on the websites of Spanish firms and find that the degree to which firms disclose voluntary information is positively correlated with industry concentration. While all of these findings are suggestive, directly testing our theory is the subject of future research.

Early work on disclosure theory suggests that market forces are sufficient to induce full disclosure. Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) suggest that in the absence of disclosure costs or asymmetric information, firms adhere to the Full Disclosure Principle. Adverse selection prompts high-valued firms to distinguish themselves from others by disclosing their information. This reduces the expected prospects of the remaining non-disclosing firms, which leads to a cascade where all firms end up disclosing their information. Subsequent work challenges these results: full disclosure may not occur because disclosure is costly (e.g. Verrecchia 1983; Fishman and Hagerty 1990), some market participants are unsophisticated (e.g., Fishman and Hagerty 2003), or the market is unsure whether firms have asymmetric information (Dye, 1985; Jung and Kwon, 1988; Shin, 2003; Acharya, DeMarzo, and Kremer, 2010).

Our work builds on this literature by considering that a firm's value depends not only on its

absolute value, but its relative value as well. Moreover, the previous literature focuses on the costs of disclosure, whereas we focus on the benefits. We construct our model using Dye (1985) and Jung and Kwon (1988), but our result generalizes to other models in which discretionary disclosure is costly. For example, if we used Verreccia (1983) instead, the prospect of a prize would increase the incentive to disclose, but growing competition for that prize would reduce that incentive relative to the cost of disclosure.

Our results also contribute to papers that consider competition for attention in financial markets. For example, in Fishman and Hagerty (1989), such attention leads to price efficiency, which is valuable for firms making investment decisions. The prize in their paper is price efficiency. In fact, Fishman and Hagerty show that firms spend more on disclosure than is socially optimal when competing for that prize. Our paper adds to theirs in the following way: when more and more firms compete for this type of attention, each firm's ability to achieve price efficiency declines because traders have a fixed bandwidth when following firms. Decreasing the marginal benefit of disclosure relative to its cost, makes disclosure less attractive. Therefore, applying our model to the formulation in Fishman and Hagerty (1989), we would predict that their effect would diminish with more competition.

Finally, our work also adds to the literature on product market competition and discretionary disclosure. This literature is split on whether product market competition increases or decreases disclosure. Stivers (2004) argues that product market competition increases disclosure, as firms might even make negative disclosures if doing so will hurt competitors more. In contrast, Wagenhofer (1989), Darrough (1993), Clinch and Verrecchia (1997), and Board (2009) argue that firms may avoid disclosure to conceal private information from competitors. Similarly, Darrough and Stoughton (1990), Feltham and Xie (1992), and Pae (2002) argue that firms may conceal such private information to prevent new entry into the market. Finally, Dye and Sridhar (1995) find that product market competition may increase or decrease disclosure depending on whether the information the firms receive is private or industry-related. In many of these papers, disclosure itself affects the nature of competition, which in turn affects the tendency to disclose. While we do not address these considerations specifically, we do add to this literature by considering product market competition simultaneously with competition for attention, finding again that increased competition often reduces management's tendency to reveal private information once the number of firms reaches a very modest threshold.

The rest of the paper is organized as follows. Section 2 introduces our base model, the action set presented to firms, and the corresponding payoffs. Section 3 characterizes the equilibrium, explores

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key comparative statics, and considers the welfare effects of our analysis. Additionally, we show the results are robust to other prize structures besides a fixed prize specification. In Section 4, we add product market competition to the model. We show that competing for the same resources, customers, etc. can initially lead to an increase in information revelation, but once the number of firms reaches an economically reasonable level, additional competition leads to less disclosure. Section 5 offers some concluding remarks. Proofs of all propositions are deferred to Appendix A, as are some technical definitions and lemmas.

2 Base Model

We consider a single period model in which a group of N risk-neutral firms, indexed by $j \in \{1, \ldots, N\}$, compete in a game of discretionary "disclosure".[4] Each firm experiences a random change in fundamental valuation, $\tilde{x}_j \sim F$. We assume $\tilde{x}_j$ has a probability density function $f(x) > 0$ for all $x \in \mathbb{R}$, and that $E[\tilde{x}_j] = 0$.[5] Realizations are independent and identically distributed for each firm, and their distribution does not depend on the number of firms competing in the market, though we will relax this assumption in Section 4.

In each firm, a manager observes the true realization of $\tilde{x}_j$ with probability p. With probability $(1 - p)$, the manager observes nothing. The parameter p measures the degree of asymmetric information in the market. When p is high, it is likely that firms have more information about their value than outsiders. When p is low, it is unlikely that firms have learned anything new. As such, p is also a measure of strong form market (in-)efficiency.

Definition 1. *The possible information events are*

$$I_j \equiv \textit{firm } j \textit{ is } \textbf{informed } \textit{of } \tilde{x}_j$$

$$U_j \equiv \textit{firm } j \textit{ remains } \textbf{uninformed } \textit{of } \tilde{x}_j.$$

Firms that observe $\tilde{x}_j$ may either conceal its value, or may credibly reveal it to investors. Firms that do not observe a value are not permitted to fabricate one. Implicitly, we suppose that investors

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[4]Our use of quotes is purposeful here because what we have in mind is more general than solely accounting disclosure alone. Disclosure is meant to refer to any instance in which information might be revealed. Likewise, our use of "firm" here is meant to represent any party that competes in the market for attention, which includes individuals (e.g., CEO's).

[5]We consider this specification to be sufficiently general since for any distribution in which $E[\tilde{x}_j] \neq 0$, rational investors would update their valuations of the firms to take this into consideration. Therefore, by setting $E[\tilde{x}_j] = 0$, we are considering the news that investors cannot readily predict before any announcements are made.

can freely verify and penalize false claims of $\tilde{x}_j$, but cannot determine whether a non-disclosing firm is in fact concealing information.[6]

Definition 2. *If firm j is informed, its available actions are $\{D_j, C_j\}$, where*

$$D_j \equiv \text{firm } j \textbf{ discloses } \text{its observation } x_j$$

$$C_j \equiv \text{firm } j \textbf{ conceals } \text{its observation } x_j.$$

The event that firm j either conceals its observation or is legitimately uninformed is given by P_j (i.e., firm j pools).

For the moment, we allow firms to choose non-deterministic strategies, although we will soon show deterministic strategies are dominant, almost surely. Formally, we describe each firm's strategy by a "disclosure policy."

Definition 3. *An informed firm j determines its action using a **disclosure policy**, a mapping $\sigma_j : \mathbb{R} \to [0, 1]$. Given any observation $x \in \mathbb{R}$, firm j discloses with probability $\sigma_j(x)$ and conceals with probability $1 - \sigma_j(x)$.*

All informed firms act simultaneously and without knowing which of their competitors are also informed. What we have in mind is that a market-wide event arises in which investors expect informed firms to make disclosures over a short time horizon. However, we explore a sequential game of disclosure in Section 3.3.4. We define $\sigma \equiv \{\sigma_1, \ldots, \sigma_N\}$ as the collection of strategies used by all firms in the market. After acting, each falls into one of three categories: uninformed firms, informed firms that reveal information, and informed firms that pool with the uninformed.

Investors are competitive, risk-neutral, and have rational expectations about firm behavior. If firm j pools (event P_j), investors weigh the odds that it is uninformed against the odds that it is concealing a poor x_j and adjust its price by Bayesian inference. Firm utility is determined by these price changes, so this expectation is also the utility an informed firm obtains by concealing its information,

$$u_j^C \equiv E[\tilde{x}_j | P_j, \sigma, p].$$

After all disclosures have been made, investors award a prize ϕ to the firm with the highest disclosed value. This prize value represents the convex gain in value that accrue to the high-

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[6]As such, we follow the previous literature on discretionary disclosure and do not consider the firms' tendency to make false statements here. To address this, an alternative set-up might include a cheap talk game (e.g., Crawford and Sobel, 1982), but we do not analyze this directly. Further, as mentioned in the introduction, Cheng (2010) shows that firms' tendency to lie increases with competition.

performing firm. We take this prize to be given exogenously, but consider in Section 3.2 what happens when this prize is endogenously determined.

Including this exogenous prize, firm j's expected utility of revealing x is

$$u_j^D(x) \equiv x + \phi W_j(x, p, \sigma_{-j}),$$

where we define $W_j(x, p, \sigma_{-j})$ as the probability that firm j has the highest disclosure. This value depends on the probability p of competing firms being informed, and on the strategies σ_{-j} they employ.

Therefore, this model extends the Jung and Kwon (1988) refinement of the Dye (1985) model, in which uncertainty about a single firm's information induces an incomplete disclosure equilibrium. Dye's model corresponds to ours in the specific case where we have only one firm and no prize ($N = 1$ and $\phi = 0$).

3 Equilibrium Disclosure

With Lemmas 1, 2, and 3, we determine the game's unique subgame-perfect Nash equilibrium.

Lemma 1. *In any subgame-perfect Nash equilibrium, each firm acts according to a **disclosure threshold** $t_j < 0$,*

$$\sigma_j(x_j) = \begin{cases} 1 & \text{for } x_j > t_j \\ 0 & \text{for } x_j < t_j. \end{cases}$$

The threshold is implicitly defined by the condition that a firm observing $x_j = t_j$ be indifferent between disclosing and concealing,

$$u_j^D(t_j) = u_j^C.$$

According to Lemma 1, each informed firm simply compares the expected utility it can obtain by revealing information to what it obtains by pooling. If it conceals its signal, then the actual realization certainly cannot affect the valuation investors assign, so u_j^C is constant with respect to the observed value x_j. In contrast, the value obtained by revealing information is increasing in x_j. If the two utilities are equal when the firm observes its threshold value $\tilde{x}_j = t_j$, then the firm should reveal any observation greater than t_j and conceal any lesser observation.

Note that the threshold t_j is lower than the distribution mean, which we've assumed to be zero. The average $\tilde{x}_j$ for an uninformed firm is simply the distribution mean, and because firms disclose their best observations, rational investors expect the average concealed observation to be negative. The weighted average assigned to pooling firms must therefore be below the distribution mean.

Since disclosure yields strictly greater utility than the value disclosed, no firm will ever conceal an above-average observation. So if firm j is indifferent between revealing and concealing a value $x_j = t_j$, then $t_j < 0$.

Lemma 2. *Every firm uses the same disclosure threshold, defined as t^*.*

In other words, there cannot be an equilibrium in which some firms are more "honest" than others. This is not too surprising, since all firms draw the observations from identical and independent distributions. One can imagine an alternate model in which there was some endogenous benefit of being perceived as trustworthy, and such a model might induce a heterogeneous equilibrium. In our equilibrium, Lemma 2 ensures that any two firms will behave identically with any given observation.

