Exponential Individuals, Hyperbolic Households

Andrew Hertzberg*

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Abstract

This paper introduces a model of household consumption and savings in which household members have imperfectly aligned altruistic preferences. Specifically, member A values his own consumption more than member B values A's consumption. Each period, members independently choose the amount of household wealth to consume as Nash best responses. At each point in time, the household consumes a higher fraction of wealth than under the full commitment Pareto optimum. Ex-ante Pareto optimal household consumption plans are not subgame perfect because both members wish to deviate to increase their own consumption. As a result the household is willing to pay for a technology that commits them to an optimal lifetime consumption plan. Despite both members individually having time consistent exponential discount rates, equilibrium household consumption dynamics are captured by a single representative agent with a hyperbolic discount factor that is microfounded in the degree of preference misalignment within the household.

*Columbia Business School. E-mail: ah2692@columbia.edu. I thank Patrick Bolton, Pierre-Andre Chiappori, Daniel Gottlieb, Daniel Paravisini, Bryan Routledge, Tano Santos, Daniel Wolfenzon, and seminar participants at Columbia Business School, Stockholm School of Economics, as well as conference participants at the Western Finance Association Conference and Household Finance NBER Summer Institute. All errors are my own.

Are households able to carry out optimal consumption and savings plans? Recent evidence shows that households value technologies that allow them to commit to increase savings and that these raise savings rates (Thaler and Benartzi 2004; Ashraf, Karlan, and Yin 2006). This is inconsistent with standard models of consumption and savings based on individual maximization. One explanation is that individuals have hyperbolic discount factors or self control problems that render optimal savings plans time inconsistent so that individuals will, ex-post, wish to save less than planned (for example: Thaler Shefrin 1981; Laibson 1997; Laibson, Repetto, and Tobacman 1998; Harris and Laibson 2001). This paper takes a different approach and shows that the same under-saving and time inconsistency arises endogenously in a household where the individual members place more weight on the utility from their own consumption than their partner does. This occurs despite the individual members of the household being fully rational and having time consistent preferences.

There is abundant evidence that household members do not have perfectly aligned preferences. For example, household consumption decisions are different when money is received by one partner or the other (Lundberg, Pollak, and Wales 1997, Phipps and Burton 1998, Ashraf 2009). In the model I propose here the household is comprised of two members who each choose how much of the combined household wealth to spend on their own private consumption. The crucial assumption I make is that household members have imperfectly aligned altruistic preferences. Specifically, member A cares more about the utility from his consumption than B cares about A's consumption and vice versa. Both members have the same exponential time preferences and agree on the optimal savings rate for the household. I characterize the household's equilibrium consumption path without commitment as a subgame perfect Nash equilibrium in consumption choices. This is the equilibrium that obtains when household members are unable to enforce contracts conditional on their consumption choices. The household is unable to carry out the optimal consumption plan ex-post because both members wish to deviate and increase their own consumption. This intuition is closely

related to the theoretical literature on dynamic commons problems that has been used to study national underinvestment (Lancaster 1973, Tornell and Velasco 1992), overexploitation of natural resources (Levhari Mirman 1980), and sovereign debt (Amador 2008).

The household is willing to pay for a technology that allows them commit to any Pareto optimal consumption plan. The model allows me to numerically calculate the value of commitment. I show that it is increasing in the degree to which members value their own utility over their partner's.

Next I find the preferences of a single representative agent that would achieve the same time path of consumption as the household. This representative agent is shown to have time preferences with the same exponential discount factor as the household members and a hyperbolic discount factor. This is despite both household members individually having the same time consistent exponential discount rates and not being hyperbolic discounters. The hyperbolic discount factor is microfounded in the misalignment of preferences between the two household members. It is decreasing (i.e. is "more hyperbolic") when household members have more divergent interests.

I extend the basic model in several ways. First, I generalize the results beyond the case of log utility functions to allow individual members of the household to have CRRA preferences. I show that the distortion to the full commitment path and the value of commitment are increasing in the elasticity of intertemporal substitution. Next, I introduce a public non-rival consumption good that is shared by both members of the household. I show that it is only in the consumption of private goods that the commitment problem occurs. As a result the intertemporal inefficiency and the value of commitment are strictly decreasing in the importance of these public goods in the household.

One interpretation of this paper is that it provides a microfoundation for hyperbolic savings and consumption behavior at the household level. However the psychological evidence for hyperbolic discounting is conducted primarily at the level of the individual (Ainslie 2001). To accommodate this, I extend the model to allow the individual members of the household

to have hyperbolic time preferences. Not surprisingly, this exacerbates the inefficiency of the household consumption path and further increases the value of commitment technologies. More interestingly, I show that hyperbolic individual preferences amplify the household problem and that the value of commitment when both problems are combined is larger than its value when both problems are considered in isolation. As such, the goal of the paper is not to replace individual hyperbolic preferences as a description of decision making. Rather my purpose is to show that household decision making also naturally renders optimal intertemporal plans time inconsistent and that in combination both channels can produce sizeable distortions to optimal savings plans and create large demands for commitment technologies.

This paper is closely related to the theoretical literature that incorporates misaligned preferences within the household (see Lundberg and Pollack 2007; Browning, Chiappori, and Lechene 2006 for comprehensive surveys of the literature). In these papers, household decision making is modeled as the outcome of a bargaining process and the focus is directed to determining what determines the threat points and bargaining weights of each household member. As a result, allocations are assumed to be Pareto optimal both within any period and over time. However, it is not obvious that households are able to enforce bargained outcomes ex-post. This is supported by recent evidence (Mazzocco 2007). The equilibrium I study in this paper is the one that obtains when no such commitment is possible and I use this to fully characterize the household's demand for intertemporal commitment. It is not my objective to argue that households suffer the time inconsistency problem without taking actions to mitigate it. Rather the goal is to show that households have an inherent tendency to undersave and to provide a framework for assessing the types of strategies that households may employ to overcome this problem.

The paper proceeds as follows. Section I sets up the base model. Section II characterizes the equilibrium consumption choices of the household, compares them to the full commitment solution, and computes the value of commitment for the household. Section III characterizes the preferences of the household's representative agent. Section IV generalizes the basic

model in several ways, by allowing household member to have CRRA preferences, to consume a shared public consumption good, and to individually have hyperbolic time preferences. Section V studies an alternate consecutive move version of the model designed to show that the equilibrium studied in the base model is the only one robust to the assumed timing of consumption within a period. Section VI concludes.

I Model of Household Consumption

The household has two members indexed by i labeled A and B. Time is discrete and indexed by t. The household is formed at the beginning of period $t = 1$. Both household members live for T years. Each year contains $N \geq 1$ periods so that there are NT periods in total. For the bulk of the analysis it is sufficient to think of $N = 1$ however in Section V I will consider the limiting case as consumption decisions are made in continuous time by letting $N \to \infty$. I assume that the household remains together for their entire lives with certainty.

A Preferences

Each period household member i consumes a single consumption good. Let $C_{i,t}$ denote the amount of this good consumed by member i in period t. The utility derived by member i from their own consumption in period t is

$$u_{i,t} = \ln C_{i,t}. \tag{1}$$

Note that member i does not directly derive utility from member j's consumption. Later in Section IV I extend the model to also allow for a non-rival public consumption good which is consumed jointly by both household members.

Both household members discount utility from future consumption using exponential discount factor $\beta^{\frac{1}{N}} \in (0, 1)$. The individual discounted utility of household member i in period

t is

$$U_{i,t} = \sum_{x=0}^{T-t} \beta^{\frac{x}{N}} u_{i,t+x}. \tag{2}$$

Thus $U_{i,t}$ is the utility of household member i absent any concern for the utility of the other household member. Note that these are standard time preferences so that when considered on their own the optimal consumption plan for each household member will be time consistent. Only in Section IV do I extend the model to also allow the individual members of the household to have time inconsistent preferences.

One of the defining characteristics of the household is that its members are altruistic. I capture this by supposing that member i places weight $\delta_i \in (0,1)$ on their own utility and weight $1-\delta_i$ on the utility of the other member. I focus on the case where the altruism between household members is imperfect by assuming that

$$\Delta \equiv \delta_A - (1-\delta_B) \geq 0. \tag{3}$$

In words, Δ measures the degree to which member i places more weight on her own discounted utility $U_{i,t}$ than member $j \neq i$ places on $U_{i,t}$. When $\Delta = 0$, both members agree on the weights to place on their own individual utility with the simplest case being $\delta_A = \delta_B = \frac{1}{2}$. The framework can also be used to study the case where members care more about each other than themselves ($\Delta < 0$) however since the evidence on household consumption decisions suggests this is generally not the case I will not focus on this scenario.

The total discounted utility of member i at t is

$$V_{i,t} = \delta_i U_{i,t} + (1-\delta_i) U_{j,t}. \tag{4}$$

This is the object each household member will maximize when taking actions at t.

B Household Budget Constraint

The present value of all combined household wealth at the beginning of $t = 1$ is W_1. I set aside household labor supply decisions and take W_1 as given. The second defining characteristic of the household is that all wealth is combined so that both household members have full access to the remaining combined wealth in each period. For simplicity I normalize the price of the consumption goods consumed by both household member to one. Any wealth not consumed by the household is saved between periods at a gross risk free interest rate of $R^{\frac{1}{N}}$. Household wealth evolves according to the following

$$W_{t+1} = R^{\frac{1}{N}} (W_t - X_t) \tag{5}$$

where

$$X_t = C_{A,t} + C_{B,t} \tag{6}$$

is total household expenditure in period t. The wealth of the household at t can be interpreted as the present value of lifetime earnings less the present value of all consumption prior to period t. In effect, I assume that both household members can borrow and lend against the combined lifetime income of the household in a frictionless capital market at gross annual interest rate R. As a result in any period, the household is able to spend at most the total value of all remaining household wealth W_t.

C Decision Making

Household members cannot commit to a path of consumption. As a result, household members are unable to enforce mutually agreed levels of consumption, either in the present or the future. Household members non-cooperatively simultaneously decide how much of the household wealth W_t to spend on their own private consumption $C_{i,t} \geq 0$ each period. The dynamic equilibrium path of consumption will be the Nash subgame perfect solution to the

consumption game between these two members. Let a single "*" denote the non-cooperative equilibrium quantities $C^*_{i,t}$.

Since both members make consumption decisions simultaneously it is possible that both members could attempt to spend more than total household wealth. To avoid this problem I assume that both members are able to consume at most half the total household wealth in any single period:

$$C_{i,t} \leq \frac{W_t}{2}. \tag{7}$$

This condition can be made arbitrarily weak by making N large. For example, (7) implies that within a year one member can withdraw up to $W_t\left(1-\frac{1}{2^N}\right)$. As $N \to \infty$ this implies that all wealth can be withdrawn in any finite period of time. By imposing (7) I ensure $C_{A,t}+C_{B,t} \leq W_t$ and hence have a well defined budget constraint for each household member's consumption problem each period. I show in the Appendix that (7) does not bind in any period $t < NT$ if and only if

$$|\delta_A - \delta_B| \leq \beta^{\frac{1}{N}}. \tag{8}$$

I assume that (8) holds. Note that when N is large $\beta^{\frac{1}{N}} \to 1$ and this constraint places almost no limit on parameters.

I have chosen to require (7) in order to avoid imposing arbitrary tie breaking rules to deal with situations where members attempt, in total, to spend more than W_t. Depending on the rule chosen other possible equilibria may arise in the simultaneous move consumption game. In Section V I revisit this problem by assuming that household members make consecutive consumption decisions. In that setting a simple one person budget constraint in which members are able to spend up to the full amount of remaining household wealth is imposed. I show that the equilibrium studied here is arbitrarily close to the unique equilibrium from that model as $N \to \infty$ thus demonstrating that this assumption is not crucial for the results studied below. Note that, in equilibrium, condition (7) will only bind in the final period of the household's life and will ensure that in that period $C_{i,NT} = \frac{W_{NT}}{2}$.

D *Full Commitment Problem and the Value of Commitment*

To evaluate the optimality of the non-cooperative equilibrium consumption path, I compare it to the consumption path that would be achieved if the household was able to fully commit to consumption choices at the start of $t = 1$. Consider the problem the household would face in setting a full commitment path. Whenever $\delta \neq \frac{1}{2}$ household members will disagree over the optimal allocation. However any allocation that they would choose must be Pareto optimal and hence I characterize the solution to the following full commitment Pareto problem:

$$\max_{\left\{\{C_{i,t}\}_{t=1}^{t=NT}\right\}_{i\in\{A,B\}}} \Pi = \eta V_{A,1} + (1-\eta)\, V_{B,1} \tag{9}$$

$$\text{subject to} \quad W_1 - \sum_{x=1}^{NT} R^{-\frac{x-1}{N}} \left[C_{A,x} + C_{B,x}\right] \geq 0 \text{ and} \tag{10}$$

$$\{C_{A,t}, C_{B,t}\}_{t=1}^{t=NT} \geq 0. \tag{11}$$

where $\eta \in [0,1]$ is the pareto weight placed on the objective of member A. Let a double "**" denote the full commitment pareto optimal consumption quantities $C_{i,t}^{**}$ that solve this problem.