This result justifies our description of the non-disclosing firms as a "pool." Since they all have identical disclosure thresholds and distributions F, investors value all non-disclosing firms identically. In a hypothetical equilibrium where some firms are more honest, investors would assign a higher valuation to an honest non-disclosing firm than to a firm that has a reputation for concealing poor valuations. But when no firm distinguishes itself by its honesty, investors assign the same value u^C to every non-disclosing firm in the pool.

Lemma 3. *The common disclosure threshold t^* is unique.*

Not only does each individual firm have a unique optimal response to the other firms' strategies, but there is only one viable choice for the entire group. So for given model parameter values, we can theoretically determine the unique disclosure threshold. This allows us to predict how disclosure behavior responds to exogenous parameter changes. Most importantly, we will demonstrate how equilibrium disclosure responds to an increase in the number of competing firms.

Taken together, Lemmas 1, 2, and 3 establish Proposition 1.

Proposition 1. *There exists a unique and non-trivial subgame-perfect Nash equilibrium, in which every firm discloses according to a common threshold t^* defined implicitly by*

$$t^* + \phi\big(1 - p + pF(t^*)\big)^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} x f(x) \, dx. \qquad (1)$$

Further, the threshold t^* lies below the unconditional mean of $\tilde{x}_j$, i.e. $t^* < 0$.

The left side of (1) can be understood as the utility a firm that observes $\tilde{x}_j = t^*$ expects if it reveals its information. The firm immediately receives its own value $\tilde{x}_j = t^*$, and can also win

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the prize ϕ if its disclosure is the highest. But since competing firms never reveal values below t^*, any other disclosing firm will have a higher value almost surely. Firm j can win, therefore, only if all other firms pool. Each competing firm pools if either it is uninformed or it is informed with an observation below the threshold. These events occur with probabilities $(1-p)$ and $pF(t^*)$, respectively. The left side therefore is the $u_j^D(t^*)$ term from Lemma 1.

The right side is utility a firm obtains by concealing its observation, which equals the expected value change of a pooling firm. Such a firm could be uninformed and have a zero expected value for its observation, or could be hiding an observation lower than the threshold. The weighted average of these possibilities yields the right side of Equation 1.

3.1 Comparative Statics

Proposition 1 implicitly defines the equilibrium disclosure threshold t^*, so we could use the Implicit Function Theorem to determine comparative statics on t^*. However, a more empirically relevant characterization would describe the frequency with which firms opt to reveal private information. That is, what is the ex ante probability that a firm, if it observes its value change, will choose to share its observation with investors?

Definition 4. *We define the equilibrium ex ante probability of an informed firm disclosing by*

$$\omega^* \equiv \Pr(D|I).$$

*We refer to this probability as the equilibrium **disclosure frequency**.*

When firms follow threshold disclosure strategies, as described in Lemma 1, there is a one-to-one relationship between the disclosure threshold and the corresponding disclosure frequency,

$$\omega^* = Pr(\tilde{x} > t^*) = 1 - F(t^*).$$

Note that disclosure threshold and frequency move in opposite directions. That is, if a firm lowers its threshold, it discloses more of its realized values, and vice-versa.

Definition 5. *Let $\hat{\omega}$ be the equilibrium disclosure frequency when $\phi = 0$, defined implicitly by*

$$x(\hat{\omega}) = \frac{p \int_{\hat{\omega}}^1 x(\Omega) \, d\Omega}{1 - p\hat{\omega}}. \tag{2}$$

Let $\hat{t}$ be the corresponding disclosure threshold, used in equilibrium when $\phi = 0$, so that

$$\hat{\omega} = 1 - F\left(\hat{t}\right).$$

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Dye (1985) and Jung & Kwon (1988) consider disclosure with no strategic interaction (i.e., $N = 1$) and no prize ϕ. Their equilibrium condition is equivalent to Equation 2. The chance to win $\phi > 0$ offers firms a greater incentive to disclose, so in equilibrium, we have

$$\omega^* \geq \hat{\omega} \qquad \forall \phi \geq 0, \forall N < \infty.$$

That is, $\hat{\omega}$ is a lower bound for ω^* over all ϕ and N. In fact, it is the largest possible lower bound. Note that $\hat{t}$ is similarly an upper bound for t^*, which by Proposition 1 must lie below $E[\tilde{x}] = 0$. We therefore also have

$$t^* \leq \hat{t} < 0 \qquad \forall \phi \geq 0, \forall N < \infty.$$

Because ω^* and t^* are informationally equivalent, we may conduct our investigation using either variable. We therefore rewrite Proposition 1 in terms of disclosure frequency, instead of a disclosure threshold.

Proposition 2. *There exists a unique and non-trivial subgame-perfect Nash equilibrium, in which every firm discloses its highest observations with a common disclosure frequency ω^* defined implicitly by*

$$F^{-1}(1 - \omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{\omega^*}^1 F^{-1}(1 - \Omega) \, d\Omega. \qquad (3)$$

Furthermore, $\omega^* \geq \hat{\omega}$.

Like Proposition 1, this proposition defines the equilibrium frequency ω^* by an indifference between disclosure and non-disclosure for a firm that observes the threshold value, $\tilde{x} = t(\omega^*)$.

Proposition 3. *Equilibrium disclosure frequency is:*

(i) decreasing in the number of competing firms, N.

(ii) increasing in the prize value, ϕ.

As $N \to \infty$, ω^* converges to $\hat{\omega}$.

Proposition 3(i) is the simplest possible statement of our main result. When firms disclose competitively to win positive attention, increased competition reduces disclosure. This defies the general economic intuition that tightening competition drives firms to give up their small advantages in the interest of providing a competitive product. Rather, increasing competition drives firms to

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hoard their informational advantage over investors. The result has immediate application in the financial sector, where disclosure is critical and where top-performing firms enjoy large rewards.

Mathematically, the cause of the competition effect is straightforward. As more firms enter the market, each firm's chance of making the highest disclosure diminishes. With it, their chances of receiving positive attention from their disclosure decreases. Since the disclosure decision is a trade-off between the desire to win the prize and the desire to conceal bad signals, additional firm entry tips the balance in favor of concealing. In the sections that follow, we will show this effect to be robust to other types of prizes.

Proposition 3(ii) states simply that firms will be more inclined to disclose when the prize they can win is large. Again, a larger prize tips the balance between the competing desires to pool and to compete openly for positive investor attention. This concept is also robust to our alternative model specifications.

The comparative statics in p turn out to be trickier. Jung and Kwon (1988) consider the special case where $N = 1$ and $\phi = 0$, and find disclosure to be strictly increasing in p. We are able to confirm this result by computing our comparative statics with $\phi = 0$. But when there is a prize, the situation becomes more complicated.

Hypothetically, if p increases and firms fail to adjust their disclosure strategies, there would be two sources of change in firm utility. First, the increase in asymmetric information would increase the Bayesian probability of a firm having inside information. Rational investors would respond by reducing their assessment u^C of pooling firms. Second, the increase in p means competing firms are more likely to be informed. Since being informed is a prerequisite to disclosing, the increase in p makes any given disclosing firm less likely to win the prize by default. Mathematically, a higher p decreases $W_j(x)$, which implies a lower expected utility of disclosure.

These two effects of increased p work against each other. To determine whether ω^* will increase or decrease, we need to know which of these effect impact firm utility more. If the reduction in $u^C(\omega^*)$ is larger than the reduction in $u^D(\omega^*)$, then disclosure becomes more appealing. Firms will then respond to an increase in p by disclosing more frequently. Conversely, if the reduction in $u^D(\omega^*)$ dominates, then firms respond with less frequent disclosure.

If the prize value ϕ is small or zero, then the reduction in $W_j(x)$ is unimportant, so the reduction in $u^C(\omega^*)$ dominates, and equilibrium disclosure increases. This echoes the Jung and Kwon (1988) result. The same result follows when N is very large, in which case the probability of winning the prize is low from the outset. In contrast, when ϕ is large and N is modest, the reduction in $W_j(x)$ is critical. The second effect dominates, so overall the incentive to disclose is reduced more than

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the incentive to pool. Consequently, firms pool more often, reducing the equilibrium disclosure frequency.

These comparative statics yield novel and testable empirical predictions. Our model predicts that, in markets in which rank-based remuneration is paid, performance announcements, advertising, and discretionary disclosure should decline with competition, *ceteris paribus*. This implies, for example, that in the mutual fund industry where firms compete for Morningstar ratings (Del Guercio and Tkac, 2008), advertising expenditure and discretionary disclosure should be negatively correlated with Herfindahl Index. The analysis also implies that discretionary disclosure should be negatively correlated with the size of the investment sector under study (e.g., technology versus manufacturing). Such predictions might be tested cross-sectionally or with a time series, while controlling for other fund characteristics. Our model also predicts that when asymmetric information increases, discretionary disclosure should increase in industries with low status-based prizes. However, in industries with high status rewards, discretionary disclosure should decline when information asymmetry increases. This prediction might also be tested in the mutual fund industry with a time-series that includes periods of high volatility.

3.2 Transparency and Welfare

According to Proposition 3, competition decreases transparency because each firm has a lower incentive to disclose private information. In this section, we consider whether this adversely affects per capita welfare and market efficiency. In what follows, we show that this depends on whether it is only necessary to identify the top firm in the market, or whether it is optimal to screen all firms in the market. We begin with the following proposition.

Proposition 4. *Define Z_N as the ex ante probability that zero of the N competing firms disclose. Then,*

(i) Z_N is decreasing in N;

(ii) $\lim_{N \to \infty} Z_N = 0$.

According to Proposition 4, the probability that investors learn something from at least one firm increases with competition. Two opposing effects account for this result. As N increases, individual disclosure frequency decreases. However, there are also more aggregate draws from $F(\cdot)$. The second effect dominates the first. In the limit, as the market for attention becomes perfectly

competitive, equilibrium disclosure is minimized, but the probability that no information is revealed at all tends to zero.

The ramifications of Proposition 4 are especially important if the social planner or a residual claimant wishes to identify one, and only one, firm to reward a prize ϕ. As N grows, this probability increases (i.e., $\frac{\partial Z}{\partial N} < 0$) and occurs almost surely with perfect competition (i.e., $N \to \infty \Rightarrow Z \to 0$).

Notwithstanding this, there are reasonable economic situations where competition decreases per capita welfare because transparency drops. This is especially true when it is efficient to screen all market participants to determine whether each of them should be allocated a scarce good (e.g., a portfolio allocation problem). To explore this, we consider an example in which a bank screens potential borrowers and can set the prize ϕ optimally to maximize profits. The analysis that follows could be applied to many other economic scenarios.

Consider a monopolistic, risk-neutral lender that is unconstrained in how many firms she can do business with. We map our base model of N symmetric firms into this framework, where each firm represents a potential borrower. The monopolist sorts through the firms that make a disclosure and chooses whether to offer each of them a loan ℓ. It is efficient for the monopolist to allocate ℓ to firms with a realization of $\tilde{x}$ above a threshold $\underline{x}$, and not do business with firms below $\underline{x}$. By construction, firms that do not make disclosures, do not receive investment capital. We denote the net gain from the loan as $G(\ell)$. Because the lender is monopolistic, we assume that she extracts $G(\ell)$ from investment.

Consider first that the prize is equal to zero. If $\underline{x} \geq \hat{t}$, the monopolist can always efficiently screen the borrowers because $\hat{t} \geq t^*$. However, when $\underline{x} < \hat{t}$, there may exist a region of inefficient under-investment. In this case, for any $\tilde{x} \in [\underline{x}, t^*]$, an informed borrower withholds their information, despite qualifying for the loan ℓ. For a given threshold t^*, the ex ante expected welfare loss from the forgone investment is

$$\mathcal{L} = pN \left[F(t^*) - F(\underline{x}) \right] G(\ell). \tag{4}$$

The difference $F(t^*) - F(\underline{x})$ is the the expected fraction of borrowers that falls in the interval $[\underline{x}, t^*]$. Of this group of firms, a fraction p will be informed. Therefore, of the N potential borrowers, the monopolistic lender expects $pN \left[F(t^*) - F(\underline{x}) \right]$ of them to be qualified for ℓ, but fail to be identified.

Now, consider that the monopolist can encourage additional disclosure by offering a prize ϕ. We assume that ϕ is a transfer between the monopolist and the firm with the highest disclosure; therefore, the size of ϕ does not affect aggregate welfare in and of itself. Again by construction, it is clear that no prize is needed when $\underline{x} \geq \hat{t}$. However, when $\underline{x} < \hat{t}$, the monopolist's goal is to

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set the optimal prize ϕ to minimize the forgone investment. Specifically, the monopolist desires to lower the firms' threshold t^* closer to $\underline{x}$. As such, we denote the threshold as a function of ϕ, $t(\phi)$. Obviously, there is no additional benefit to increasing the prize once $t(\phi) = \underline{x}$. Hence, we define $\underline{\phi}$ to be the prize that leads to that equality. The monopolist's problem for a given N is

$$\min_{\phi \in \mathbb{R}^+} pN \left[F(t(\phi)) - F(\underline{x}) \right] G(\ell) + \phi. \tag{5}$$

Proposition 5. *The optimal prize for the monopolist to offer is*

$$\phi^* = \begin{cases} 0 & \text{if } N < \frac{1}{G(\ell)pF'W^{N-1}} \\ \min \left(G(\ell)W - \frac{1}{NpF'W^{N-2}}, \underline{\phi} \right) & \text{if } N \geq \frac{1}{G(\ell)pF'W^{N-1}} \end{cases}, \tag{6}$$

where $W = \left(1 - p + pF(t) \right)$ *and* $F' = \frac{\partial F}{\partial t}$.

The condition in (6) can be appreciated as follows. First, it always holds that $\phi^* < G(\ell)$. That is, the monopolist never has to give up the entire surplus that she earns from lending to the best quality borrower. Second, when the gain from investing is low (small $G(\ell)$), the probability of being informed is low (small p), and the distribution is relatively flat locally around the threshold (low F'), the lender optimally avoids paying the cost of offering a prize. In such cases, capital allocation is the least efficient.

The effect of competition on incentives to offer the prize may be non-monotonic for some N, but is strictly decreasing once N reaches a threshold. To see this, consider the limit when $N \to \infty$: it is optimal for the lender to offer no prize. Mathematically, we can re-write the condition in (6) and compute

$$\lim_{N \to \infty} NG(\ell)pF'W^{N-1} = 0 < 1.$$

Economically, when $N \to \infty$, the prize becomes ineffective: borrowers minimize disclosure because their chances to win the prize tends to zero. In turn, the lender optimally chooses not to offer a prize and efficiency is minimized.

As this example suggests, in some cases competition not only decreases transparency, but also lowers per capita welfare. Admittedly, this is not a general statement about welfare as the effects depend on the particular type of economic situation of interest. However, the problem we explore may arise anytime a business, a planner, or an investor has market power and desires to screen all potential business partners. It points out that competition's effect on transparency and efficiency needs to be carefully considered before policy decisions are made.

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3.3 Other Prize Structures

We now show that competition's effect of reducing disclosure is robust to alternative model specifications. Because the probability of winning ϕ declines exponentially with firm entry, the effect tends to trump other simultaneous considerations, especially when N is large. Neither multiple prizes nor prizes that decrease with competition alter this result. Prizes that grow with N can change things, but only for small N or for prizes that (rather implausibly) grow exponentially ad infinitum. For prizes awarded based on percentile, we show that perfect competition minimizes transparency; however, as will become clear shortly, for practical purposes it is not possible to show that disclosure declines monotonically with N for a finite number of firms.

3.3.1 Multiple Prizes

Allowing only a single firm to win the prize ϕ has simplified our analysis, but it seems reasonable for the second-highest discloser to receive some positive investor attention as well. Perhaps the top ten firms all deserve some sort of status prize. A more egalitarian prize structure is sensible if firms are limited by capacity or distribution constraints. In such cases, some prize should be reserved for the firms with slightly less impressive disclosures. In what follows, we consider that a finite number of prizes K are awarded to the top firms. In Section 3.3.3, we show that the analysis remains robust to considering prizes awarded by percentile.

Definition 6. *A disclosure game with a* **progressive prize structure** *is one in which the firms that make the K highest disclosures each win a prize. The firm that makes the kth highest disclosure wins ϕ_k. We require the prizes to be positive and strictly monotonic,*

$$\phi_1 > \phi_2 > \ldots > \phi_K > 0.$$

Compared to a model with a single prize of $\phi = \phi_1$, the addition of prizes for runners-up naturally induces greater disclosure. But although the change to a progressive prize structure may increases disclosure for any particular N, our central result remains unchanged:

Proposition 6. *Under any particular progressive prize structure, equilibrium disclosure frequency strictly decreases as competition increases. That is, $\omega_{N+1}^* < \omega_N^*$ for any N.*

This result justifies our simplification in working with a single prize ϕ. Although additional prizes may change the quantitative predictions of equilibrium disclosure, the qualitative comparative statics remain unchanged. The chance of winning a lesser prize decreases with competition just as

the chance of winning a single prize does. Competition therefore reduces disclosure in this setup as well.

3.3.2 Increasing/Decreasing Prize Values

A reasonable consideration is that the prize depends on N, which we denote as ϕ_N. A case might be made for either increasing or decreasing prize values. Prize value might decrease when additional firms enter because investor attention is diluted over a larger population of firms. More commonly, though, the prize might shrink because of increasing competition for a scarce resource. For example, as competition for bank financing, supplier credit, or labor resources increases, the relative bargaining power of these residual claimants grows, thereby lowering the value that the top firm gets when winning the competition. It is straightforward to see, based on Proposition 3, that a prize that decreases in N only augments our result. If the addition of further competitors causes an exogenous reduction of prize value (i.e., lower ϕ), then equilibrium disclosure falls even faster then if the prize remained constant.

The argument that prizes increase with competition is a more challenging one. Such a case can be made when disclosures are easy to compare, and when choosing the very best firm has high value to potential clients and investors. One might argue that investment funds fit this description. A single fund can accommodate large increases in the amount of money it manages, and there is very little reason to be content with the second best fund (net of fees, at least). If potential investors believe that the disclosed information is a strong predictor of future performance (e.g., Berk and Green, 2004), then being the best in a large field of competitors may bring a larger prize than being the best in a small field.

Continuously increasing prize value can overcome the effect of competition, at least when N is small. But the following proposition shows that unless the prize grows exponentially by a factor of at least $1/(1 - p\hat{\omega})$, disclosure will eventually decrease once N reaches some critical value.

Proposition 7. *If ϕ_N increases with N and*

$$lim_{N \to \infty} \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}}, \tag{7}$$

then there exists some $\overline{N} \in \mathbb{R}$ such that $N > \overline{N}$ implies that $\omega_{N+1}^* < \omega_N^*$.

To gain intuition for this result, consider the case in which prize value per firm remains constant:

$$\phi_N \equiv N\phi_1.$$

In this case,

$$\lim_{N \to \infty} \frac{\phi_{N+1}}{\phi_N} = 1 < \frac{1}{1 - p\hat{\omega}},$$

so the condition in (7) is satisfied, and disclosure decreases with competition for large N.

Intuitively, the chance of winning the prize declines exponentially in N, so unless the prize grows forever at the same exponential rate, the expected winnings will eventually decline in N. One must ask, however, where a prize that increases exponentially with firm entry would come from. The value of high status may well increase exponentially as the number of competing firms increases from, say $N = 1$ to $N' = 10$. But it is difficult to believe the same exponential increase could continue from $N = 10$ to $N' = 50$. We conjecture that exponentially increasing status prizes are uncommon at best, and may never occur in industries with a large number of firms.

3.3.3 Prizes based on percentiles

Rather than awarding a fixed number of prizes, we can instead use a firm's relative ranking. For example, each firm in the top 20% of the N firms could be awarded a prize, so that the $N/5$ highest disclosures each receive an additional ϕ. This variation introduces some complications that prevent us from showing the claim from the main model, "equilibrium disclosure ω_N^* is strictly decreasing in N." Because the number of prizes is discrete, it cannot increase in exact proportion with N. For example, when 20% of the firms receive a prize, a single prize is awarded when $N = 5, 6, 7, 8, 9$, and we numerically find that $\omega_5^* > \omega_6^* > \ldots > \omega_9^*$. But for $N = 10$, we suddenly award a second prize, which can mean that $\omega_9^* < \omega_{10}^*$. We must therefore content ourselves with the result that disclosure decreases to its minimum possible frequency under perfect competition.