To quantify the welfare loss incurred by the household under the non-cooperative equilibrium I calculate how much the household would be willing to pay at $t = 1$ for a technology that allowed them to commit to an optimal consumption path. Let $V_{i,1}^{*}(W_1)$ be the discounted lifetime utility that will be achieved by household member i absent commitment as a function of initial household wealth. Let $V_{i,1}^{**}(W_1(1-\phi), \eta)$ be the counterpart for the case where the household has spent a fraction ϕ of their initial wealth W_1 to achieve the full commitment plan that places weight η on the preferences of member A. The value of commitment ϕ^{**} is defined as the most that the household will pay while ensuring that there exists a weight η so that the purchase is a pareto improvement for both members. Formally

ϕ^{**} solves:

$$\begin{aligned} & \phi^{**} = \max_{\phi,\eta} \phi \\ \text{subject to} \quad & V_{i,1}^{**}\left(W_1\left(1-\phi\right),\eta\right) \geq V_{i,1}^{*}\left(W_1\right) \text{ for } i \in \{A,B\}, \text{ and } \eta \in [0,1]. \end{aligned} \tag{12}$$

An analytical solution for ϕ^{**} is intractable in most cases so this will be solved for numerically.

II Consumption Choices and the Value of Commitment

A Non-Cooperative Equilibrium Consumption Choices

The equilibrium consumption path is solved in the Appendix. The equilibrium level of consumption by member i in period $t < NT$ is

$$C_{i,t}^{*} = \frac{\delta_i}{1+\Delta+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}} W_t. \tag{13}$$

By assumption the equilibrium consumption in period $t = NT$ is $C_{i,t}^{*} = \frac{W_t}{2}$. This is the unique interior equilibrium.[1] The allocation of consumption within any period is determined the weights each member place on their own utility

$$\frac{C_{i,t}^{*}}{C_{j,t}^{*}} = \frac{\delta_i}{\delta_j}.$$

If member A places more weight on his own utility than B places on hers then A will have a larger share of consumption in each period. Total equilibrium household expenditure in

[1] There is another set of trivial equilibria in which both members set $C_{i,t}^{*} = 0$. This is optimal only becasue of the log utiltiy assumption. If instead I assume the period utility function to be $u_{i,t} = \ln\left(C_{i,t}+\varepsilon\right)$ for any arbitrarily small $\varepsilon > 0$ this equilibrium would not exist. No equivalent of this equilibria exists in the consecutive move version of the model in Section V. Hence I ignore this for the rest of the paper.

period t is

$$X_t^* = \frac{1}{1 + \frac{1}{1+\Delta}\sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t. \tag{14}$$

The share of wealth that is spent in any period $t < NT$ is strictly increasing in Δ, the degree to which household members weigh their own individual utility more than their partner's. The dynamics of equilibrium consumption between periods is

$$\frac{X_{t+1}^*}{X_t^*} = (R\beta)^{\frac{1}{N}} \left(\frac{\sum_{x=0}^{NT-(t+1)} \beta^{\frac{x}{N}}}{\Delta + \sum_{x=0}^{NT-(t+1)} \beta^{\frac{x}{N}}} \right). \tag{15}$$

The higher is Δ the more downward sloping is the equilibrium consumption path. The dynamics of household consumption is determined only by Δ and not the particular values of δ_A and δ_B that give rise to that degree of misalignment. So two households in which both household members are care slightly more for themselves with $\delta_A = \delta_B = 0.6$ will have an identical path of total consumption as one in which one member cares more for himself and the other cares equally for both with $\delta_A = 0.7$ and $\delta_B = 0.5$. The only difference in these household will be the consumption share of each member within a period but the total level of consumption will be identical.

B Comparison to Full Commitment Consumption Path

To assess the optimality of the equilibrium consumption path that the household will achieve without commitment, I compare it to the set of Pareto optimal consumption paths, one of which would be chosen if the household had access to perfect commitment at $t = 1$. The total level of consumption that the household would commit to in any period is

$$X_t^{**} = \frac{1}{1 + \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t. \tag{16}$$

The optimal total level of consumption is not affected by the Pareto weight η given to each member in the planning problem. Comparing the full commitment solution to the equilibrium

level of consumption leads directly to the following proposition:

Proposition 1: *If $\Delta > 0$ then in any period $t < NT$ the non-cooperative equilibrium level of consumption is higher than the amount that the household would commit to conditional on entering the period with wealth W_t.*

The proof of Proposition 1 comes directly by comparing (14) with (16). The intuition for this result is as follows. When making their consumption choices each household member trades off the benefit of a dollar spent on consumption for themselves versus a the benefit of saving a dollar for the combined household. Both household members place weight δ_i on the utility from their own consumption relative to a combined weight of unity for the discounted value of household savings. Thus, in total, the household acts as though it places weight $1+\Delta$ on it's current self relative to the combined future interest of the household. The social planner, for any pareto weight η always places the same weight on the discounted utility of the household in each period. Put differently, the full commitment solution is not subgame perfect because at least one household member will wish to unilaterally deviate from this allocation by spending slightly more on themselves. There is no distortion to savings only when member i cares about her own utility as much as j does ($\Delta = 0$). In this case both members have the same objective and the non-cooperative growth rate of consumption is identical to the full commitment equilibrium.

The dynamics of consumption under the full commitment consumption path are given by:

$$\frac{X^{**}_{t+1}}{X^{**}_t} = R\beta. \tag{17}$$

Thus a direct corollary of Proposition 1 is that the slope of the consumption path in the non-cooperative equilibrium is strictly below the slope of the full commitment consumption path whenever $\Delta > 0$. This is seen immediately by comparing (15) and (17).

The equilibrium consumption path is compared to the full commitment solution in Figure 1 assuming $T = 50$, $\beta = 0.95$, $R = \frac{1}{0.95}$, and $N = 1$. The figure illustrates that early in

the life of the household the equilibrium level of consumption is higher than under the full commitment solution. When both members place 60% weight on their own utility ($\delta_A = \delta_B = 0.6$) in total the household spends over 18% more than it would under the full commitment solution in the first year. If this altruism is reduced so that $\delta_A = \delta_B = 0.7$ then the household overspends by more than 37% in the first year of it's life. The under-provision of savings means that later in the households life they consume much less than under the full commitment optimum. If $\delta_A = \delta_B = 0.6$ then the household consumption is less than 60% of the level that the household would like to commit to for each of the last five years of the households life.

In the full commitment solution, only the allocation of consumption within each period is determined by the pareto weight assigned to each household member in the planning problem. For a given pareto weight η the ratio of each members consumption in any period $t < NT$ is

$$\frac{C^{**}_{A,t}}{C^{**}_{B,t}} = \frac{\eta\delta_A + (1-\eta)(1-\delta_B)}{(1-\eta)\delta_B + \eta(1-\delta_A)}.$$

From this expression we see that if member i was given full control to chose the consumption path of both members then the ratio of her consumption to her partner's would be $\frac{\delta_i}{1-\delta_i}$ in each period.

C The Value of Commitment

Having shown that the allocation of consumption achieved in the non-cooperative solution is inefficient I now turn to quantifying this inefficiency. To do this I ask what fraction ϕ^{**} of the household's initial wealth would both household members agree to spend in order to achieve a pareto efficient allocation. Due to the assumption of log utility this fraction will be independent of the level of initial household wealth. Despite this, an analytical solution for ϕ^{**} is in most cases intractable. Instead I solve for this fraction numerically in Figure 2. Panel A shows that the value of commitment increases monotonically with the weight that

household members place on their own utility relative to the utility of the other. A household in which both members place weight $\delta_i = 0.6$ on their own utility will be prepared to pay up to 1.61% of the present value of their total wealth to achieve full commitment. If both members place weight $\delta_i = 0.7$ on their own utility the undersaving problem is more severe and they would be willing to pay up to 5.62% of total household wealth to eliminate this inefficiency.

Panel B shows that the value of commitment varies non-monotonically with the discount rate of the household members. This stems from the fact that there are two countervailing forces. First, when β is larger, both household members care more about the future and hence are willing to pay more to avoid the effect that undersaving will have on their future consumption levels. Conversely, increasing β raises both household members individual desire to save and thus mitigates the problem. Panel B shows that this first force dominates for most values of β and is only reversed by the second force when β is very close to unity.[2]

Panel C considers the effect of changing the pattern of self interest within the household holding Δ constant at 0.2. When $\delta_A = 0.6$ then both household members are equally self interested since $\delta_B = 0.6$. Conversely when $\delta_A = 0.7$ then $\delta_B = 0.5$ implying that A cares more about his own utility than B's but that B cares equally for both. Recall that (14) and (15) show that the dynamic path of consumption for the household is identical under both scenarios, it is just the allocation of that consumption within each period that is different. The results in Panel C indicate that the value of commitment is increasing with the degree of asymmetry of self interest in the household. However, noting the y-axis, the magnitude of this effect is second order when compared to changes in Δ as shown in Panel A.

[2] Qualitatively, the same non-monotonic relationship obtains for all other reasonable paramter choices.

III Representative Agent

Typically household savings and consumption decisions are modeled as if they are made by a single optimizing representative agent. If the interests of household members are perfectly aligned ($\Delta = 0$) then this assumption involves no loss of generality since both members have the same objective function. In this case the representative agent will have the same time preferences as the individual household members. In this section I find the representative agent for a household in which the interests of its members are not perfectly aligned. Of particular interest, I ask whether it is possible to find a representative agent that would achieve the same consumption path and what time preferences would this agent have. We know already that the time preferences of the representative agent must be different to that of its individual members since those preferences are time consistent and would give rise a consumption path identical to the full commitment solution.

Since the primary focus of this paper is the intertemporal choices of the household I consider a representative agent with preferences over the level of total household consumption X_t. Matching the allocation of consumption within each period between $C_{A,t}$ and $C_{B,t}$ is not interesting because in equilibrium these are consumed in a constant ratio.[3] Consider the problem of a single representative agent who chooses the level of X_t, each period. The period utility of the representative agent is

$$u_{r,t} = \ln X_t. \tag{18}$$

The discounted utility of the representative agent at time t is

$$U_{r,t} = u_{r,t} + \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} u_{r,t+x} \tag{19}$$

where $\beta_r \in (0,1]$ is a standard exponential discount factor and $\Omega_r \in (0,1]$ is a quasi hyperbolic

[3] This can be achieved by using a more general period utility function over $C_{A,t}$ and $C_{B,t}$ of the form $u_{r,t} = \mu_{A,r} \ln C_{A,t} + (1 - \mu_{A,r}) \ln C_{B,t}$ does not alter the time preferences found in Proposition 2.

discount factor of the type introduced by Laibson (1997). While more general utility functions and discount functions could be considered the results below show that this form is sufficiently flexible to represent the household. The representative agent faces the same intertemporal budget constraint as the household (5).

As stressed by Laibson (1997), when $\Omega_r < 1$ any optimal path of consumption from the perspective of the representative agent at t will be time inconsistent. When considering the representative agent without commitment, I study the problem where the agent is aware of this time inconsistency and takes it into account when making consumption choices each period. As a result the consumption path chosen by the representative agent will be found by backward induction where consumption choices are subgame perfect best responses given the resulting choices that they will lead to in the future. The consumption path of the representative agent with and without commitment is solved in the Appendix.

Proposition 2: *The representative agent without commitment has an identical path of total consumption as the household without commitment if:*

$$i.\ \beta_r = \beta, \text{and}$$
$$ii.\ \Omega_r = \frac{1}{1+\Delta}.$$

Whenever $NT > 2$ and is finite this is the unique set of time preferences that replicate he consumption path of the household.

Proposition 2 is proved in the Appendix. The key result in Proposition 2 is that $\Delta > 0$ implies $\Omega_r < 1$. Thus, when household members care more about their own utility than their partner does, the representative agent for the household must have hyperbolic time preferences. This microfoundation for the hyperbolic discount factor of the representative agent captures the central intuition for the household undersavings result documented in Proposition 1. At any point in time, when a household member decides how much to spend

on private consumption he places weight δ_i on the utility from this consumption relative to unity for the combined marginal utility of an additional dollar of savings. Since in equilibrium both members are making the same trade-off, in total they act as though they are currently worth $1+\Delta$ relative to unity for their combined marginal utility from future savings. In total, despite the fact both members of the household have standard exponential time preferences, the household acts as if it always discounts the entire future with hyperbolic discount factor $\Omega_r = \frac{1}{1+\Delta} < 1$. Note that if both household members care about their own utility as much as their partner does ($\Delta = 0$) then only in this case does the representative agent also have standard exponential time preferences ($\Omega_r = 1$). Thus, even if we believe that individuals have time consistent exponential preferences, modelling households savings and consumption decisions as if the household has standard time consistent preferences is valid only if we assume that household members have perfectly aligned objectives.