Proposition 8. *Suppose that for any N, a fixed fraction λ of the competing firms win the prize ϕ. Further, suppose that $\lambda \le p\hat{\omega}$. Then, disclosure converges to its lower bound in the perfectly competitive limit:*

$$\omega_N^* \longrightarrow \hat{\omega} \ \ as \ N \to \infty.$$

According to Proposition 8, as the market becomes perfectly competitive, disclosure is minimized. It should be noted that the condition that $\lambda < p\hat{\omega}$ is weak in the sense that it allows for a large number of firms to receive prizes. If $\lambda = p\hat{\omega}$ when $N \to \infty$, this would mean that all firms that observed a value above $\hat{t}$ would receive a prize. Therefore, we limit the fraction of prizes ($\lambda < p\hat{\omega}$) to keep the analysis realistic and economically interesting.

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3.3.4 Sequential Disclosure

As a further robustness check, consider an alternate model specification in which firms act sequentially instead of simultaneously. Firms are randomly ordered, and each in turn observes its shock value $\tilde{x}$ with probability p, then chooses whether to disclose.

Since each firm makes a unique, history-dependent decision, we no longer have a single symmetric, deterministic disclosure threshold. Rather, each firm will have a random disclosure threshold that depends upon the disclosures of the preceding firms and on the number of firms remaining to act. Let ν_j be the ex ante probability that the jth firm to act will disclose if they are informed. The average of these probabilities is the analogue of the disclosure frequency in the main model,

$$\bar{\nu}_N = \frac{1}{N} \sum_{j=1}^{N} \nu_j.$$

In the perfectly-competitive limit, we can show that every individual j^{th} firm discloses with frequency $\hat{\omega}$, the minimum possible. We can also show the slightly stronger claim that the average frequency of disclosure over all N firms converges to the minimum $\hat{\omega}$.

Proposition 9. *In sequential equilibrium with N firms,*

(i) The ex ante probability that the jth firm discloses converges to the minimum with perfect competition:

$$\lim_{N \to \infty} \nu_j = \hat{\omega}.$$

(ii) The ex ante probability that a randomly selected informed firm discloses also converges to the minimum with perfect competition:

$$\lim_{N \to \infty} \bar{\nu}_N = \hat{\omega}.$$

4 Disclosure Under Product Market Competition

Our base model is most appropriate for firms that compete for the prize only indirectly. That is, it models the competition for ϕ between firms in different industries. Although such firms do not compete directly for customers or revenue, they both vie for the same status-based prize (i.e., recognition from investors). The base model is less appropriate when considering firms that compete directly in the product market, as well as for attention through their disclosures.

In this section, we present an alternative model that better characterizes the latter case. That is, we suppose that firms compete for customers as well as for positive attention from investors. The fundamental aspect of competition relevant here is that firm revenue decreases with the entry of additional firms. Accordingly, signals that affect firm value have a smaller absolute impact, even if they alter firm value by the same fraction. For example, an internal audit that reveals an increase in efficiency has a greater absolute effect if it happens in a firm with high revenue and market value.

4.1 Simple Product Market Competition

To capture this effect simply, we model product market competition by using a distribution of value signals that becomes compressed with the entry of additional firms. When N firms compete in the product market as well as for investor attention, we exchange the original distribution of signals $x \sim F$ for a compressed distribution $x_N \sim F_N$. Specifically, whenever a firm would have drawn a signal x in the original model, they instead draw a scaled-down event x/N in the new model.

Formally, we write the new distribution as

$$F_N(x) \equiv F(Nx).$$

An increase in N has the effect of "squishing" the distribution of news events while leaving the support unchanged. If $x = \$10k$ had been a 90th-percentile result with $N = 5$, $x = \$1k$ would be the new 90th-percentile with $N = 50$. Increasing N scales down expectations while preserving the concavity and any other peculiarities of the value distribution. We refer to this as "equal shares competition" for earnings.

We wish to stress that this is not intended as a realistic model of competition. A plethora of microeconomic papers have dealt with such issues before us, so we initially gloss over other aspects of pricing, capital, and entry costs. Our goal is simply to show how the value-scaling effect of competition affects disclosure, using the simplest model that captures the relevant effect. In Section 4.2, though, we consider more general models of competition, and show that our general result holds for most plausible models of competition and revenue.

In Section 2, we found that as N increases, the incentive to disclose falls as the probability of winning the prize decreases. With product market competition, though, potential revenue declines as well, which reduces the incentive to pool. These two effects oppose one another. Which effect dominates depends upon the number of competing firms.

For what values of N, then, does competition reduce disclosure? If we were to find the necessary number of firms to be in the millions, for example, then our point here would only be academic

and not of practical import. To gain a sense of how many firms is "enough," consider the following proposition.

Proposition 10. *If $N > 1/(p\hat{\omega})$, then $\omega^*_{N+1} < \omega^*_N$. Further, as $N \to \infty$,*

(i) Average per-firm earnings π_N converge to zero.

(ii) The equilibrium disclosure frequency ω^*_N converges to $\hat{\omega}$.

To appreciate Proposition 10, suppose the distribution $\tilde{x} \sim F$ is symmetric, so the fact that $\hat{t} < E[x]$ implies that

$$\hat{\omega} = 1 - F(\hat{t}) > 1 - F(E[x]) = 0.5. \tag{8}$$

Then the necessary condition becomes $N > 2/p$. If, for example, firm information arrives with probability 20%, then $N = 2/(20\%) = 10$ firms is enough competition that further entry will only reduce disclosure. The higher p is, the fewer firms that are required to assure that further competition decreases disclosure. We conjecture that in many industries (e.g., financial sectors), there are already enough competitors present so that disclosure responds negatively to additional competition.

Proposition 10 also shows that ω^*_N actually converges to $\hat{\omega}$ under perfect competition, while industry profits converge to zero. Product prices decrease to their lowest possible values, which maximizes social welfare. However, perfect competition in disclosure induces firm to retain their maximum degree of asymmetric information. Thus, while perfect competition drives product prices to their most socially efficient level, it drives firm prices to their least informationally efficient. To better appreciate this, we provide the following example.

Example 1. *Consider the disclosure game where $\tilde{x}$ is Gaussian with $\mu = 0$, $\sigma = 5$ and $\phi = 1$, $p = .3$ and product market competition characterized by $F_N(x) = F(Nx)$. Figure 1 shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit $\hat{\omega} \approx .556$.*

Although the above condition of $N > 1/(p\hat{\omega})$ is mild enough, the condition is indeed only sufficient for competition to decrease disclosure, not necessary. Typically, an even smaller number of firms will suffice. We therefore derive the constraint on N that is both necessary and sufficient for further entry to reduce disclosure.

Consider the position of a firm j that draws the threshold value, $x_j = F^{-1}(1 - \omega^*)$. With N firms competing for the prize, firm j is indifferent between disclosing and herding. If a $(N + 1)^{th}$
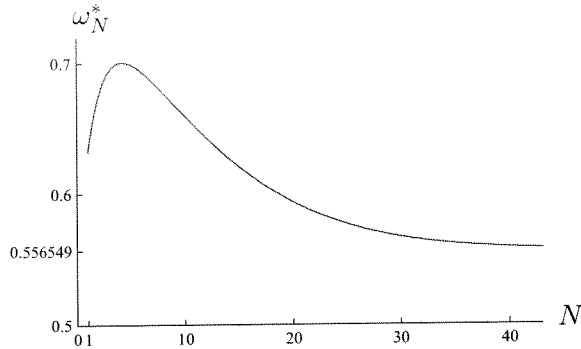
Figure 1: Disclosure game with product market competition. The random variable is $\tilde{x} \sim N(0,5)$ and $\phi = 1$ and $p = .3$. The product market competition characterized by $F_N(x) = F(Nx)$. The curve shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit $\hat{\omega} \approx .556$.

competitor enters, and firm j observes the same x_j, how do the firm's prospects change? Should it disclose, the entry reduces its expected prize winnings by a factor of $(1 - p\omega)$ because

$$\phi W(\omega; N) = \phi(1 - p\omega)^{N-1} \tag{9}$$

is exponentially decreasing in N. But the other terms, $F_N^{-1}(1 - \omega)$ and $u_N^C(\omega)$, decline by a factor of $N/(N + 1)$, as demonstrated in Lemma A2 in the appendix. As N rises, then, this linear effect diminishes in significance compared to the exponential effect on the expected prize value. Intuitively it seems that there is a critical number of firms at which additional competition makes herding more attractive than competing for the prize.

Proposition 11. *Disclosure frequency decreases with firm entry if and only if the number of competing firms exceeds some threshold.:*

$$N > \frac{p\omega_N^*}{1 - p\omega_N^*} \equiv \overline{N} \iff \omega_{N+1}^* < \omega_N^*. \tag{10}$$

Because the ex-ante probability of a firm disclosing is $P(D_j|N) = p\omega_N^*$, we may equivalently write

$$N > \frac{P(D_j|N)}{P(P_j|N)} \iff \omega_{N+1}^* < \omega_N^*. \tag{11}$$

According to Proposition 11, if N exceeds the threshold specified by the relative probabilities of disclosing and pooling, then the exponential effect overwhelms the linear effect. So, the net effect of firm entry is a reduction in the incentive to disclose, which results in $\omega_{N+1}^* < \omega_N^*$. Note,

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however, that the threshold for N established by Proposition 11 is changing with N. That is, as N increases, ω_N^* varies, and so the probability ratio in Equation 11 may also increase. Therefore, although this proposition details the necessary and sufficient condition for N, it does not provide a tighter unconditional bound than in Proposition 10.

4.2 Generalized Product Market Competition

Now let us consider more general models of competition. That is, consider a model in which firm entry reduces the value of competing firms in a more sophisticated way than the simple $\frac{1}{N}$ rule used in the previous section. Instead of $F_N(x) \equiv F(Nx)$, we define the distribution as a function of N by

$$F_N(x) \equiv F\left(\frac{x}{\alpha_N}\right),$$

for some decreasing sequence $\{\alpha_N\}$. Note that if α_N decreases rapidly, then firm entry has a dramatic effect on the revenue of competing firms. If α_N decreases more slowly, then the effect is less pronounced.

Proposition 12. *If, under generalized competition with $F_N(x) = F(\frac{x}{\alpha_N})$,*

$$\lim_{N \to \infty} \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega},$$

then there exists some $\overline{N} \in \mathbb{R}$ such that $N > \overline{N} \Rightarrow \omega_{N+1}^* < \omega_N^*$.

The proof follows nearly the same structure as the proof of Proposition 11. Note, however, that in this case, we need an additional restriction on the sequence $\{\alpha_N\}$ in order to complete the proof. Roughly stated, the requirement above is that competition not reduce firm value too "quickly" as additional firms enter.

Thus, the question becomes one of whether the per-firm revenue can decrease ad-infinitum at such a rate with the entry of additional firms. Although one can posit such a model, exponentially decreasing revenue is not a common feature of microeconomic models of competition.

Example 2. *Consider a Cournot competition with linear pricing. In such a model, per-firm earnings (and hence firm value) declines as N grows:*

$$\pi_N = \frac{\pi_1}{N^2}.$$

Therefore, $\alpha_N = \frac{1}{N^2}$. This sequence satisfies the criterion in Proposition 12 because

$$\lim_{N \to \infty} \frac{\alpha_{N+1}}{\alpha_N} = \lim_{N \to \infty} \frac{N^2}{(N+1)^2} = 1 > 1 - p\hat{\omega}.$$

So under linear Cournot competition, disclosure does indeed decline with competition for large N.

5 Concluding Remarks

The primary result in this paper is that increased competition in financial markets reduces disclosure when convex payoffs are present. We show this both in a parsimonious model, as well as in more sophisticated extensions. The fundamental idea, that firm entry makes attaining top status more difficult, is straightforward. But the exponential relationship between the number of competing firms and the probability of winning the prize is mathematically powerful. The result is a robustness that makes our central result widely generalizable.

As our model shows, we cannot always appeal to the Invisible Hand to make markets transparent. While competition in product markets often has a favorable effect on prices, driving firms to lower and more socially efficient prices, it can have the opposite effect on disclosure in financial markets. Specifically, when a large part of the firm incentive to disclose is due to the prize value of high status, the entry of additional firms can reduce this incentive. In the asymptotic limit of perfect competition, prices converge to their most efficient values, but disclosure falls to its least efficient.

One application of our analysis would be disclosure during the recent financial crisis. Indeed, terms like "sub-prime mortgage" and "collateralized debt obligation" became household language and massive devaluations made these "toxic assets" infamous in the eyes of the public. Investors and regulators were eager to know firms' exposure to these assets, but many firms did not know their own positions. This maps well onto our model, since investors could not be certain which firms accurately knew the contents of their balance sheets. Informed firms could choose either to conceal their asset positions, or to disclose the gruesome details. Financial markets might have been able to right themselves more quickly had the position of insured and insuring firms been known to investors. A great deal of media and political attention has focused on the need to mitigate or avoid such problems in the future.

While past models have focused on the "Lemons Problem" of corporate disclosure, little attention has been paid to the exogenous benefits of positive investor attention. Essentially, existing disclosure theory casts disclosures from Wells Fargo in the same light as disclosures from a local grocer. But in financial sectors, where investor attention and high status make a large contribution to firm value, the comparison is simply inappropriate. If firms do in fact take the benefits of status into account when considering discretionary disclosure, then competition may make them less inclined to disclose private information.

In the end, our analysis implies that policy makers should consider the type of competition that

takes place in markets when deciding whether to regulate them. Competition may not always cure market ailments, and may even exacerbate them.

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Appendix A

Proof of Lemma 1

Suppose firm j observes the event x. In a subgame-perfect Nash equilibrium, the firm must disclose optimally given the value of x. That is, it discloses when $u_j^D(x) > u_j^C$ and conceal x when $u_j^C < u_j^D(x)$.

If the firm discloses, then it is eligible to with the prize ϕ. So its new market valuation is x, plus an additional ϕ if no competing firm makes a higher disclosure,

$$u_j^D(x) = x + \phi W_j(x),$$

where $W_j(x)$ is the probability that no competing firm discloses a higher value than x:

$$W_j(x) = \prod_{k \neq j} (1 - P(I_k)P((x_k > x) \cap D_j))$$

$$= \prod_{k \neq j} \left(1 - p \int_x^\infty \sigma_k(z) f(z) \, \mathrm{d}z \right) \tag{A1}$$

Note that $u_j^D(x)$ is differentiable, and therefore continuous. Furthermore, for any x,

$$u_j^D(x) \leq x + \phi \qquad \text{and} \qquad x \leq u_j^D(x)$$

Evaluating at $x = u_j^C - \phi$ and $x = u_j^C$, these inequalities yield

$$u_j^D(u_j^C - \phi) \leq u_j^C \qquad \text{and} \qquad u_j^C \leq u_j^D(u_j^C).$$

So if firm j observes $x_j = u_j^C - \phi$, then disclosure yields a lower expected utility than u_j^C; and if it observes $x_j = u_j^C$, then disclosure yields a higher expected utility than u_j^C. Because $u_j^D(x)$ is continuous, the Intermediate Value Theorem assures us there is a potential observation $t_j \in [u_j^C - \phi, u_j^C]$ for which

$$u_j^D(t_j) = u_j^C. \tag{A2}$$

This t_j is the disclosure threshold for firm j, where the firm is indifferent between disclosing and pooling. Since $u_j^D(x)$ is strictly monotonic in x, we further obtain

$$x > t_j \implies u_j^D(x) > u_j^C$$
$$x < t_j \implies u_j^D(x) < u_j^C.$$

The subgame-optimal response of firm j is therefore to disclose any values above the threshold t_j and to conceal any values below, as desired.

Now to show that $t_j < 0$, we derive the value u_j^C that investors assign if the firm conceals its observation. In a rational expectations equilibrium, the beliefs of the investors with respect to the strategy must be consistent with the strategy actually used,

$$
\begin{aligned}
u_j^C = E[x|P_j] &= \frac{P(U_j)E[x|U_j] + P(I_j \cap C_j)E[x|I_j \cap C_j]}{P(U_j) + P(I_j)P(C_j|I_j)} \\
&= \frac{(1-p) \cdot 0 + pP(x < t_j)E[x|x < t_j]}{(1-p) + pP(x < t_j)} \\
&= \frac{pF(t_j)}{1-p+pF(t_j)}E[x|x < t_j] \qquad \text{(A3)} \\
&< \frac{pF(t_j)}{1-p+pF(t_j)}E[x] = 0.
\end{aligned}
$$

$$
\therefore \qquad u_j^C < 0 < u_j^D(0). \qquad \text{(A4)}
$$

Because $u_j^D(x)$ is monotonically increasing in x, the threshold t_j must be below zero. That is, all average or better values of x will be disclosed in equilibrium. ∎

Proof of Lemma 2

Write Equation (A3) as an integral, then apply integration by parts,

$$
\begin{aligned}
u_j^C &= \frac{p}{1-p+pF(t_j)} \int_{-\infty}^{t_j} xf(x)\, \mathrm{d}x \qquad \text{(A5)} \\
&= \frac{p}{1-p+pF(t_j)} \left(\left[xF(x) \right]_{-\infty}^{t_j} - \int_{-\infty}^{t_j} F(x)\, \mathrm{d}x \right) \\
&= \frac{p}{1-p+pF(t_j)} \left(t_j F(t_j) - \int_{-\infty}^{t_j} F(x)\, \mathrm{d}x \right).
\end{aligned}
$$

Using this expression, some algebraic manipulation transforms $u_j^D(t_j) = u_j^C$ into

$$
\phi W_j(t_j)(1-p+pF(t_j)) = (1-p)(-t_j) - p \int_{-\infty}^{t_j} F(x)\, \mathrm{d}x. \qquad \text{(A6)}
$$

Now suppose for contradiction that a non-symmetric equilibrium exists. That is, suppose an equilibrium exists in which firms j and k use different thresholds. Without loss of generality, assume that $t_k < t_j$. Equation (A6) holds for firm k as well as for j. Subtracting these yields

$$
\phi \left(W_j(t_j)(1-p+pF(t_j)) - W_k(t_k)(1-p+pF(t_k)) \right)
$$

$$
= (1-p)(-t_j + t_k) - p \int_{t_k}^{t_j} F(x)\, \mathrm{d}x < 0.
$$

$$
\therefore W_j(t_j)(1-p+pF(t_j)) < W_k(t_k)(1-p+pF(t_k)). \qquad \text{(A7)}
$$

But we can obtain a contradiction by deriving the opposite inequality. We simplify Equation (A1) with the assumption that all firms use threshold strategies, then evaluate at t_j:

$$
\begin{aligned}
W_j(t_j) &= \prod_{i \neq j} \left(1 - p \int_{t_j}^{\infty} \sigma_i(z) f(z) \, \mathrm{d}z \right) \\
&= \prod_{i \neq j} \left(1 - p + pF\big(\max(t_i, t_j) \big) \right) \\
&= \left(1 - p + pF\big(\max(t_j, t_k) \big) \right) \prod_{i \neq j,k} \left(1 - p + pF\big(\max(t_i, t_j) \big) \right).
\end{aligned}
$$

(A8)

The same holds for firm k, so we obtain

$$
W_k(t_k) = \left(1 - p + pF\big(\max(t_j, t_k) \big) \right) \prod_{i \neq j,k} \left(1 - p + pF\big(\max(t_i, t_k) \big) \right).
$$

Since $t_j > t_k$, these equations show that $W_j(t_j) > W_k(t_k)$. Therefore,

$$
W_j(t_j)\big(1 - p + pF(t_j)\big) > W_k(t_k)\big(1 - p + pF(t_k)\big).
$$

This directly contradicts Equation (A7), so the hypothesized asymmetric equilibrium cannot exist.
■

Proof of Lemma 3

Suppose for contradiction there exist two distinct equilibrium thresholds t^* and t^{**}. Without loss of generality, assume $t^* < t^{**}$. Equation (A6) holds at both thresholds. Subtracting, we obtain

$$
\phi\Big(W(t^*)\big(1 - p + pF(t^*)\big) - W(t^{**})\big(1 - p + pF(t^{**})\big) \Big)
$$

$$
= (1 - p)(t^{**} - t^*) + p \int_{t^{**}}^{t^*} F(x) \, \mathrm{d}x < 0.
$$

$$
\therefore \quad W(t^*)\big(1 - p + pF(t^*)\big) < W(t^{**})\big(1 - p + pF(t^{**})\big).
$$

(A9)

We now obtain a contraction by deriving the opposite inequality. Since strategies are symmetric, $t_i = t_j$ in Equation (A8), so the equation simplifies to

$$
W(t^*) = \big(1 - p + pF(t^*)\big)^{N-1}.
$$

(A10)

And the same holds for the other equilibrium threshold,

$$
W(t^{**}) = \big(1 - p + pF(t^{**})\big)^{N-1}.
$$

Because $t^* > t^{**}$, these equations show that $W(t^*) > W(t^{**})$. Therefore,

$$
W(t^*)\big(1 - p + pF(t^*)\big) > W(t^{**})\big(1 - p + pF(t^{**})\big),
$$

directly contradicting Equation (A9). By this contradiction, we conclude that a second distinct equilibrium threshold t^{**} cannot exist. ■