IV Generalizing Household Preferences

This section studies several extensions to the preferences assumed for household members in the model studied so far. First, I allow household members to have CRRA period utility functions to study how the results vary with the elasticity of intertemporal substitution of the household members. Next, I study how the presence of public consumption goods within the household impacts equilibrium savings. Finally, I allow the individual members of the household to have time inconsistent hyperbolic time preferences of the type emphasized by Laibson (1997). The goal is to show how these extensions affect household savings decisions and the value of commitment. I will consider each extension one at a time and in isolation so as to highlight the differences from the base model results presented above.

A CRRA Utility

The model presented so far, by assuming log period utility, has implicitly concentrated on the case where the elasticity of intertemporal substitution (EIS) for both household members is unity. The literature which has sought to estimate the EIS has produced mixed results. Estimates range between being close to zero (Hall 1988, Dynan 1993) to being as high as two (Blundell, Browning and Meghir 1994; Mulligan 2002; and Gruber 2006). I study how the household saving problem changes with different values of the EIS by replacing the log period utility function in (1) with a CRRA utility function of

$$u_{i,t} = \frac{C_{i,t}^{1-\frac{1}{\zeta}} - \frac{1}{\zeta}}{1-\frac{1}{\zeta}}. \tag{20}$$

Here ζ is the EIS of each household member. The log utility case studied so far is a special case of this utility function where $\zeta = 1$. The rest of the framework remains the same as before.[4]

An analytical solution to the non-cooperative equilibrium is fully characterized in the Appendix. In this generalized setting the solutions for equilibrium consumption choices are generally intractable. To avoid this I focus on numerical examples illustrating the resulting equilibrium household consumption path. These are presented in Figure 3. Panel A shows how the equilibrium consumption path of the household varies with the EIS and compares it to the full commitment consumption path. In each case I assume that both members place weight $\delta_i = 0.6$ on their own utility. Note first that since these are drawn for $\beta = \frac{1}{R}$, the full commitment consumption path is flat and identical for each value of ζ. The panel shows that degree of undersavings is increasing in the EIS. For these parameters, when $\zeta = 0.5$, the household spends over 8% more than the full commitment level of consumption in the first year of its life. If instead, $\zeta = 1.5$ then household consumption is more than 30% higher

[4] I generalize assumption (8) and assume $\left|(\delta_A)^{\frac{1}{\rho}} - (\delta_B)^{\frac{1}{\rho}}\right| \leq \left(\beta^{\frac{1}{N}} R^{\frac{1-\rho}{N}} 2^{\rho-1}\right)^{\frac{1}{\rho}}$. As demonstrated in the appendix this is necessary and sufficient to ensure (7) does not bind for any $t < NT$.

than optimal level. Of course when the EIS is higher the utility cost from an intertemporal inefficiency of a fixed size is also lower. So as we increase the EIS the size of intertemporal inefficiency increases but the utility cost of a given distortion falls.

Panel B shows which of these countervailing forces dominates by showing how the value of commitment varies with the EIS. As in Panel A, this is drawn assuming $\delta_i = 0.6$ for both household members. The clear comparative static result from Panel B is that the value of commitment increases with the EIS. Despite the fact that the utility cost of a given distortion is lower, the increased size of the intertemporal inefficiency dominates this effect. For the parameters assumed in Figure 3, the household will be willing to pay 0.77% of household wealth to achieve full commitment if $\zeta = 0.5$. If instead $\zeta = 1.5$ the household will be willing to pay 2.54% of household wealth to achieve full commitment.

A-1 Representative Agent with CRRA Utility

I now show that under certain conditions the representative agent results of Section III can be generalized to the case where individual household members have CRRA utility. To do this consider the same representative agent as before except now replace the log period utility function in (18) with

$$u_{r,t} = \frac{X_t^{1-\frac{1}{\zeta}} - \frac{1}{\zeta}}{1 - \frac{1}{\zeta}}. \tag{21}$$

Proposition 3: *Assume the household is symmetric so that $\delta_A = \delta_B$. Assume also that household members and the representative agent have CRRA utility as per (20) and (21). The representative agent without commitment has an identical path of total consumption as the household without commitment if:*

$$i.\ \beta_r = \beta, \text{and}$$
$$ii.\ \Omega_r = \frac{1}{1+\Delta}.$$

Whenever $NT > 2$ and is finite and $\Delta > 0$, then the consumption path of the household cannot be replicated by a representative agent with any constant discount factor β_r and $\Omega_r = 1$.

Thus the logic of the representative agent from the log case carries over to the more general CRRA case in the case where the household is perfectly symmetric. Outside of the symmetric case it is not possible to find two constant for β_r and Ω_r that will replicate the consumption path of the household.

B Public Consumption Good

B-1 Setup with Public Consumption

So far I have assumed that all consumption goods are consumed individually by one member or the other. As such, $C_{A,t}$ only contributes utility to household member B in so far as B cares about the utility of A. However one advantage of being in a household is that it allows the household members to share non-rival public consumption goods such as housing, children, and consumer durables. To study how this impacts the intertemporal consumption that the household will achieve suppose that there is a second good, H_t, that provides utility directly to both household members. The total level of public consumption is the sum of the amount purchased in each period by both household members

$$H_t = H_{A,t} + H_{B,t}$$

where $H_{i,t}$ is the amount of the public consumption good purchased by member i in period t. Assume now that the period utility of member i is

$$u_{i,t} = \mu \ln C_{i,t} + (1 - \mu) \ln H_t$$

where $\mu \in [0,1]$ captures the relative weight that household members place on private consumption relative to public consumption.[5] This period utility function replaces the simple period utility function in (1) which is just a special case were $\mu = 1$. Apart from this change the preferences of the household members remain the same as described for the base model in (2), (3), and (4).

I assume that public consumption is also continuous and decided non-cooperatively. Each period both members simultaneously chose how much of the remaining household wealth to spend on $C_{i,t} \geq 0$ and $H_{i,t} \geq 0$. As before, consumption choices are chosen non-cooperatively as Nash equilibrium subgame perfect best responses to each other. To avoid the possibility that household members spend more than total household wealth I adapt (7) to assume that

$$C_{i,t} + H_{i,t} \leq \frac{W_t}{2}. \tag{22}$$

To ensure this condition never binds outside of $t = NT$ I adapt (8) to now assume

$$|\delta_A - \delta_B| \leq \frac{1 - \mu + \beta^{\frac{1}{N}}}{\mu} \tag{23}$$

which, since $\mu < 1$, is less restricitve than (8). Total expenditure in period t is now

$$X_t = C_{A,t} + C_{B,t} + H_{A,t} + H_{B,t}. \tag{24}$$

The intertemporal budget constraint (5) remains the same as before. The benchmark full commitment planning problem is amended in the same way to incorporate the public consumption good.

[5] The model can be extended to allow each member to place different weights on public versus private cosnumption. If we assume that members are unable to reverse the consumption decision of the other ($H_{i,t} \geq 0$) then the level of public consumption will be determined by the level desired by the member with the highest weight on public consumption (lowest value of μ_i).

B-2 Non-Cooperative Equilibrium Consumption Choices with Public Consumption

The model with public household consumption is solved in the Appendix. The primary focus is to study how the presence of this shared consumption goods affects the intertemporal decisions of the household. The equilibrium level of consumption by member i in period $t < T$ is

$$C^*_{i,t} = \frac{\delta_i \mu}{1 + \mu\Delta + \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t. \tag{25}$$

Since it doesn't matter who buys a given unit of the public consumption good the individual choices of $H_{A,t}$ and $H_{B,t}$ are not uniquely determined in equilibrium. However the total level of public consumption is uniquely determined in equilibrium and is

$$H^*_t = \frac{1 - \mu}{1 + \mu\Delta + \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t. \tag{26}$$

Total equilibrium consumption in each period is

$$X^*_t = \frac{1}{1 + \frac{1}{1+\mu\Delta} \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t.$$

The full commitment optimal level of total consumption is the same as before as described in (16).

Proposition 4: *In the model with public consumption, if $\mu\Delta > 0$ then in any period $t < NT$ the non-cooperative equilibrium level of consumption is higher than the amount that the household would commit to conditional on entering the period with wealth W_t.*

This highlights the importance of private consumption in the intertemporal distortion to household savings. This is the decision which is distorted because it requires each member to trade off between their own utility and the combined interests of the household. Since both household members value themselves more than the other this creates a distortion whereby

each member doesn't fully internalize the benefit of savings relative to the utility gain from private consumption. When deciding on the level of public consumption each member trades off the combined interest of the household today versus the future combined interest. This trade-off is not distorted by the self interest of the individual household members. In the extreme, if all consumption were public ($\mu = 0$) then both members would have the same objective and would choose an intertemporally efficient consumption path even if they cared very little for their partner (i.e. if Δ was large). This intuition is captured in the intertemporal preferences of the representative agent for household with public consumption.

Proposition 5: *In the model with public consumption, the representative agent without commitment has an identical path of total consumption as the household without commitment if:*

$$i.\ \beta_r = \beta, \text{and}$$
$$ii.\ \Omega_r = \frac{1}{1+\mu\Delta}.$$

Whenever $NT > 2$ and is finite and $\mu\Delta > 0$, then the consumption path of the household cannot be replicated by a representative agent with any constant discount factor β_r and $\Omega_r = 1$.

Thus the representative agent for the household remains a single agent with a hyperbolic discount factor. The size of the hyperbolic discount factor is now microfounded in the degree to which household members disagree over the relative weight they assign to each others *private* consumption. The larger the fraction of household consumption that is private, the smaller will be Ω_r and hence the further will the household be from the time consistent consumption path it would like to commit to. This intuition is captured in Figure 4 which plots the value of commitment as a function of μ. The value of commitment is strictly increasing in the weight that both household members place on private versus public consumption. This

demonstrates that the savings problem is less severe in a household where members draw a larger fraction of their utility directly from the same things. This suggests that increases in the importance of shared consumption, say through having children, may also reduce the savings distortion. In addition, it implies that one reason why household are more likely to form amongst people with more shared interests is that this helps alleviate the over consumption problem.

C Hyperbolic Household Members

The central message of the paper is that even if a household is comprised of members who individually have time consistent preferences that the household overall will typically be unable to carry out optimal consumption plans. To emphasize this point I have studied a model in which individuals have standard exponential time preferences and hence, left to themselves, optimal consumption plans will be time consistent. However, most of psychological evidence for hyperbolic time preferences is done at the level of the individual (Ainslie 1992). I now study how time inconsistency in the individual time preferences interacts with the time inconsistency exhibited by the combined household.

To do this I re-examine the base model introduced in section I. The only change to that setup is to the time preferences of both household members so that (2) is replaced with

$$U_{i,t} = u_{i,t} + \Psi \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} u_{i,t+x} \tag{27}$$

where $\Psi \leq 1$ is the hyperbolic discount factor used by both household members to discount future utility relative to the present.[6]

The equilibrium of the model is solved in the Appendix along with the full commitment

[6] I also amend the assumption in (8) with $|\delta_A - \delta_B| \leq \Psi\beta^{\frac{1}{N}}$ to ensure (7) does not bind in any period $t < NT$.

optimal allocation. The equilibrium level of consumption by member i in period $t < NT$ is

$$C^*_{i,t} = \frac{\delta_i}{1 + \Delta + \Psi \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t \tag{28}$$

and total consumption

$$X^*_t = \frac{1}{1 + \frac{\Psi}{1+\Delta} \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} W_t. \tag{29}$$

For a given level of wealth W_t the household will consume more in a period if the members are more hyperbolic (Ψ lower) and have more misaligned preferences (Δ larger). For all periods after $t = 1$ the optimal level of household consumption is still given by (16). Direct comparison shows that the household consumes more than the full commitment fraction of wealth in every period $t > 1$ whenever $\frac{\Psi}{1+\Delta} > 1$. This is the analog to Proposition 1 but the commitment problem is now exacerbated by the inconsistent time preferences of the household members.

By comparing (29) to the case where $\Psi = 1$ it is clear that the representative agent result of Section III can be extended directly to the case where household members have hyperbolic time preferences.

Proposition 6: *In the model where household members discount future utility with a hyperbolic discount factor $\Psi \leq 1$, the representative agent without commitment has an identical path of total consumption as the household without commitment if:*

$$i.\ \beta_r = \beta, \text{and}$$
$$ii.\ \Omega_r = \frac{\Psi}{1 + \Delta}.$$

Whenever $NT > 2$ and is finite and $\frac{\Psi}{1+\Delta} > 0$, then the consumption path of the household cannot be replicated by a representative agent with any constant discount factor β_r and $\Omega_r = 1$.