Proof of Proposition 1

Taken together, Lemmas 1, 2 and 3 show that all firms use a common and unique disclosure threshold defined implicitly by

$$u_j^D(t^*) = u^C(t^*).$$

We expand this equivalence using $u_j^D(t^*) = t^* + \phi W(t^*)$, Equation (A10), and Equation (A5) to obtain the desired expression

$$t^* + \phi\big(1 - p + pF(t^*)\big)^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} x f(x)\, \mathrm{d}x.$$

Finally, we find $t^* < 0$ by the same argument that shows $t_j < 0$ in Lemma 1. ∎

Proof of Proposition 2

Consider the integral in the right hand side of Equation (1). The substitution $\Omega = 1 - F(x)$ yields

$$f(x)\, \mathrm{d}x = -\, \mathrm{d}\Omega, \qquad x = F^{-1}(1 - \Omega) = t(\Omega).$$

Therefore,

$$\int_{-\infty}^{t^*} x f(x)\, \mathrm{d}x = \int_{1-F(-\infty)}^{1-F(t^*)} F^{-1}(1 - \Omega)(-\, \mathrm{d}\Omega) = -\int_{1}^{\omega^*} F^{-1}(1 - \Omega)\, \mathrm{d}\Omega = \int_{\omega^*}^{1} F^{-1}(1 - \Omega)\, \mathrm{d}\Omega.$$

Equation (1) therefore becomes the desired expression,

$$F^{-1}(1 - \omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{\omega^*}^{1} F^{-1}(1 - \Omega)\, \mathrm{d}\Omega.$$

∎

Definition A1. *For any disclosure frequency ω, define the corresponding disclosure threshold by $t(\omega)$. That is,*

$$t(\omega) \equiv F^{-1}(1 - \omega)$$

Definition A2. *Define $B(\omega)$ as the benefit of disclosing the threshold value relative to concealing, assuming that all firms disclose with frequency ω,*

$$B(\omega) \equiv u^D(\omega) - u^C(\omega),$$

where

$$u^D(\omega) \equiv E[u_j^D | x_j = t(\omega)]$$
$$u^C(\omega) \equiv E[x_j | P_j, t_j = t(\omega)].$$

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Note that this definition does not require that ω be the equilibrium frequency, which we denote distinctly by ω^*.

Lemma A1. *The equilibrium disclosure frequency is defined implicitly by*

$$B(\omega^*) = 0.$$

Further, for any $\omega > \hat{\omega}$,

(i) $\frac{\partial B}{\partial \omega} < 0$

(ii) $B(\omega) > 0 \Rightarrow \omega < \omega^*$

(iii) $B(\omega) < 0 \Rightarrow \omega > \omega^*$

Proof of Lemma A1

Using Definitions A1 and A2, we can write $B(\omega)$ as

$$B(\omega) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_\Omega^1 t(\Omega) \, d\Omega.$$

Comparing this to Corollary 2 reveals that $B(\omega^*) = 0$ is algebraically equivalent to Equation 3. So $B(\omega^*)$ defines the equilibrium disclosure frequency, as desired.

(i) Note that

$$\frac{\partial}{\partial \omega} t(\omega) = \frac{\partial}{\partial \omega} F^{-1}(1 - \omega) = \frac{-1}{f(F^{-1}(1 - \omega))} < 0,$$

so the first term is decreasing in ω. Clearly the second term is also decreasing in ω. In the third term,

$$\frac{\partial}{\partial \omega} \left(-\frac{p \int_\omega^1 t(\Omega) \, d\Omega}{1 - p\omega} \right) = \frac{-p^2 \int_\omega^1 t(\Omega) \, d(\Omega) + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2}$$

and the integrand $t(\Omega)$ is decreasing in Ω, so

$$\cdots < \frac{-p^2(1 - \omega)t(\omega) + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2}$$

$$= \frac{-p^2 + p^2\omega + p - p^2\omega}{(1 - p\omega)^2} t(\omega)$$

$$= \frac{p(1 - p)}{(1 - p\omega)^2} t(\omega).$$

By our assumption that $\omega > \hat{\omega}$, we know that $t(\omega) < \hat{t} < E[\tilde{x}] = 0$, and so the derivative of the third term is also negative. Thus, $B(\omega)$ is strictly decreasing in ω for all $\omega > \hat{\omega}$.

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(ii) Since $\omega^* > \hat{\omega}$, $\partial B / \partial \omega < 0$. So because $B(\omega^*) = 0$, we have

$$B(\omega) = B(\omega) - B(\omega^*) = \int_{\omega^*}^{\omega} \frac{\partial B(\Omega)}{\partial \Omega} \, d\Omega < 0.$$

The contrapositive of this statement is the desired expression,

$$B(\omega) > 0 \Rightarrow \omega < \omega^*.$$

(iii) Proven as in part (ii).

∎

Proof of Proposition 3

For clarity, let us write $B(\omega) \equiv u^D(\omega) - u^C(\omega)$ explicitly in terms of the model parameters:

$$B(\omega, \phi, p, N) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_{\omega}^{1} t(\Omega) \, d\Omega.$$

For any set of parameter values (ϕ, p, N), the equilibrium disclosure frequency is uniquely defined by $B(\omega^*, \phi, p, N) = 0$. Because B is differentiable with respect to each of its parameters, the Implicit Function Theorem tells us how the equilibrium frequency changes with the parameter values. For each parameter $\theta \in \{\phi, p, N\}$, the IFT gives

$$\frac{\partial \omega^*}{\partial \theta} \equiv \frac{\partial \omega}{\partial \theta}\bigg|_{B=0} = -\frac{\frac{\partial B}{\partial \theta}\big|_{B=0}}{\frac{\partial B}{\partial \omega}\big|_{B=0}}.$$

Lemma A1 tells us that $\frac{\partial B}{\partial \omega} < 0$ for all $\omega > \hat{\omega}$. Differentiating with respect to the other model parameters yields

$$\frac{\partial B}{\partial \phi} = (1 - p\omega)^{N-1} > 0$$

$$\frac{\partial B}{\partial N} = \phi(1 - p\omega)^{N-1} \ln(1 - p\omega) < 0$$

$$\frac{\partial B}{\partial p} = -\omega\phi(N-1)(1 - p\omega)^{N-2} - \frac{1}{(1 - p\omega)^2} \int_{\omega}^{1} t(\Omega) \, d\Omega.$$

Note that $\int_{\omega}^{1} t(\Omega) \, d\Omega < 0$ is the expected value of x for a non-disclosing firm, which is negative. So the second term of $\frac{\partial B}{\partial p}$ is positive, while the first is negative. Which term dominates depends on the parameter values.

Applying the Implicit Function Theorem yields the desired comparative statics:

$$\frac{\partial B}{\partial \phi} > 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial \phi} = -\frac{\partial B / \partial \phi}{\partial B / \partial \omega} > 0,$$

$$\frac{\partial B}{\partial N} < 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial N} = -\frac{\partial B / \partial N}{\partial B / \partial \omega} > 0.$$

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As shown already, ω is decreasing in N. Since any monotonic bounded sequence of real numbers converges[7], and since we know $\omega_N^* > \hat{\omega}$ for all N, ω_N^* converges as $N \to \infty$. Let us refer to its limit as

$$\omega_\infty = \lim_{N \to \infty} \omega^* \tag{A11}$$

The function $B(\cdot)$ is continuous in ω_N^* and N, and $B_N(\omega_N^*) = 0$ for all N. The sequence $\{B_N(\omega_N^*)\}$ therefore converges to zero as well:

$$0 = \lim_{N \to \infty} B_N(\omega_N^*) \tag{A12}$$

$$= \lim_{N \to \infty} x(\omega_N^*) + \lim_{N \to \infty} \phi(1 - p\omega_N^*)^N - \lim_{N \to \infty} \frac{p \int_{\omega_N^*}^1 x(\Omega)\, \mathrm{d}\Omega}{1 - p\omega_N^*} \tag{A13}$$

$$= x(\omega_\infty) + 0 - \frac{p \int_{\omega_\infty}^1 x(\Omega)\, \mathrm{d}\Omega}{1 - p\omega_\infty} \tag{A14}$$

That is,

$$x(\omega_\infty) = \frac{p \int_{\omega_\infty}^1 x(\Omega)\, \mathrm{d}\Omega}{1 - p\omega_\infty}, \tag{A15}$$

and therefore $\omega_\infty = \hat{\omega}$. ∎

Proof of Proposition 4

For part (i), note that for all N and corresponding thresholds t_N^*, the threshold condition says

$$0 = t_N^* + \phi\big(1 - p + pF(t_N^*)\big)^{N-1} + \frac{p}{1 - p + pF(t_N^*)} \int_{-\infty}^{t_N^*} x f(x)\, \mathrm{d}x$$

Multiplying by $\big(1 - p + pF(t_N^*)\big)$ yields

$$0 = t_N^*\big(1 - p + pF(t_N^*)\big) + \phi\big(1 - p + pF(t_N^*)\big)^N - p \int_{-\infty}^{t_N^*} x f(x)\, \mathrm{d}x$$

$$= t_N^*\big(1 - p + pF(t_N^*)\big) + \phi Z_N - p \int_{-\infty}^{t_N^*} x f(x)\, \mathrm{d}x$$

$$\therefore \quad \phi Z_N = -t_N^*\big(1 - p + pF(t_N^*)\big) + p \int_{-\infty}^{t_N^*} x f(x)\, \mathrm{d}x$$

Differentiating with respect to N yields

$$\phi \frac{\mathrm{d}Z_N}{\mathrm{d}N} = -\frac{\partial t_N^*}{\partial N}\big(1 - p + pF(t_N^*)\big) - t_N^* p f(t_N^*) \frac{\partial t_N^*}{\partial N} + p \frac{\partial}{\partial N}\left(\int_{-\infty}^{t_N^*} x f(x)\, \mathrm{d}x\right) \frac{\partial t_N^*}{\partial N}$$

$$= -\frac{\partial t_N^*}{\partial N}\big(1 - p + pF(t_N^*)\big)$$

---

[7] Rudin, Theorem 3.14, "Principles of Mathematical Analysis."

Since $\frac{\partial t_N^*}{\partial N} > 0$, we have $\frac{\partial Z_N}{\partial N} < 0$, so Z_N is decreasing in N as desired.