This proposition makes clear that the time inconsistency of the households overall consumption path is amplified when the individual members are themselves hyperbolic. This point is made clearer by considering how the value of commitment is effected when the individual members of the household are hyperbolic. This is shown in Figure 5 which plots the value of commitment considers a household in which both members place symmetric weight on their own utility versus their partners (i.e. $\delta = \delta_A = \delta_B$). The relationship between the value of commitment and δ is shown for for $\Psi = 0.85$ and $\Psi = 1$. When household members place the same weight on each other's utility ($\delta_A = \delta_B = 0.5$) the value of commitment with $\Psi = 0.85$ is 1.19% of household wealth. This is the force that Laibson (1997) documents showing that hyperbolic individuals will value commitment to resolve the time inconsistency in their optimal consumption plans. Conversely, when $\delta_A = \delta_B = 0.65$ and $\Psi = 1$ the household is willing to spend 3.38% of its lifetime wealth to achieve full commitment and overcome the inefficiency due purely to the divergence in both member's objectives. A household which combines both forces, so that $\delta_A = \delta_B = 0.65$ and $\Psi = 0.85$ will pay 8.67% of household wealth for commitment. This is 4.10 percentage points higher than the sum of the value of commitment (1.19% + 3.38%) from considering both of these forces in isolation. Having hyperbolic individuals amplifies the household's problem because it further distorts each member's value of the future combined wealth of the household which they trade off against the current value of their own private utility. Thus, even if the degree of time inconsistency introduced by either of these forces is mild on its own there may be sizable welfare costs once these problems are considered in combination.

V Consecutive Consumption Choices

The model studied above has highlighted the way that non-cooperative consumption decisions by household members who care more about their own utility than their partner does will lead to over consumption relative to the optimal full commitment solution. This has been

shown in a standard model of intertemporal consumption and savings with the only innovation being that the household has two members with imperfectly aligned altruistic preferences and that they share the same pool of wealth. Because I assumed that both household members decided consumption simultaneously this creates the theoretical possibility that household members could attempt to spend more than the total amount of all household wealth. To avoid specifying arbitrary tie breaking rules to deal with such a scenario I assumed in (7) that each member is able to spend no more than half of the household's wealth in any period. Since the number of periods per year N is potentially very large this is not a very restrictive assumption. However it is still artificial and thus it is important to demonstrate the results are robust to alternate ways of dealing with this problem.

The focus of this section is consider the same model but to remove this restriction and instead assume that household members make consumption decisions consecutively within any period. By assuming consecutive moves the model can revert to a standard budget constraint whereby each member can spend up to the full amount of remaining household wealth each time they consume. The purpose of this section is to study this alternate assumption and establish that when N is large the simultaneous move equilibrium studied above is the limiting case of the unique equilibria reached in the consecutive move setup.

A Consecutive Move Setup

Assume that the preferences of the household members is unchanged from the setup in Section I. The timing of decisions and the budget constraint facing each member is now as follows. The household starts the period with wealth of W_t. Without loss of generality, assume that member A is able to decide her own level of consumption first subject to

$$C_{A,t} \leq W_t. \tag{30}$$

Thus A is free to spend up to all of the household's remaining wealth. After this decision is made, the interim level of household wealth is

$$\widetilde{W}_t = W_t - C_{A,t}. \tag{31}$$

Member B learns how much wealth the household has remaining and chooses her own consumption level subject to

$$C_{B,t} \leq \widetilde{W}_t. \tag{32}$$

Thus B is able to spend up to the full amount of remaining household wealth. From one period to the next wealth evolves in the same way as before as specified in (5). As before consumption choices are chosen non-cooperatively and are found as subgame perfect best responses at each point in time.

B Non-Cooperative Equilibrium Consumption Choices

The consecutive move version of the model is solved in the Appendix. The unique equilibrium consumption choice of member's A and B as a function of W_t are

$$\begin{aligned} C_{A,t}^{*Con} &= \frac{\delta_A}{1+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}} W_t \text{ and} \\ C_{B,t}^{*Con} &= \left(\frac{\delta_B}{\delta_B+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right)\left(\frac{1+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}-\delta_A}{1+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right) W_t. \end{aligned} \tag{33}$$

The unique equilibrium level of total consumption in any period is

$$X_t^{*Con} = \left(\frac{1}{1+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right)\left(\frac{(1+\Delta)\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}+\delta_B}{\delta_B+\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right) W_t. \tag{34}$$

The equilibrium consumption choices are slightly complicated because of the stackelberg leader and follower dynamics within each period. This encourages A to consume slightly more to strategically lower the amount of consumption from B. Apart from this within

period strategic consumption motive the forces governing both consumption decisions are identical to before. As the length of each period becomes arbitrarily small (i.e. N gets large) then the magnitude of these within period strategic incentives will diminish as well. This is established formally in the following Proposition which is proved in the Appendix.

Proposition 7: *As $N \to \infty$ the equilibrium consumption choices of the consecutive move game become arbitrarily close to the simultaneous move equilibrium as defined in (13) and (14). Formally,*

$$\lim_{N\to\infty} \frac{C_{i,t}^{*Con}}{C_{i,t}^{*}} = 1 \text{ and } \lim_{N\to\infty} \frac{X_{t}^{*Con}}{X_{t}^{*}} = 1.$$

Proposition 7 establishes that the equilibrium studied in the simultaneous move model is not a by-product of the arbitrary expenditure limits assumed in (7). Moreover, any additional equilibria that might have arisen in that model were a different assumption made to deal with potential overdrawing would not be robust to minor variations in the timing of consumption decisions.

VI Conclusion

This paper introduces a model of household consumption and savings in which household members have imperfectly aligned altruistic preferences. I show that the household is unable to achieve the optimal consumption path without commitment. I have not addressed the specific strategies that the household will employ to mitigate this problem. Some strategies such as investing in illiquid assets have already been studied in the context of individuals with hyperbolic preferences (Laibson 1997). Strategies specific to the household problem studied here will also be effective. For example, punishment strategies between the household members or separate bank accounts for each member may improve the efficiency of the consumption path if they are credible. In addition, restrictions on borrowing that require the consent of both partners, as is common with 401K loans (Choi Laibson Madrian Metrick

2004), can also alleviate the problem. A detailed consideration of these strategies using the framework provided here is left for future work.

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VII Appendix

A Non-Cooperative Equilibrium Household Consumption with Log Utility

This section of the Appendix solves for subgame perfect equilibrium non-cooperative household consumption decisions. I solve a generalized model in which both members have a period utility function that places weight μ on the utility from private consumption and $1-\mu$ on the utility from public consumption H_t as introduced in Section IV. I also allow individual members to have a hyperbolic discount factor Ψ (as introduced in Section IV). The results for the rest of the paper will be special cases of the results I find here where $\mu = 1$ and/or $\Psi = 1$.

A-1 Equilibrium at $t = NT$

In the final period $t = NT$ member i takes $C_{j,NT}$ and $H_{j,NT}$ as given and solves the following problem:

$$\max_{C_{i,NT},H_{i,NT}} \quad \delta_i \left[\mu \ln C_{i,NT} + (1-\mu)\ln\left(H_{i,NT}+H_{j,NT}\right)\right] \tag{35}$$
$$+ (1-\delta_i)\left[\mu \ln C_{j,NT} + (1-\mu)\ln\left(H_{i,NT}+H_{j,NT}\right)\right]$$
subject to
$$\frac{W_{NT}}{2} - C_{i,NT} - H_{i,NT} \geq 0 \text{ and} \tag{36}$$
$$C_{i,NT}, H_{i,NT} \geq 0. \tag{37}$$

Since (35) is strictly increasing in $C_{i,NT}$ and $H_{i,NT}$ it follows that (36) will bind with equality and hence can be substituted into the objective. Ignoring terms which i takes as given we can rewrite her problem as

$$\max_{H_{i,T}} \delta_i \mu \ln\left(\frac{W_{NT}}{2} - H_{i,NT}\right) + (1-\mu)\ln\left(H_{i,NT} + H_{j,NT}\right) \tag{38}$$

subject to

$$\frac{W_{NT}}{2} - H_{i,NT} \geq 0 \text{ and} \tag{39}$$

$$H_{i,NT} \geq 0. \tag{40}$$

Start by ignoring the boundary conditions (39) and (40) on $H_{i,NT}$. The first order condition for the unconstrained problem rearranges to give:

$$H_{i,NT} = \frac{(1-\mu)\frac{W_{NT}}{2} - \delta_i \mu H_{j,NT}}{1-\mu(1-\delta_i)}. \tag{41}$$

Since the objective is strictly concave in $H_{i,NT}$, using the boundary conditions (39) and (40) on $H_{i,NT}$ gives that i's unique best response to any possible choice of $H_{j,NT} \geq 0$ is

$$\begin{aligned} H^{BR}_{i,NT}\left(H_{j,NT}\right) &= \left\{ \begin{array}{cc} b_{i,NT}\frac{W_{NT}}{2} - m_{i,NT}H_{j,NT} & \text{if } H_{j,NT} \leq \frac{1-\mu}{\delta_i \mu}\frac{W_{NT}}{2} \\ 0 & \text{if } H_{j,NT} > \frac{1-\mu}{\delta_i \mu}\frac{W_{NT}}{2} \end{array} \right\} \\ \text{where } b_{i,NT} &\equiv \frac{1-\mu}{1-\mu(1-\delta_i)} > 0, \text{ and} \\ m_{i,NT} &\equiv \frac{\delta_i \mu}{1-\mu(1-\delta_i)} \in (0,1). \end{aligned} \tag{42}$$

Note that $H^{BR}_{i,NT}\left(H_{j,NT}\right)$ is weakly decreasing and hence the most that i will spend on public consumption is

$$H^{BR}_{i,NT}(0) = \frac{1-\mu}{1-\mu(1-\delta_i)}\frac{W_{NT}}{2}$$

which is strictly less than the upper bound $\frac{W_{NT}}{2}$ since $\delta_i > 0$. Thus ((39)) can be ignored. Note that $H^{BR}_{i,T}(0) > 0$ and hence $H_{A,T} = H_{B,T} = 0$ cannot be a Nash equilibrium. If $b_{i,NT} \geq \frac{b_{j,NT}}{m_{j,NT}}$ then $H^*_{i,NT} = b_{i,NT}\frac{W_{NT}}{2}$ and $H^*_{j,NT} = 0$ is a Nash equilibrium. In this case equilibrium, private consumption will be

$$C^*_{i,NT} = (1-b_{i,NT})\frac{W_{NT}}{2} \text{ and } C^*_{j,NT} = \frac{W_{NT}}{2}.$$

Since $m_{i,T}, m_{i,T} < 1$ then this equilibrium is unique. A symmetric argument, applies when $b_{i,NT} \leq m_{i,NT}b_{j,NT}$. Finally, if $b_{i,NT} \in \left(m_{i,NT}b_{j,NT}, \frac{b_{j,NT}}{m_{j,NT}}\right)$ then there is an interior nash equilibrium. This is found by substituting the interior portion of j's reaction function into

the reaction function of i:

$$H^*_{i,NT} = \frac{b_{i,NT} - m_{i,NT}b_{j,NT}}{1 - m_{i,NT}m_{j,NT}} \frac{W_{NT}}{2}.$$

To total expenditure on public consumption in this interior solution is

$$H^*_{NT} = \left(\frac{b_{i,NT}\left(1 - m_{j,NT}\right) + b_{j,NT}\left(1 - m_{i,NT}\right)}{1 - m_{i,NT}m_{j,NT}} \right) \frac{W_{NT}}{2}.$$

The equilibrium level of private consumption in this interior solution is

$$C^*_{i,T} = \left(1 - \frac{b_{i,NT} - m_{i,NT}b_{j,NT}}{1 - m_{i,NT}m_{j,NT}} \right) \frac{W_T}{2}.$$

Thus the equilibrium value of member i's objective function is

$$V_{i,NT} = \ln W_{NT} + k_{i,NT}$$

where $k_{i,NT}$ is a constant term that depends on parameters in the following way

$$k_{i,NT} \equiv \left\{ \begin{array}{ll} \left[\begin{array}{l} \delta_i \mu \ln\left(1 - b_{i,NT}\right) \\ +(1-\mu)\ln\left(b_{i,NT}\right) - \ln 2 \end{array} \right] & \text{if } b_{i,NT} \leq m_{i,NT}b_{j,NT} \\ \left[\begin{array}{l} \delta_i \mu \ln\left(1 - \frac{b_{i,NT} - m_{i,NT}b_{j,NT}}{1 - m_{i,NT}m_{j,NT}}\right) \\ + \left(1 - \delta_i\right)\mu \ln\left(1 - \frac{b_{j,NT} - m_{j,NT}b_{i,NT}}{1 - m_{i,NT}m_{j,NT}}\right) \\ +(1-\mu)\ln\left(\frac{b_{i,NT}\left(1-m_{j,NT}\right) + b_{j,NT}\left(1-m_{i,NT}\right)}{1 - m_{i,NT}m_{j,NT}}\right) - \ln 2 \end{array} \right] & \text{if } b_{i,NT} \in \left(m_{i,NT}b_{j,NT}, \frac{b_{j,NT}}{m_{j,NT}}\right) \\ \left[\begin{array}{l} \left(1 - \delta_i\right)\mu \ln\left(1 - b_{j,T}\right) \\ +(1-\mu)\ln\left(b_{j,T}\right) - \ln 2 \end{array} \right] & \text{if } b_{i,NT} \geq \frac{b_{j,NT}}{m_{j,NT}} \end{array} \right\}.$$

A-2 Solve for Subgame Perfect Consumption Path by Induction

I conjecture the following form for the subgame perfect household allocation.