For part (ii), note that the probability of any single firm failing to disclose converges to a number less than 1:

$$\lim_{N \to \infty} \left(1 - p + pF(t_N^*)\right) = 1 - p + pF(t_\infty) < 1$$

So the probability of zero firms disclosing converges to zero as $N \to \infty$,

$$\lim_{N \to \infty} Z_N = \lim_{N \to \infty} \left(1 - p + pF(t_\infty)\right)^N = 0$$

■

Proof of Proposition 5

We begin solving the monopolist's problem by adopting the following shorthand notation, $W \equiv (1 - p + pF(t))$ and $F' \equiv \frac{\partial F}{\partial t}$. Using first-order conditions with respect to ϕ we obtain,

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \phi} \left[\mathcal{L} + \phi\right] \\
&= \frac{\partial}{\partial \phi} \left[pN \left[F(t(\phi)) - F(\underline{x})\right] G(\ell) + \phi\right] \\
&= pNF'G(\ell)\frac{\partial t}{\partial \phi} + 1
\end{aligned}
\tag{A16}
$$

Recall from Proposition 3 we obtained $\frac{\partial t}{\partial \phi}$ from the Implicit Function Theorem using the net benefit of disclosure formula B. That is, $-\frac{\partial B/\partial \phi}{\partial B/\partial t}\Big|_{B=0} = \frac{\partial t}{\partial \phi}$. Therefore, we can rewrite Equation A16 as,

$$= -pNF'G(\ell)\left(\frac{\partial B/\partial \phi}{\partial B/\partial t}\Big|_{B=0}\right) + 1.$$

A rearrangment yields,

$$
\begin{aligned}
0 &= -\frac{\partial B}{\partial t}\Big|_{B=0} + pNF'G(\ell)\frac{\partial B}{\partial \phi}\Big|_{B=0} \\
&= -\left[1 + \phi(N-1)pF'W^{N-2} + \frac{p^2 F' \int_0^t x\, dF}{W^2} - \frac{ptF'}{W}\right] + pNF'G(\ell)W^{N-1} \\
&= -\left[1 + \phi(N-1)pF'W^{N-2} + \frac{pF'}{W}E[\tilde{x}_j|ND] - \frac{pF'}{W}t\right] + pNF'G(\ell)W^{N-1} \\
&= -\left[1 + \phi(N-1)pF'W^{N-2} + pF'\phi W^{N-2}\right] + pNF'G(\ell)W^{N-1} \\
&= -\left[1 + \phi NpF'W^{N-2}\right] + pNF'G(\ell)W^{N-1}.
\end{aligned}
$$

Solving for the optimal prize we obtain,

$$\phi = G(\ell)W - \frac{1}{NpF'W^{N-2}}.$$

However, if $\phi > \underline{\phi}$, then it is a dominant strategy for the monopolist to offer $\underline{\phi}$, because incentivizing firms to disclose below $\underline{x}$ yields no additional benefit. This yields that the optimal prize is equal to $\phi^* = \min\left(G(\ell)W - \frac{1}{NpF'W^{N-2}}, \underline{\phi} \right)$.

The prize that the monopolist can offer is positive so long as as $N \geq \frac{1}{G(\ell)pF'W^{N-1}}$. The monopolist is restricted to $\phi \in \mathbb{R}^+$, so when $N < \frac{1}{G(\ell)pF'W^{N-1}}$ the prize is equal to zero. $\blacksquare$

Proof of Proposition 6

Define a firm's "rank" according to the firms place among realized disclosures by competing firms. That is, if there are $k - 1$ higher disclosures, the firm has rank k and receives ϕ_k. A disclosing firm's rank is therefore a stochastic function of its disclosed value. We define $\tilde{r}(\omega)$ accordingly:

$$\tilde{r}(\omega) = \text{rank of a firm that discloses } x = F^{-1}(1 - \omega).$$

Using this notation, we would write the expected utility of disclosure in the base model as

$$
\begin{aligned}
u^D(\omega) &= t(\omega) + \phi W(\omega) \\
&= t(\omega) + \phi P\big(\tilde{r}(\omega) = 1\big).
\end{aligned}
$$

(Single Prize ϕ)

With prizes for the top K firms, the expected payout becomes

$$u^D(\omega) = t(\omega) + \sum_{k=1}^{K} \phi_k P\big(\tilde{r}(\omega) = k\big).$$

(Multiple Prizes $\{\phi_k\}$)

We wish to show that this value is decreasing in N. Unfortunately, we cannot claim that $P\big(\tilde{r}(\omega) = k\big)$ is decreasing in N without some further restrictions. Although the chance of having *at least* the kth-highest disclosure is strictly decreasing in N, the chance of having *exactly* the kth-highest disclosure may be increasing in N, at least for certain parameter values. We therefore rearrange the sum in order to write it in terms we know to be unconditionally decreasing in N,

$$
\begin{aligned}
u^D(\omega) &= t(\omega) + \sum_{k=1}^{K} \phi_k \Big(P\big(\tilde{r}(\omega) \leq k\big) - P\big(\tilde{r}(\omega) \leq k - 1\big) \Big) \\
&= t(\omega) + \sum_{k=1}^{K} (\phi_k - \phi_{k+1}) P\big(\tilde{r}(\omega) \leq k\big).
\end{aligned}
$$

Note that $P\big(\tilde{r}(\omega) \leq k\big)$, the probability of having at least the kth-highest disclosure, is strictly decreasing in N. Since prizes are strictly decreasing in rank, we also have $(\phi_k - \phi_{k+1}) > 0$. Therefore, $u^D(\omega)$ is unconditionally decreasing in N. We conclude that disclosure frequency decreases in N under a progressive prize structure. $\blacksquare$

Proof of Proposition 7

We consider the base model with prizes ϕ_N that increase with N according to some sequence $\{\phi_N\}$. Then the benefit of disclosing relative to concealing is a function of N,

$$B_N(\omega) = t(\omega) + \phi_N(1 - p\omega)^{N-1} - u^C(\omega).$$

The same holds for $(N+1)$ firms, so we can subtract the two equations to obtain

$$
\begin{aligned}
B_{N+1}(\omega) - B_N(\omega) &= \phi_{N+1}(1 - p\omega)^N - \phi_N(1 - p\omega)^{N-1} \\
&= \phi_N(1 - p\omega)^{N-1}\left(\frac{\phi_{N+1}}{\phi_N}(1 - p\omega) - 1\right).
\end{aligned}
\tag{A17}
$$

Under our assumption that $\lim_{N\to\infty}\frac{\phi_{N+1}}{\phi_N} < \frac{1}{1-p\hat{\omega}}$, there exists some $\overline{N}$ such that

$$N > \overline{N} \;\Rightarrow\; \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}},$$

so evaluating Equation (A17) at $\omega = \omega_N^*$ for any $N > \overline{N}$ yields

$$B_{N+1}(\omega_N^*) - 0 < \phi_N(1 - p\omega_N^*)^{N-1}\left(\frac{\phi_{N+1}}{\phi_N}(1 - p\hat{\omega}) - 1\right) < 0.$$

By Lemma A1, we obtain the desired $\omega_{N+1}^* < \omega_N^*$. ∎

Proof of Proposition 8

Let t_N^* be the equilibrium disclosure threshold with N firms. Suppose that a firm j observes and discloses exactly $x_j = t_N^*$. Then the probability q that any other given opponent observes a higher value is given by

$$q \equiv p\left(1 - F\left(t_N^*\right)\right) = p\omega_N^*.$$

Any such realization above the threshold will certainly be disclosed, so the number of firms who disclose values higher than t_N^* is a binomial random variable $\tilde{S} \sim B(N, q)$. The probability that firm j wins a prize is bounded by the probability that fewer than λN other firms disclose values higher than $\hat{t}$. That is,

$$W_j(t_N^*) \le P\left(\tilde{S} \le \lambda N - 1\right).$$

This probability is the weight of a left tail of the binomial distribution of $\tilde{S}$. We may bound it using Hoeffding's inequality (Hoeffding, 1963), which states that the sum $\tilde{s}$, of any N random variables, has the probabilistic bound

$$P\left(|\tilde{s} - E[\tilde{s}]| \ge c\right) \le 2\exp\left(\frac{-2c^2}{\sum_{i=1}^{N}(b_i - a_i)^2}\right) \tag{A18}$$

where the ith random variable is contained by the interval $[a_i, b_i]$. In our application, $\tilde{S}$ is the sum of $(N-1)$ identically-distributed Bernoulli trials with success probability q, so

$$a_i = 0, \quad b_i = 1, \quad E\left[\tilde{S}\right] = q(N-1).$$

We first transform our probability into the same form as Hoeffding's inequality,

$$\begin{aligned} W_j\left(t_N^*\right) &= P\left(\tilde{S} \leq \lambda N - 1\right) \\ &= P\left(\tilde{S} - E\left[\tilde{S}\right] \leq \lambda N - 1 - q(N-1)\right) \\ &\leq P\left(\left|\tilde{S} - E\left[\tilde{S}\right]\right| \geq (q-\lambda)N - q + 1\right). \end{aligned}$$

We then can apply the (A18) with $c = (q-\lambda)N - q + 1$ to obtain

$$W_j\left(t_N^*\right) \leq 2\exp\left(-\frac{2\left((q-\lambda)N - q + 1\right)^2}{N}\right). \tag{A19}$$

Note that firms will always disclose values above $\hat{t}$, so any equilibrium threshold t_N^* must be below $\hat{t}$. We therefore have

$$q = p(1 - F(t_N^*)) > p\left(1 - F\left(\hat{t}\right)\right) = p\hat{\omega} > \lambda.$$

This ensures that as $N \to \infty$, the exponential in (A19) goes to $-\infty$ and the right hand side goes to zero for any sequence of thresholds $\{t_N^*\}$. Since firms optimally respond to $W = 0$ by concealing all realizations below $\hat{t}$, the disclosure frequency converges to $\hat{\omega}$, as desired. ∎

Proof of Proposition 9

By our definition of $\hat{t}$, any firm with a realization $x_j > \hat{t}$ discloses even if they have no chance of winning the prize. This establishes a lower bound for both limits:

$$\lim_{N \to \infty} \nu_j \geq \hat{\omega} \quad \text{and} \quad \lim_{N \to \infty} \bar{\nu}_N \geq \hat{\omega}. \tag{A20}$$

(i) Suppose firm j discloses a lower value, $x_j < \hat{t}$. If any of the $N - j$ firms yet to act observes a value above $\hat{t}$, they will certainly disclose it. The probability that firm j wins the prize is therefore bounded above by

$$W_j\left(x_j\right) \leq W_j\left(\hat{t}\right) = \left(1 - p\left(1 - F(\hat{t})\right)\right)^{N-j} = (1 - p\hat{\omega})^{N-j}.$$

So as $N \to \infty$, the probability of winning the prize converges to zero. In this limit, so firm j will optimally conceal any values below $\hat{t}$, disclosing no more frequently than $\hat{\omega}$. Together with (A20), this establishes the desired result.

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(ii) Again, note that a firm that realizes $x_j < \hat{t}$ will not disclose unless it has a positive probability of winning the prize. Specifically, it will not disclose if any preceding firm has already disclosed a value above $\hat{t}$. That is, the probability of disclosing a value below $\hat{t}$ cannot possibly be larger than the probability that no preceding firm i has disclosed $x_i > \hat{t}$. This allows us to place a very loose upper bound on ν_j:

$$\nu_j = P(\tilde{x}_j < \hat{t}) \cdot P(D_j | x_j < \hat{t}) \quad + P\left(D_j | x_j > \hat{t}\right) \cdot P(\tilde{x}_j > \hat{t})$$

$$\leq (1 - \hat{\omega}) \cdot \prod_{i=1}^{j-1} P(U_i \text{ or } x_i < \hat{t}) + \hat{\omega} \cdot 1$$

$$= (1 - \hat{\omega})(1 - p\omega)^{j-1} + \hat{\omega}.$$

Averaging over all j yields

$$\bar{\nu}_N \leq \frac{1}{N} \sum_{j=1}^{N} \left((1 - \hat{\omega})(1 - p\hat{\omega})^{j-1} + \hat{\omega}\right)$$

$$= (1 - \hat{\omega}) \frac{1}{N} \left(\frac{1 - (1 - p\hat{\omega})^N}{1 - (1 - p\hat{\omega})}\right) + \hat{\omega}$$

$$= \frac{1 - \hat{\omega}}{p\hat{\omega}} \left(\frac{1 - (1 - p\hat{\omega})^N}{N}\right) + \hat{\omega}.$$

As $N \to \infty$, the first term vanishes, so $\lim_{N \to \infty} \bar{\nu}_N \leq \hat{\omega}$. Together with (A20), this yields the desired result.

■

Lemma A2. *Under equal shares competition, the signal that corresponds to a given probability ω, previously written as $x(\omega)$ becomes*

$$t_N(\omega) = \frac{1}{N} x(\omega). \tag{A21}$$

Similarly,

$$u_N^C(\omega) = \frac{1}{N} u^C(\omega) \tag{A22}$$

$$u_N^D(\omega) = \frac{1}{N} x(\omega) + \phi(1 - p\omega)^{N-1}. \tag{A23}$$

Proof of Lemma A2

Under the definition,

$$F_N(x) = F(Nx),$$

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we find, for any $p \in [0, 1]$, that

$$p = F_N\left(F_N^{-1}(p)\right) \equiv F\left(N F_N^{-1}(p)\right),$$

which can be rearranged to

$$F_N^{-1}(p) = \frac{1}{N} F^{-1}(p),$$

so for $p = 1 - \omega$, we have

$$F_N^{-1}(1 - \omega) = \frac{1}{N} F^{-1}(1 - \omega).$$

$$\therefore \qquad t_N(\omega) = \frac{1}{N} t(\omega).$$

Using this first result, the others follow quickly

$$u_N^D(\omega) \equiv t_N(\omega) + \phi(1 - p\omega)^{N-1}$$
$$= \frac{1}{N} t(\omega) + \phi(1 - p\omega)^{N-1}$$

$$u_N^C(\omega) \equiv \frac{p}{1 - p\omega} \int_\omega^1 t_N(\Omega) \, d\Omega$$
$$= \frac{p}{1 - p\omega} \int_\omega^1 \frac{t(\Omega)}{N} \, d\Omega$$
$$= \frac{1}{N} u^C(\omega).$$

(i) This follows immediately from our definition $F_N(x) \equiv F(Nx)$. As $N \to \infty$, $F_N(x) \to F(0)$ for every $x \in \mathbb{R}$.

(ii) For any $N > \frac{1}{p\hat{\omega}}$, Proposition 10 tells us that ω_N^* is decreasing in N. Since any monotonic bounded sequence of real numbers converges[8], and since we know $\omega_N^* > \hat{\omega}$ for all N, ω_N^* converges. Let us refer to its limit as

$$\omega_\infty = \lim_{N \to \infty} \omega_N^* \tag{A24}$$

---

[8]Rudin, Theorem 3.14, "Principles of Mathematical Analysis."

The function $B(\cdot)$ is continuous in ω_N^* and N, and $B_N(\omega_N^*) = 0$ for all N. The sequence $\{B_N(\omega_N^*)\}$ therefore converges to zero as well:

$$0 = \lim_{N \to \infty} B_N(\omega_N^*) \tag{A25}$$

$$= \lim_{N \to \infty} x(\omega_N^*) + \lim_{N \to \infty} \phi(1 - p\omega_N^*)^N - \lim_{N \to \infty} \frac{p \int_{\omega_N^*}^1 x(\Omega) \, d\Omega}{1 - p\omega_N^*} \tag{A26}$$

$$= x(\omega_\infty) + 0 - \frac{p \int_{\omega_\infty}^1 x(\Omega) \, d\Omega}{1 - p\omega_\infty} \tag{A27}$$

That is,

$$x(\omega_\infty) = \frac{p \int_{\omega_\infty}^1 x(\Omega) \, d\Omega}{1 - p\omega_\infty}, \tag{A28}$$

and therefore $\omega_\infty = \hat{\omega}$.

■

Proof of Proposition 10

*N*ote: This proof calls upon Proposition 11, which comes next in the appendix. Clearly $P(P_j|N) < 1$, so

$$NP(D_j|N) > 1 \quad \Rightarrow \quad NP(D_j|N) > P(P_j|N),$$

which implies $\omega_{N+1}^* < \omega_N^*$ by Proposition 11. We can achieve this inequality by a sufficient assumption,

$$N > \frac{1}{p\hat{\omega}} \quad \Rightarrow \quad NP(D_j|N) > 1.$$

■

Proof of Proposition 11

Applying Proposition A2 to the definition of $B(\omega)$ under equal shares competition yields

$$B_N(\omega) = u_N^D(\omega) - u_N^C(\omega)$$
$$= \frac{1}{N} t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{1}{N} u^C(\omega).$$
$$\therefore \quad NB_N(\omega) = t(\omega) + N\phi(1 - p\omega)^{N-1} - u^C(\omega). \tag{A29}$$

The same holds for $N + 1$. That is,

$$(N + 1)B_{N+1}(\omega) = t(\omega) + (N + 1)\phi(1 - p\omega)^N - u^C(\omega). \tag{A30}$$

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Subtracting Equation (A29) from Equation (A30) yields

$$(N+1)B_{N+1}(\omega) - NB_N(\omega) = (N+1)\phi(1-p\omega)^N - N\phi(1-p\omega)^{N-1}$$
$$= \phi(1-p\omega)^{N-1}\big((N+1)(1-p\omega) - N\big)$$
$$= \phi(1-p\omega)^{N-1}\big(N+1 - Np\omega - p\omega - N\big)$$
$$= \phi(1-p\omega)^{N-1}\big((1-p\omega) - Np\omega\big). \tag{A31}$$

If we at $\omega = \omega_N^*$, then $B_N(\omega_N^*) = 0$, so Equation (A31) reduces to

$$B_{N+1}(\omega_N^*) = \frac{\phi(1-p\omega_N^*)^{N-1}}{N+1}\Big((1-p\omega_N^*) - Np\omega_N^*\Big).$$

Focusing on the sign of the term in parenthesis, we find

$$N > \frac{1-p\omega_N^*}{p\omega_N^*} \quad \Rightarrow \quad B_{N+1}(\omega_N^*) < 0 \quad \Rightarrow \quad \omega_{N+1}^* < \omega_N^*, \tag{A32}$$

where the second implication is due to Lemma A1. That is, disclosure at the frequency ω_N^* gives $B < 0$, so the marginal disclosure loses value. The equilibrium frequency ω_{N+1}^* must be lower.

This shows that the entry of the $(N+1)^{th}$ firm reduces disclosure when N is large. When N is smaller than the threshold, the inequalities in Equation (A32) are reversed, as shown by the same logic. This completes the equivalence.

Finally, note that the ex ante probability of a firm j disclosing is the joint probability that it observes x_j and that x_j exceeds its threshold. Given the number of competing firms, this means

$$P(D_j|N) = p\omega_N^* \quad \text{and} \quad P(P_j|N) = 1 - p\omega_N^*,$$

so the same equivalency holds using the threshold $N > \frac{P(P_j|N)}{P(D_j|N)}$, as desired. ∎

Proof of Proposition 12

Under generalized competition with N firms, we have

$$B_N(\omega) = t_N(\omega) + \phi(1-p\omega)^{N-1} - u_N^C(\omega)$$
$$= \alpha_N t(\omega) + \phi(1-p\omega)^{N-1} - \alpha_N u^C(\omega).$$
$$\therefore \quad \frac{1}{\alpha_N}B_N(\omega) = t(\omega) + \frac{1}{\alpha_N}\phi(1-p\omega)^{N-1} - u^C(\omega).$$

The same holds for $N+1$, so we can subtract the two equations to obtain

$$\frac{1}{\alpha_{N+1}}B_{N+1}(\omega) - \frac{1}{\alpha_N}B_N(\omega) = \frac{1}{\alpha_{N+1}}\phi(1-p\omega)^N - \frac{1}{\alpha_N}\phi(1-p\omega)^{N-1}$$
$$= \frac{1}{\alpha_{N+1}}\phi(1-p\omega)^{N-1}\left((1-p\omega) - \frac{\alpha_{N+1}}{\alpha_N}\right).$$

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Evaluating at ω_N^* and rearranging terms yields

$$B_{N+1}(\omega_N^*) = (1 - p\omega_N^*)^{N-1}\phi\left((1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N}\right).$$

Under our assumption that $\lim_{N\to\infty}\frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega}$, there exists some $\overline{N}$ such that

$$N > \overline{N} \Rightarrow \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega}.$$

So for $N > \overline{N}$, we obtain

$$\begin{aligned}
B_{N+1}(\omega_N^*) &= (1 - p\omega_N^*)^{N-1}\phi\left((1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N}\right) \\
&< (1 - p\omega_N^*)^{N-1}\phi\big((1 - p\omega_N^*) - (1 - p\hat{\omega})\big) \\
&= (1 - p\omega_N^*)^{N-1}\phi\left(\hat{\omega} - \omega_N^*\right).
\end{aligned}$$

$$\therefore \quad B_{N+1}(\omega_N^*) < 0.$$

By Lemma A1, we conclude that $\omega_{N+1}^* < \omega_N^*$, as desired. ∎