Conjecture 1 *The subgame perfect equilibrium housheold allocation from t until NT is proportional to W_t. That is, for any period $t \in \{1, ..., NT\}$ the subgame perfect equilibrium levels of private and public consumption can be written as $C^*_{i,t+x} = g_{i,t+x}W_t$ and $H^*_{t+x} = h_{i,t+x}W_t$ for $x \in \{0, 1, ..., NT - t\}$ where $g_{i,t+x}$ and $h_{i,t+x}$ are strictly positive constants independent of W_t.*

I will establish this conjecture by induction below. Consider the problem that each household member faces in period $t < NT$. Member i takes $C_{j,t}$ and $H_{j,t}$ as given and solves the

following:

$$\max_{C_{i,t},H_{i,t}} \delta_i \mu \ln C_{i,t} + (1-\mu)\ln\left(H_{i,t}+H_{j,t}\right) + (1-\delta_i)\,\mu \ln C_{j,t} \tag{43}$$
$$+\Psi \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left[\delta_i \mu \ln C^*_{i,t+x} + (1-\delta_i)\,\mu \ln C^*_{j,t+x} + (1-\mu)\ln\left(H^*_{t+x}\right)\right]$$
subject to
$$W_{t+1} = R^{\frac{1}{N}}\left(W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}\right), \tag{44}$$
$$\frac{W_t}{2} - C_{i,t} - H_{i,t} \geq 0, \tag{45}$$
$$C_{i,t} \geq 0, \text{ and} \tag{46}$$
$$H_{i,t} \geq 0. \tag{47}$$

Conjecture 1 implies that

$$\Psi \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left[\delta_i \mu \ln C^*_{i,t+x} + (1-\delta_i)\,\mu \ln C^*_{j,t+x} + (1-\mu)\ln\left(H^*_{t+x}\right)\right] = Y_{t+1}\ln W_{t+1} + k_{i,t}$$
$$\text{where } Y_{t+1} \equiv \Psi \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}$$

and $k_{i,t}$ is a constant. In equilibrium the budget constraint will bind. Log utility will ensure $C^*_{i,t} > 0$ in equilibrium and hence (46) can be ignored for now and verified later. Ignoring terms that í takes as given in t and substituting (44) into the objective, i's problem can be rewritten as

$$\max_{C_{i,t},H_{i,t}} \delta_i \mu \ln C_{i,t} + (1-\mu)\ln\left(H_{i,t}+H_{j,t}\right) \tag{48}$$
$$+Y_{t+1}\ln\left(W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}\right)$$
subject to
$$\frac{W_t}{2} - C_{i,t} - H_{i,t} \geq 0 \text{ and} \tag{49}$$
$$H_{i,t} \geq 0. \tag{50}$$

Start by ignoring (49) and (50). The first order conditions for the unconstrained problem are

$$C_{i,t} \quad : \quad \frac{\delta_i \mu}{C_{i,t}} - \frac{Y_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0 \tag{51}$$
$$H_{i,t} \quad : \quad \frac{1-\mu}{H_{i,t}+H_{j,t}} - \frac{Y_{t+1}}{W_t - C_{i,t} - C_{j,t} - H_{i,t} - H_{j,t}} = 0 \tag{52}$$

The first order condition for $H_{i,t}$ implies that

$$H_t = H_{i,t} + H_{j,t} = \frac{1-\mu}{1-\mu+Y_{t+1}}\left[W_t - C_{i,t} - C_{j,t}\right]. \tag{53}$$

Hence for any given level of W_t, $C_{i,t}$, and $C_{j,t}$ both members agree on the optimal level of H_t. Since it is funded jointly they are indifferent as to who pays for it. Equation (51) implies

$$C_{i,t} = g_{i,t}\left[W_t - C_{j,t} - H_t\right]. \tag{54}$$
$$\text{where } g_{i,t} \equiv \frac{\delta_i \mu}{Y_{t+1} + \delta_i \mu} \in (0,1).$$

Substituting j's analog of (54) into ((54)) gives

$$C_{i,t} = \frac{g_{i,t}\left(1 - g_{j,t}\right)}{1 - g_{i,t}g_{j,t}}\left[W_t - H_t\right]. \tag{55}$$

Combining (53) and (55) gives the equilibrium level of public consumption

$$H_t^* = \left(\frac{1-\mu}{1 + \mu\Delta + \Psi\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right) W_t. \tag{56}$$

Combining (55) and (56) gives the equilibrium level of private consumption for each member

$$C_{i,t}^* = \left(\frac{\delta_i \mu}{1 + \mu\Delta + \Psi\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right) W_t. \tag{57}$$

Equilibrium total expenditure is thus

$$X_t^* = H_t^* + C_{A,t}^* + C_{B,t}^* = \left(\frac{1}{1 + \frac{\Psi}{1+\mu\Delta}\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}\right) W_t. \tag{58}$$

These solutions were derived for the unconstrained problem ignoring (49) and (46). The expression for $C_{i,t}^*$ in (57) demonstrates that (46) is slack. It just remains to show that (49) is not violated for either household member. First note that $X_t^* < W_t$ for any $t < NT$ and hence the expenditure limit can at most be violated for one household member. Since both members agree on the level of public consumption and are indifferent who pays for it then (49) will be satisfied if and only if $C_{i,t}^* \leq \frac{W_t}{2}$ for both members. This requires

$$\delta_i \leq \frac{1 + \mu\Delta + \Psi\sum_{x=1}^{NT-t}\beta^{\frac{x}{N}}}{2\mu}.$$

This constraint is more restrictive the higher is t and hence holds in every period if it is true for the household member with the largest δ_i in period $t = NT - 1$. This requires

$$\max\left\{\delta_A, \delta_B\right\} \leq \frac{1 + \mu\Delta + \Psi\beta^{\frac{1}{N}}}{2\mu}.$$

Using the fact that $\Delta = \delta_A + \delta_B - 1$ this re-arranges to

$$|\delta_A - \delta_B| \leq \frac{1 - \mu + \Psi\beta^{\frac{1}{N}}}{\mu}. \tag{59}$$

I assume (59) holds and hence (49) is also slack. Thus, conditional on Conjecture 1 being true (56), (57) and (58) are the unique subgame perfect equilibrium consumption choices.

The final step of the derivation is to prove Conjecture 1 by induction. As the first step, note that Conjecture 1 is verified for $t = NT$ above. Next observe that (56) and (57) give equilibrium consumption levels that are proportional to W_t. Observe also that using (58) we can compute W_{t+1} as

$$W_{t+1} = R^{\frac{1}{N}} \left(\frac{\frac{\Psi}{1+\mu\Delta} \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}}{1 + \frac{\Psi}{1+\mu\Delta} \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}}} \right) W_t$$

which is also proportional to W_t. By extension of (56) and (57) this implies that $H^*_{t+1}, C^*_{A,t+1}, C^*_{B,t+1}$ are also proportional to W_t. The same argument applies for any period $x > t$. Hence this establishes Conjecture 1 by induction.

B Solution to Household Allocation with Full Commitment

This section of the Appendix solves for the full commitment Pareto optimal household allocation. I solve a generalized model in which both members have a period utility function that places weight μ on the utility from private consumption and $1 - \mu$ on the utility from public consumption H_t as introduced in Section B. Also, I allow individual members to have a hyperbolic discount factor Ψ as introduced in Section C. The results for the rest of the paper will be special cases of the results I find here where $\mu = 1$ and/or $\Psi = 1$.

The problem is to solve

$$\max_{\{C_{A,t}, C_{B,t}, H_t,\}_{t=1}^{t=NT}} \Pi = \eta V_{A,1} + (1 - \eta) V_{B,1} \tag{60}$$

$$\text{subject to} \qquad W_1 - \sum_{x=0}^{NT-1} R^{-\frac{x}{N}} \left[C_{A,1+x} + C_{B,1+x} + H_{1+x} \right] \geq 0 \text{ and} \tag{61}$$

$$\{C_{A,t}, C_{B,t}, H_{A,t}, H_{B,t}\}_{t=1}^{t=NT} \geq 0. \tag{62}$$

The objective of this problem can be re-written as

$$\Pi = (1 - \theta) U_{A,1} + \theta U_{B,1} \tag{63}$$

$$\text{where } \theta \equiv \delta_B + \eta (1 - \delta_A - \delta_B) \tag{64}$$

using the expressions for $U_{A,1}$ and $U_{B,1}$ (63) becomes

$$\begin{aligned}\Pi &= (1-\theta)\,\mu\left[\ln C_{A,1}+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln C_{A,1+x}\right] \\ &+\theta\mu\left[\ln C_{B,1}+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln C_{B,1+x}\right] \\ &+(1-\mu)\left[\ln H_1+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln H_{1+x}\right].\end{aligned} \tag{65}$$

I will start by ignoring the non-negativity constraints in (62) and verify that these hold later. Writing the Lagrangian for the remaining problem with $\Gamma \geq 0$ being the multiplier on the resource constraint we have

$$\begin{aligned}&\max_{\left\{C_{A,t},C_{B,t},H_t,\right\}_{t=1}^{t=T}}(1-\theta)\,\mu\left[\ln C_{A,1}+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln C_{A,1+x}\right] \\ &+\theta\mu\left[\ln C_{B,1}+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln C_{B,1+x}\right] \\ &+(1-\mu)\left[\ln H_1+\Psi\sum_{x=1}^{NT-1}\beta^{\frac{x}{N}}\ln H_{1+x}\right] \\ &+\Gamma\left[W_1-\sum_{x=0}^{NT-1}R^{-\frac{x}{N}}\left[C_{A,1+x}+C_{B,1+x}+H_{1+x}\right]\right].\end{aligned} \tag{66}$$

The first order conditions give the optimal level of expenditure on each type of consumption in every period as a function of Γ:

$$C_{A,1} \;:\; C_{A,1}^{**}=\frac{(1-\theta)\,\mu}{\Gamma} \tag{67}$$

$$C_{A,1+x} \;:\; C_{A,1+x}^{**}=\frac{(1-\theta)\,\mu\Psi\beta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{68}$$

$$C_{B,1} \;:\; C_{B,1}^{**}=\frac{\theta\mu}{\Gamma} \tag{69}$$

$$C_{B,1+x} \;:\; C_{B,1+x}^{**}=\frac{\theta\mu\Psi\beta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{70}$$

$$H_1 \;:\; H_1^{**}=\frac{1-\mu}{\Gamma} \tag{71}$$

$$H_{1+x} \;:\; H_{1+x}^{**}=\frac{(1-\mu)\,\Psi\beta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}} \tag{72}$$

where $x \in \{1,2,...,NT-1\}$ and "**" indicates solution to the full commitment problem. In

the first period, the optimal level of total expenditure is

$$X_1^{**} = C_{A,1}^{**} + C_{B,1}^{**} + H_1^{**} = \frac{1}{\Gamma}. \tag{73}$$

For any period after the first, the optimal level of total expenditure is

$$X_{1+x}^{**} = \frac{\Psi \beta^{\frac{x}{N}}}{\Gamma R^{-\frac{x}{N}}}. \tag{74}$$

Since the optimal allocation will exhaust the household budget constraint it must be that

$$W_1 = X_1^{**} + \sum_{x=1}^{NT-1} \frac{X_{1+x}^{**}}{R^{\frac{x}{N}}} = \frac{1}{\Gamma}\left[1 + \Psi \sum_{x=1}^{NT-1} \beta^{\frac{x}{N}}\right]$$

which implies that

$$\Gamma^{**} = \frac{1 + \Psi \sum_{x=1}^{NT-1} \beta^{\frac{x}{N}}}{W_1}. \tag{75}$$

Combining (75) with (73) and (74) gives

$$X_1^{**} = \frac{1}{1 + \Psi \sum_{x=1}^{NT-1} \beta^{\frac{x}{N}}} W_1 \tag{76}$$

and for $t > 1$

$$X_t^{**} = \frac{\Psi \beta^{\frac{t-1}{N}}}{1 + \Psi \sum_{x=1}^{NT-1} \beta^{\frac{x}{N}}} R^{\frac{t-1}{N}} W_1.$$

Note that under the full commitment allocation household wealth evolves as

$$\begin{aligned} W_t &= R^{\frac{t-1}{N}} W_1 - R^{\frac{t-1}{N}} X_1^{**} - \sum_{x=2}^{t-1} R^{\frac{t-x}{N}} X_x^{**} \\ &= R^{\frac{t-1}{N}} W_1 \Psi \left[\frac{\sum_{k=t-1}^{NT-1} \beta^{\frac{k}{N}}}{1 + \Psi \sum_{k=1}^{NT-1} \beta^{\frac{k}{N}}}\right] \end{aligned}$$

and so

$$R^{\frac{t-1}{N}} W_1 = \frac{W_t}{\Psi}\left(\frac{1 + \Psi \sum_{k=1}^{NT-1} \beta^{\frac{k}{N}}}{\sum_{k=t-1}^{NT-1} \beta^{\frac{k}{N}}}\right).$$

Hence for $t > 1$, X_t^{**} can be re-written as

$$X_t^{**} = \frac{\Psi \beta^{\frac{t-1}{N}}}{1 + \Psi \sum_{x=1}^{NT-1} \beta^{\frac{x}{N}}} \frac{W_t}{\Psi}\left(\frac{1 + \Psi \sum_{k=1}^{NT-1} \beta^{\frac{k}{N}}}{\sum_{k=t-1}^{NT-1} \beta^{\frac{k}{N}}}\right).$$

This simplifies to

$$X_t^{**} = \frac{1}{1 + \sum_{k=1}^{NT-t} \beta^{\frac{k}{N}}} W_t. \tag{77}$$

This fully describes the total level of consumption each period under full commitment. The optimal levels of $C_{A,t}^{**}$, $C_{B,t}^{**}$, and H_t^{**} follow immediately by using (67) through (72) to get the following constant consumption shares within each period.

$$\begin{aligned} \frac{C_{A,t}^{**}}{X_t^{**}} &= (1-\theta)\,\mu \\ \frac{C_{B,t}^{**}}{X_t^{**}} &= \theta\mu \\ \frac{H_t^{**}}{X_t^{**}} &= 1-\mu. \end{aligned}$$

Note that the optimal solution satisfies (62).

C Representative Agent

This section of the Appendix solves the problem of the representative agent without commitment. Since the representative agent is allowed to have hyperbolic time preferences I study for the subgame perfect equilibrium path X_t^{r*} where the agent rationally anticipates the consumption choices she will make later in life (i.e. does not naively and incorrectly expect to follow the optimal consumption plan for the rest of her life). The goal is to find values for β_r and Ω_r that ensure $X_t^{r*} = X_t^*$ in every period.

C-1 Equilibrium Consumption

In the final period $t = NT$ the representative agent will optimal consume all remaining wealth

$$X_{NT}^{r*} = W_{NT}.$$

In order to solve for equilibrium consumption choices for all $t < NT$ I make the following conjecture.

Conjecture 2: *The subgame perfect equilibrium household allocation of the representative agent from t until NT is proportional to W_t. That is, for any period $t \in \{1, ..., NT\}$ the subgame perfect equilibrium levels of X_t can be written as $X_{t+x}^{r*} = k_{t+x} W_t$ for $x \in \{0, 1, ..., NT - t\}$ where k_{t+x} are strictly positive constants independent of W_t.*

I establish Conjecture 2 by induction. Consider the problem that the representative agent

faces in period $t < NT$

$$\max_{X_t} \ln X_t + \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \ln X_{t+x}^{r*} \tag{78}$$

subject to

$$W_{t+1} = R^{\frac{1}{N}} (W_t - X_t), \tag{79}$$

$$X_t \leq W_t, \text{ and} \tag{80}$$

$$X_t \geq 0. \tag{81}$$

I will solve this problem ignoring (80) and (81) and verify that these are satisfied at the end. Using Conjecture 2, substituting (79) into the objective function, and ignoring constant terms transforms the problem to

$$\max_{X_t} \ln X_t + \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \ln (W_t - X_t). \tag{82}$$

The first order condition for this problem is

$$\frac{1}{X_t^{r*}} - \frac{\Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}}}{W_t - X_t^{r*}} = 0.$$

Which can be rearranged to give the equilibrium consumption choice of the representative agent in any period as

$$X_t^{r*} = \frac{1}{1 + \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}}} W_t. \tag{83}$$

I can now prove Conjecture 2 by induction. First, observe that it is verified for $t = NT$ above. Next, observe that (83) shows that X_t^{r*} is proportional to W_t. Moreover, since wealth will evolve under these equilibrium choices as

$$W_{t+1} = \frac{R^{\frac{1}{N}} \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}}}{1 + \Omega_r \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}}} W_t$$

then W_{t+1} is also proportional to W_t. By extension of (83) this implies X_{t+1}^{r*} is proportional to W_t and so on for all $\left\{X_{t+x}^{r*}\right\}_{x=1}^{NT-t}$. This establishes Conjecture 2.

C-2 Equivalence with Household Equilibrium

Comparing (58) and (83) we see that $X_t^{r*} = X_t^*$ if

$$\beta_r = \beta \text{ and } \Omega_r = \frac{\Psi}{1 + \mu\Delta}. \tag{84}$$

To establish that (84) is a necessary condition consider what is required to achieve equivalence in $t = NT - 1$ and $NT - 2$. This requires

$$t = NT - 1 : \Omega_r \beta_r^{\frac{1}{N}} = \frac{\Psi}{1 + \mu\Delta} \beta^{\frac{1}{N}} \text{ and} \tag{85}$$

$$t = NT - 2 : \Omega_r \left[\beta_r^{\frac{1}{N}} + \beta_r^{\frac{2}{N}} \right] = \frac{\Psi}{1 + \mu\Delta} \left[\beta^{\frac{1}{N}} + \beta^{\frac{2}{N}} \right]. \tag{86}$$

To satisfy (85) it must be that

$$\Omega_r = \frac{\Psi}{1 + \mu\Delta} \left(\frac{\beta}{\beta_r} \right)^{\frac{1}{N}}. \tag{87}$$

Substituting (87) into (86) gives

$$\frac{\Psi}{1 + \mu\Delta} \left(\frac{\beta}{\beta_r} \right)^{\frac{1}{N}} \left[\beta_r^{\frac{1}{N}} + \beta_r^{\frac{2}{N}} \right] = \frac{\Psi}{1 + \mu\Delta} \left[\beta^{\frac{1}{N}} + \beta^{\frac{2}{N}} \right]$$

which upon simplification uniquely requires $\beta_r = \beta$ and therefore implies that $\Omega_r = \frac{\Psi}{1+\mu\Delta}$ must also hold. Thus (84) is a necessary condition for equivalence if $NT > 2$ and finite. Note that if $NT = 2$ then any combination of Ω_r and β_r that satisfies (87) is sufficient and (84) is therefore not a necessary condition. This establishes Proposition 2.

D CRRA Utility

In this section of the Appendix I analytically characterize the non-cooperative equilibrium levels of household consumption. I then characterize the solution to the associated planners problem with full commitment. Finally, I characterize the solution for the representative agent with CRRA utility and establish equivalence in the symmetric case where $\delta_A = \delta_B$. Since these steps mirror many of the proofs in the first three sections of the Appendix I keep derivations brief. To simplify notation let $\rho = \frac{1}{\zeta}$.

D-1 Non-Cooperative Equilibrium Household Consumption with CRRA Utility

To start I conjecture a form for the value function of each member in any.

Conjecture 3: *The subgame perfect equilibrium household allocation when member's have CRRA utility gives rise to a value function for member i in period of the form*

$$V_{i,t}(W_t) = \frac{\Upsilon_{i,t}}{1 - \rho} W_t^{1-\rho}$$

where $\Upsilon_{i,t}$ is a positive constant independent of W_t.

Conjecture 3 will be proved by induction. Consider the problem faced by member i in period t using the assumed form of the value function for period $t + 1$. They will choose $C_{i,t}$ taking $C_{j,t}$ as given to solve

$$\max_{C_{i,t}} \frac{\delta_i}{1-\rho}\left(C_{i,t}\right)^{1-\rho} + \frac{1-\delta_i}{1-\rho}\left(C_{j,t}\right)^{1-\rho} + \beta^{\frac{1}{N}}\frac{\Upsilon_{i,t+1}}{1-\rho}W_{t+1}^{1-\rho} \tag{88}$$

subject to

$$W_{t+1} = R^{\frac{1}{N}}\left(W_t - C_{i,t} - C_{j,t}\right), \tag{89}$$

$$C_{i,t} \leq \frac{W_t}{2}, \text{ and} \tag{90}$$

$$C_{i,t} \geq 0. \tag{91}$$

Start by ignoring (90) and (91). Substituting (89) into (88) and ignoring terms that i takes as given allows us to rewrite her problem as

$$\max_{C_{i,t}} \frac{\delta_i}{1-\rho}\left(C_{i,t}\right)^{1-\rho} + \beta^{\frac{1}{N}}R^{\frac{1-\rho}{N}}\frac{\Upsilon_{i,t+1}}{1-\rho}\left(W_t - C_{i,t} - C_{j,t}\right)^{1-\rho}.$$

The first order condition is

$$\delta_i C_{i,t}^{-\rho} - \beta^{\frac{1}{N}}R^{\frac{1-\rho}{N}}\Upsilon_{i,t+1}\left(W_t - C_{i,t} - C_{j,t}\right)^{-\rho} = 0.$$

Rearranging this gives *i*'s best response to any $C_{j,t}$:

$$C_{i,t} = \left(W_t - C_{j,t}\right)M_{i,t} \tag{92}$$

where

$$M_{i,t} = \frac{1}{1 + \left(\frac{\beta^{\frac{1}{N}}R^{\frac{1-\rho}{N}}\Upsilon_{i,t+1}}{\delta_i}\right)^{\frac{1}{\rho}}} \in (0,1) \tag{93}$$

Solving both members best response functions simultaneously gives the subgame equilibrium consumption choices of

$$C_{i,t}^* = \frac{\left(1 - M_{j,t}\right)M_{i,t}}{1 - M_{i,t}M_{j,t}}W_t. \tag{94}$$

Total equilibrium consumption in period t is thus

$$X_t^* = \left[\frac{M_{A,t} + M_{B,t} - 2M_{A,t}M_{B,t}}{1 - M_{A,t}M_{B,t}}\right]W_t. \tag{95}$$

Using (95) gives that in equilibrium household wealth evolves according to

$$W_{t+1} = R^{\frac{1}{N}}\left(\frac{1 - M_{A,t} - M_{B,t} + M_{A,t}M_{B,t}}{1 - M_{A,t}M_{B,t}}\right)W_t. \tag{96}$$

Putting (94) and (96) into (88) we can write the value function for both member i in period t:

$$V_{i,t}\left(W_t\right) = \frac{\Upsilon_{i,t}}{1-\rho}W_t^{1-\rho} \tag{97}$$

where

$$\begin{aligned}\Upsilon_{i,t} &= \delta_A \left(\frac{(1-M_{j,t})\,M_{i,t}}{1-M_{A,t}M_{B,t}}\right)^{1-\rho} \\ &\quad + (1-\delta_i)\left(\frac{(1-M_{i,t})\,M_{j,t}}{1-M_{A,t}M_{B,t}}\right)^{1-\rho} \\ &\quad + \beta^{\frac{1}{N}}\Upsilon_{i,t+1}\left(R^{\frac{1}{N}}\left(\frac{1-M_{A,t}-M_{B,t}+M_{A,t}M_{B,t}}{1-M_{A,t}M_{B,t}}\right)\right)^{1-\rho}.\end{aligned} \tag{98}$$

In the final period equilibrium consumption will be

$$C^*_{i,NT} = \frac{W_{NT}}{2} \text{ for } i = A, B. \tag{99}$$

Thus the value function for each household member in the final period is:

$$V_{i,NT}(W_{NT}) = \frac{\Upsilon_{i,NT}}{1-\rho}(W_{NT})^{1-\rho} \text{ where } \Upsilon_{i,NT} = \left(\frac{1}{2}\right)^{1-\rho}. \tag{100}$$

This verifies Conjecture 3 for $t = NT$. Moreover, (98) and (98) show that conditional on the conjecture being true for $t+1$ then it is also true for t. Hence this establishes Conjecture 3 by an argument of induction.

Note that (100) defines $\Upsilon_{i,NT}$. Using $\Upsilon_{i,NT}$ and (98) fixes $\Upsilon_{i,NT-1}$. By the same argument a recursive application of (98) fixes the entire series $\{\Upsilon_{i,t}\}_{t=1}^{NT}$ for $i = A, B$. This series and (93) then fixes the entire series $\{M_{i,t}\}_{t=1}^{NT}$ for $i = A, B$. And this using (94) and (95) fixes the entire series of equilibrium consumption decisions $\left\{C^*_{i,t}\right\}_{t=1}^{NT}$ for $i = A, B$ and $\{X^*_t\}_{t=1}^{NT}$.

Finally, we need to check that the constraints (90) and (91) are satisfied. The solution for optimal consumption demonstrates that (91) is satisified. Since $\frac{C^*_{i,t}}{W_t}$ is increasing in t it is sufficient to show that (90) holds for $t = NT-1$ for both members. Using (94), we have that $\frac{C^*_{i,NT-1}}{W_{NT-1}} \leq \frac{1}{2}$ if and only if

$$M_{i,NT-1}(2 - M_{j,NT-1}) \leq 1. \tag{101}$$

Note that using (100) and (93) gives that

$$M_{i,NT-1} = \frac{1}{1+\left(\frac{\beta^{\frac{1}{N}}R^{\frac{1-\rho}{N}}}{\delta_i 2^{1-\rho}}\right)^{\frac{1}{\rho}}}. \tag{102}$$

Observe that $M_{i,NT-1}$ is strictly increasing in δ_i and so ensuing (101) holds for the member with highest δ_i will be necessary and sufficient. Combining (101) and (102) and rearranging gives

$$(\delta_i)^{\frac{1}{\rho}} - (\delta_j)^{\frac{1}{\rho}} \leq \left(\frac{\beta^{\frac{1}{N}}R^{\frac{1-\rho}{N}}}{2^{1-\rho}}\right)^{\frac{1}{\rho}}. \tag{103}$$

Since (103) must hold for both members we require

$$\left|(\delta_i)^{\frac{1}{\rho}} - (\delta_j)^{\frac{1}{\rho}}\right| \leq \left(\frac{\beta^{\frac{1}{N}} R^{\frac{1-\rho}{N}}}{2^{1-\rho}}\right)^{\frac{1}{\rho}}. \tag{104}$$

Note that when $\rho = 1$ (i.e. the log utiltiy case) (104) reduces to (8). Since we assume paramters satisfy this condition this verifies that (90) is satisified everywhere along the equilibrium consumption path.

D-2 Full Commitment Consumption Path with CRRA Utility

I now characterize the full commitment consumption path in the case where both members have CRRA period utility functions. I do this by supposing that W_1 is divided between member A and B so that A receives $(1-\theta)W_1$ and B receives θW_1 where θ is defined in (64). For both members the optimal path of consumption will be characterized by the standard envelope condition:

$$\frac{\partial}{\partial \Delta}\left\{\frac{\left(C^{**}_{i,t}+\Delta\right)^{1-\rho}}{1-\rho} + \beta^{\frac{1}{N}}\frac{\left(C^{**}_{i,t+1}-R^{\frac{1}{N}}\Delta\right)^{1-\rho}}{1-\rho}\right\}_{\Delta=0} = 0 \tag{105}$$

which simplifies to give the standard Euler equation relating the optimal choice of consumption in one period to the next:

$$C^{**}_{i,t+1} = (R\beta)^{\frac{1}{\rho N}} C^{**}_{i,t}. \tag{106}$$

Equation (106) implies that

$$C^{**}_{i,t} = (R\beta)^{\frac{t-1}{\rho N}} C^{**}_{i,1}. \tag{107}$$

Since the optimal allocation must exhaust the wealth allocated to A it must be that

$$C^{**}_{A,1} + \frac{C^{**}_{A,2}}{R^{\frac{1}{N}}} + \frac{C^{**}_{A,3}}{R^{\frac{2}{N}}} + ... + \frac{C^{**}_{A,NT}}{R^{\frac{NT-1}{N}}} = (1-\theta)W_1$$

which in combination with (107) gives that

$$C^{**}_{A,1} = \frac{(1-\theta)W_1}{\sum_{x=0}^{NT-1}\gamma^x} \text{ where } \gamma \equiv \left(R^{1-\rho}\beta\right)^{\frac{1}{\rho N}}. \tag{108}$$

Combining (108) and (107) gives the optimal level of consumption for member A in every period

$$C^{**}_{A,t} = (R\beta)^{\frac{t-1}{\rho N}}\left[\frac{1-\left(R^{1-\rho}\beta\right)^{\frac{1}{\rho N}}}{1-\left(R^{1-\rho}\beta\right)^{\frac{T}{\rho}}}\right](1-\theta)W_1. \tag{109}$$

By symmetry, the full commitment solution for B is

$$C_{B,t}^{**} = (R\beta)^{\frac{t-1}{\rho N}} \left[\frac{1 - (R^{1-\rho}\beta)^{\frac{1}{\rho N}}}{1 - (R^{1-\rho}\beta)^{\frac{T}{\rho}}} \right] \theta W_1. \tag{110}$$

Adding (109) and (110) gives the optimal level of total household consumption in each period:

$$X_t^{**} = (R\beta)^{\frac{t-1}{\rho N}} \left[\frac{1 - (R^{1-\rho}\beta)^{\frac{1}{\rho N}}}{1 - (R^{1-\rho}\beta)^{\frac{T}{\rho}}} \right] W_1. \tag{111}$$

This fully characterizes the optimal allocation for the household when both members have CRRA period utility functions.

D-3 Representative Agent with CRRA Utility

I establish the equivalance of the representative agent and the symmetric non-cooperative household ($\delta_A = \delta_B$) by finding necessary and sufficent conditions on β_r and Ω_r. To start, I make the following supposition:

Supposition 1: *Suppose that for some $t \in \{1, NT-1\}$ the representative agent and a symmetric household have the same consumption path from $t+1$ onwards (conditional on starting $t+1$ with the same wealth).*

The consumption path of the representative agent can be written as

$$\left\{X_x^{r*}\right\}_{x=t+1}^{x=NT} = \left\{\alpha_{t+1}^t W_{t+1}, \alpha_{t+2}^t W_{t+1}, ..., \alpha_{NT}^t W_{t+1}\right\}. \tag{112}$$

Supposition 1 implies that for both i

$$\left\{C_{i,x}^*\right\}_{x=t+1}^{x=NT} = \left\{ \frac{\alpha_{t+1}^t}{2} W_{t+1}, \frac{\alpha_{t+2}^t}{2} W_{t+1}, ..., \frac{\alpha_{NT}^t}{2} W_{t+1} \right\}. \tag{113}$$

Anticipating a consumption path of (112) for any wealth left into period $t+1$ the representative agent will choose X_t to solve

$$\max_{X_t} \frac{(X_t)^{1-\rho}}{1-\rho} + \Omega_r \left[\sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \left(\alpha_{t+x}^t\right)^{1-\rho} \right] \frac{(W_{t+1})^{1-\rho}}{1-\rho}$$
$$\text{subject to } W_{t+1} = R^{\frac{1}{N}} (W_t - X_t)$$

Substituting the constraint into the objective reduces the representative agent's problem to

$$\max_{X_t} \frac{(X_t)^{1-\rho}}{1-\rho} + \Omega_r \left[\sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \left(\alpha_{t+x}^t\right)^{1-\rho} \right] \frac{\left(R^{\frac{1}{N}} (W_t - X_t)\right)^{1-\rho}}{1-\rho}$$

which can be re-written as

$$\max_{X_t} \frac{(X_t)^{1-\rho}}{1-\rho} + \frac{\Omega_r}{1-\rho} \Upsilon_{r,t+1} (W_t - X_t)^{1-\rho}$$

where

$$\Upsilon_{r,t+1} \equiv R^{\frac{1-\rho}{N}} \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \left(\alpha_{t+x}^t\right)^{1-\rho}. \tag{114}$$

The first order condition for the representative agent's problem is

$$X_t^{-\rho} - \Omega_r \Upsilon_{r,t+1} (W_t - X_t)^{-\rho} = 0$$

and can be simplified to give the choice of conumption in period t of:

$$X_t^{r*} = \frac{1}{1 + (\Omega_r \Upsilon_{r,t+1})^{\frac{1}{\rho}}} W_t. \tag{115}$$

Now consider the problem faced by household member i in period t anticipating a consumption path of (113) to follow in $t+1$ onwards conditional on the level of wealth W_{t+1}. She will take $C_{j,t}$ as given and choose $C_{i,t}$ to solve

$$\max_{C_{i,t}} \frac{\delta_i}{1-\rho} (C_{i,t})^{1-\rho} + \frac{1-\delta_i}{1-\rho} (C_{j,t})^{1-\rho} + \left[\sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left(\frac{\alpha_{t+x}^t}{2}\right)^{1-\rho}\right] \frac{(W_{t+1})^{1-\rho}}{1-\rho}$$
$$\text{subject to } W_{t+1} = R^{\frac{1}{N}} (W_t - C_{i,t} - C_{j,t}).$$

Substituting the constraint into the objective reduces member i's problem to

$$\max_{C_{i,t}} \frac{\delta_i}{1-\rho} (C_{i,t})^{1-\rho} + \frac{1-\delta_i}{1-\rho} (C_{j,t})^{1-\rho} + \left[\sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left(\frac{\alpha_{t+x}^t}{2}\right)^{1-\rho}\right] \frac{\left(R^{\frac{1}{N}} (W_t - C_{i,t} - C_{j,t})\right)^{1-\rho}}{1-\rho}$$

which can be re-written as

$$\max_{C_{i,t}} \frac{\delta_i}{1-\rho} (C_{i,t})^{1-\rho} + \frac{1-\delta_i}{1-\rho} (C_{j,t})^{1-\rho} + \frac{\Upsilon_{i,t+1}}{1-\rho} (W_t - C_{i,t} - C_{j,t})^{1-\rho}$$

where

$$\Upsilon_{i,t+1} \equiv \left(\frac{1}{2^{1-\rho}}\right) R^{\frac{1-\rho}{N}} \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left(\alpha_{t+x}^t\right)^{1-\rho} = \left(\frac{1}{2^{1-\rho}}\right) \Upsilon_{r,t+1} \tag{116}$$

The first order condition is

$$\delta_i C_{i,t}^{-\rho} - \Upsilon_{i,t+1} (W_t - C_{i,t} - C_{j,t})^{-\rho} = 0.$$

By an argument of symmetry it must be that $C^*_{i,t} = C^*_{j,t}$ and so this can be simplified to give

$$C^*_{i,t} = \frac{1}{2 + \left(\frac{\Upsilon_{i,t+1}}{\delta_i}\right)^{\frac{1}{\rho}}} W_t.$$

Since the household is symemtric, total household expenditure in period t will be twice $C^*_{i,t}$:

$$X^*_t = \frac{2}{2 + \left(\frac{\Upsilon_{i,t+1}}{\delta_i}\right)^{\frac{1}{\rho}}} W_t. \tag{117}$$

Equating (115) and (117) shows that, conditional on starting the period with W_t, the representative agent will have the same level of consumption as the household in period t if and only if

$$2^{\rho}\Omega_r\delta_i\Upsilon_{r,t+1} = \Upsilon_{i,t+1}. \tag{118}$$

Using (114) and (116) we can rewrite (118) as

$$2\Omega_r\delta_i \sum_{x=1}^{NT-t} \beta_r^{\frac{x}{N}} \left(\alpha^t_{t+x}\right)^{1-\rho} = \sum_{x=1}^{NT-t} \beta^{\frac{x}{N}} \left(\alpha^t_{t+x}\right)^{1-\rho}.$$

Equating terms, this will hold if

$$\beta_r = \beta \text{ and } \Omega_r = \frac{1}{2\delta_i} \tag{119}$$

To show that (119) are necessary conditions, note that If $NT = 2$ then any combination of Ω_r and β_r for which $2\Omega_r\delta_i\beta_r^{\frac{1}{N}} = \beta^{\frac{1}{N}}$ will satisify this. If $NT > 2$ then for this to be true in every period requires

$$t = NT-1 : 2\Omega_r\delta_i\beta_r^{\frac{1}{N}} = \beta^{\frac{1}{N}} \tag{120}$$

$$t = NT-2 : 2\Omega_r\delta_i\left[\beta_r^{\frac{1}{N}}\left(\alpha^t_{NT-1}\right)^{1-\rho} + \beta_r^{\frac{2}{N}}\left(\alpha^t_{NT}\right)^{1-\rho}\right] = \beta^{\frac{1}{N}}\left(\alpha^t_{NT-1}\right)^{1-\rho} + \beta^{\frac{2}{N}}\left(\alpha^t_{NT}\right)^{1-\rho} \tag{121}$$

Substituting (120) into (121) shows that for both to hold requires $\beta_r = \beta$. With (120) this gives $\Omega_r = \frac{1}{2\delta_i}$ is and demonstrates that (119) is necessary and sufficient whenever $NT > 2$.

If these conditions hold then Suppostion 1 can be proved by induction. Note that the Supposition is true for $t = NT - 1$ since all remaining wealth is consumed in the final period for both the representative agent and the non-cooperative household (i.e. $\alpha^t_{NT} = 1$). This proof then establishes the conjecture for $t = NT - 2$ and so on by iteration. This establishes Proposition 3.

E Consecutive Consumption Choices

This section of the Appendix solves for the unique equilibrium consumption path in the consecutive move version of the model introduced in Section V of the paper.

E-1 Equilibrium at $t = NT$

In the final period member B will optimally consume all remaining wealth:

$$C_{B,NT}^{*Con} = \widetilde{W}_{NT}. \tag{122}$$

Anticipating (122), member A will choose $C_{A,NT}$ to solve

$$\max_{C_{A,NT}} \delta_A \ln C_{A,NT} + (1-\delta_A) \ln (W_{NT} - C_{A,NT}) \tag{123}$$

$$\text{subject to} \quad C_{A,NT} \geq 0 \text{ and} \tag{124}$$

$$W_{NT} - C_{A,NT} \geq 0. \tag{125}$$

Ignoring (124) and (125) since they will not bind at the optimal choice, A's consumption choice is characterized by the first order condition

$$\frac{\delta_A}{C_{A,NT}^*} - \frac{1-\delta_A}{W_{NT} - C_{A,NT}^*} = 0. \tag{126}$$

Rearranging (126) and combing with (122) gives the equilibrium consumption levels for A and B in $t = NT$:

$$C_{A,NT}^* = \delta_A W_{NT} \text{ and} \tag{127}$$

$$C_{B,NT}^* = (1-\delta_A) W_{NT}. \tag{128}$$

And total equilibrium consumption in $t = NT$ is simply

$$X_{NT}^* = W_{NT}. \tag{129}$$

E-2 Solve for the Subgame Perfect Consumption path by Induction

I conjecture the following form for the subgame perfect household allocation.

Conjecture 4: *The subgame perfect equilibrium household allocation from t until NT is proportional to W_t. That is, for any period $t \in \{1, ..., NT\}$ the subgame perfect equilibrium levels of $C_{A,t}^*$ and $C_{B,t}^*$ can be written as $C_{t+x}^* = g_{i,t+x} W_t$ for $x \in \{0, 1, ..., NT - t\}$ where $g_{i,t+x}$ are strictly positive constants independent of W_t.*

I establish Conjecture 4 by induction. The problem that member B solves in any period

t taking $\widetilde{W}_t$ as given is:

$$\max_{C_{B,t}} \delta_B \ln C_{B,t} + \sum_{x=1}^{TN-t} \beta^{\frac{x}{N}} \left[(1-\delta_B) \ln C_{A,t+x}^{Con*} + \delta_B \ln C_{B,t+x}^{Con*}\right] \tag{130}$$

subject to

$$W_{t+1} = R^{\frac{1}{N}} \left(\widetilde{W}_t - C_{B,t}\right), \tag{131}$$

$$C_{B,t} \geq 0, \text{ and} \tag{132}$$

$$\widetilde{W}_t - C_{B,t} \geq 0. \tag{133}$$

Conjecture 4 implies that

$$\sum_{x=1}^{TN-t} \beta^{\frac{x}{N}} \left[(1-\delta_B) \ln C_{A,t+x}^{Con*} + \delta_B \ln C_{B,t+x}^{Con*}\right] = \chi_{t+1} \ln W_{t+1} + k_{i,t} \tag{134}$$

where

$$\chi_{t+1} \equiv \sum_{x=1}^{TN-t} \beta^{\frac{x}{N}} \tag{135}$$

and $k_{i,t}$ is a constant. In equilibrium (132) and (133) will not bind and hence I ignore those constraints and verify this later. Using (134) in (130) and substituting in the intertemporal budget constraint (131) allows me to simplify B's problem to:

$$\max_{C_{B,t}} \delta_B \ln C_{B,t} + \chi_{t+1} \ln \left(\widetilde{W}_t - C_{B,t}\right).$$

The first order condition is

$$\frac{\delta_B}{C_{B,t}} - \frac{\chi_{t+1}}{\widetilde{W}_t - C_{B,t}} = 0$$

which gives B's best response for any given level of $\widetilde{W}_t$:

$$\widetilde{C}_{B,t}^* = \frac{\delta_B}{\delta_B + \chi_{t+1}} \widetilde{W}_t. \tag{136}$$

Note that (136) verifies that (132) and (133) are satisified in equilibrium.

Member A will anticipate (136) and choose $C_{A,t}$ to solve

$$\max_{C_{A,t}} \delta_A \ln C_{A,t} + (1-\delta_A) \ln \widetilde{C}_{B,t}^* + \sum_{x=1}^{TN-t} \beta^{\frac{x}{N}} \left[\delta_A \ln C_{A,t+x}^{Con*} + (1-\delta_A) \ln C_{B,t+x}^{Con*}\right] \tag{137}$$

subject to (136),

$$W_{t+1} = R^{\frac{1}{N}} \left(W_t - C_{A,t} - \widetilde{C}_{B,t}^*\right), \tag{138}$$

$$C_{A,t} \geq 0, \text{ and} \tag{139}$$

$$W_t - C_{A,t} \geq 0. \tag{140}$$

I ignore (139) and (140) and verify that they are satisified at the end. Using the analog of (132) for A and substituting (138) and (136) into (137) we rewrite A's problem (ignoring constants) as:

$$\max_{C_{A,t}} \delta_A \ln C_{A,t} + \left((1-\delta_A) + \chi_{t+1}\right) \ln\left(W_t - C_{A,t}\right).$$

The first order condition is

$$\frac{\delta_A}{C_{A,t}^{Con*}} - \frac{(1-\delta_A) + \chi_{t+1}}{W_t - C_{A,t}^{Con*}} = 0.$$

Which gives A's optimal consumption choice as

$$C_{A,t}^{Con*} = \frac{\delta_A}{1+\chi_{t+1}} W_t. \tag{141}$$

Note that (141) demonstrates that (139) and (140) are satisfied as conjectured. Substituting (141) into (136) gives B's equilibrium consumption choice as a function of W_t:

$$C_{B,t}^{Con*} = \frac{\delta_B}{\delta_B + \chi_{t+1}} \left(\frac{1+\chi_{t+1}-\delta_A}{1+\chi_{t+1}}\right) W_t. \tag{142}$$

Adding (141) and (142) gives the equilibrium level of total consumption in period t:

$$X_t^{Con*} = \left(\frac{1}{1+\sum_{x=1}^{TN-t} \beta^{\frac{x}{N}}}\right) \left(\frac{\delta_B + (\delta_A+\delta_B)\sum_{x=1}^{TN-t} \beta^{\frac{x}{N}}}{\delta_B + \sum_{x=1}^{TN-t} \beta^{\frac{x}{N}}}\right) W_t. \tag{143}$$

Note finally that Conjecture 4 was verified about for the case of $t = NT$. Moreover (141) and (142) demonstrate that it is true for $t = NT-1$ and so on by iteration. This establishes Conjecture 4 by induction.

E-3 Comparison of Consecutive and Simultaneous Move Equilibria

Comparing (14) to (143) gives

$$\frac{X_t^*}{X_t^{Con*}} = \frac{\Sigma_t^2 + (1+\delta_B)\Sigma_t + \delta_B}{\Sigma_t^2 + \left(\frac{\delta_B}{\delta_A+\delta_B} + (\delta_A+\delta_B)\right)\Sigma_t + \delta_B}$$

where $\Sigma_t \equiv \sum_{x=1}^{TN-t} \left[\beta^{\frac{1}{N}}\right]^x$. Taking the limit of this ratio as $N \to \infty$ requires finding

$$\lim_{N\to\infty} \frac{X_t^*}{X_t^{Con*}} = \lim_{N\to\infty} \frac{\Sigma_t^2 + (1+\delta_B)\Sigma_t + \delta_B}{\Sigma_t^2 + \left(\frac{\delta_B}{\delta_A+\delta_B} + (\delta_A+\delta_B)\right)\Sigma_t + \delta_B}.$$

Since both the numerator and denominator tend to infinity we can apply L'Hopital's rule to get

$$\begin{aligned}\lim_{N\to\infty}\frac{X_t^*}{X_t^{Con*}} &= \lim_{N\to\infty}\frac{\frac{\partial\Sigma_t}{\partial N}\left(2\Sigma_t+(1+\delta_B)\right)}{\frac{\partial\Sigma_t}{\partial N}\left(2\Sigma_t+\frac{\delta_B}{\delta_A+\delta_B}+(\delta_A+\delta_B)\right)}\\ &= \lim_{N\to\infty}\frac{2\Sigma_t+(1+\delta_B)}{2\Sigma_t+\frac{\delta_B}{\delta_A+\delta_B}+(\delta_A+\delta_B)}.\end{aligned}$$

Again both the numerator and denominator tend to infinity so we can re-apply L'Hopital's rule to get

$$\begin{aligned}\lim_{N\to\infty}\frac{X_t^{*S}}{X_t^*} &= \lim_{N\to\infty}\frac{2\Sigma_t+(1+\delta_B)}{2\Sigma_t+\frac{\delta_B}{\delta_A+\delta_B}+(\delta_A+\delta_B)}\\ &= \lim_{N\to\infty}\frac{2\frac{\partial\Sigma_t}{\partial N}}{2\frac{\partial\Sigma_t}{\partial N}}\\ &= 1.\end{aligned}$$

This establishes Proposition 7.

Figure 1
Equilibrium and Full Commitment Consumption Path

This plot shows the equilibrium level of total household expenditure in every period without commitment X^*_t and the optimal full commitment consumption path X^{**}_t. It is drawn using the following parameters: Initial household wealth is W_1=3,000,000, the exponential discount factor is β=0.95, the gross interest rate is R=1/0.95, the household exists for T=50 years and there are N=1 period within each year. The figure compares the scenario where household members place weight on their own utility of $\delta_A=\delta_B$ =0.6 and $\delta_A=\delta_B$ =0.7.

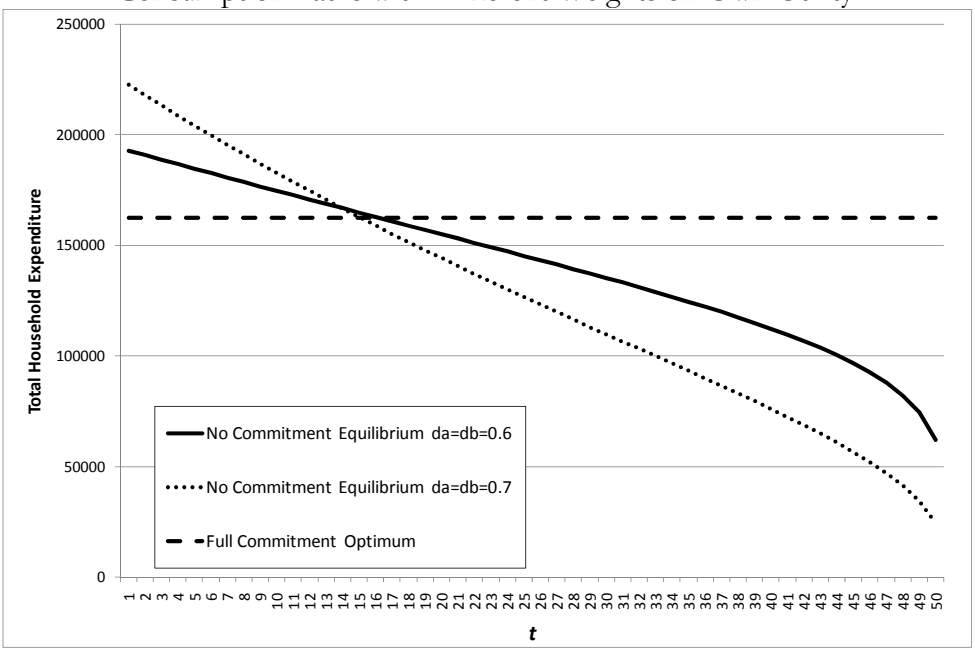
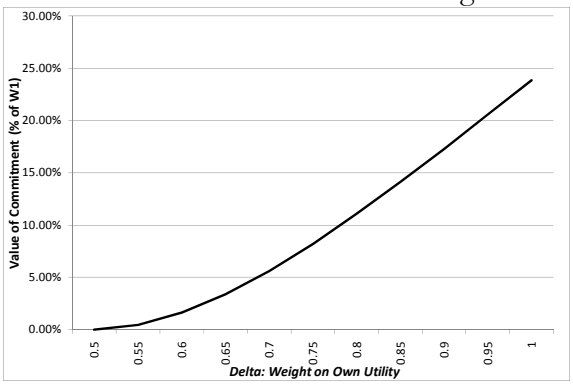


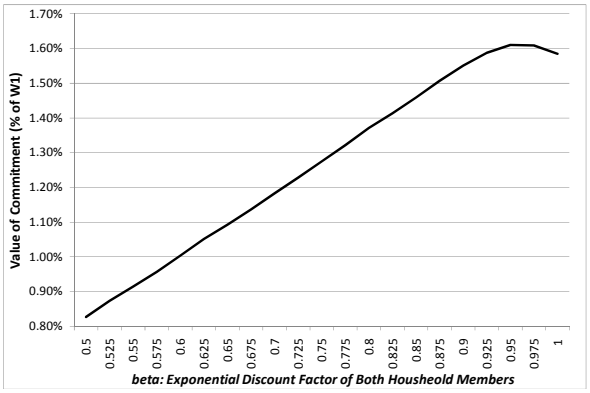
Figure 2
Comparative Statics: The Value of Commitment

These plots show the fraction of W_1 that the household would be willing to pay at t=1 to achieve the full commitment consumption path. Due to log additive utility functions this fraction is invariant to the choice of W_1. Each panel shows how the value of commitment varies with: $\delta=\delta_A=\delta_B$ the weight household members place on their own utility (Panel A); β the discount factor of each household member (Panel B), δ_A the weight member A places on her own utility holding $\Delta=\delta_A+\delta_B-1=0.2$ constant. Apart from the variable on the x-axis, each plot is drawn using the following parameters: the weight both household members place on their own utility is $\delta_A=\delta_B=0.6$, their exponential discount factor is $\beta=0.95$, the gross interest rate is $R=1/0.95$ and the household exists for $T=50$ years with $N=1$ periods per year.

Panel A: The Value of Commitment and the Weight on Own Utility δ



Panel B: The Value of Commitment and Household Member Discount Factor β



Panel C: The Value of Commitment Varying δ_A Holding $\Delta=\delta_A+\delta_B-1$ Constant at 0.2

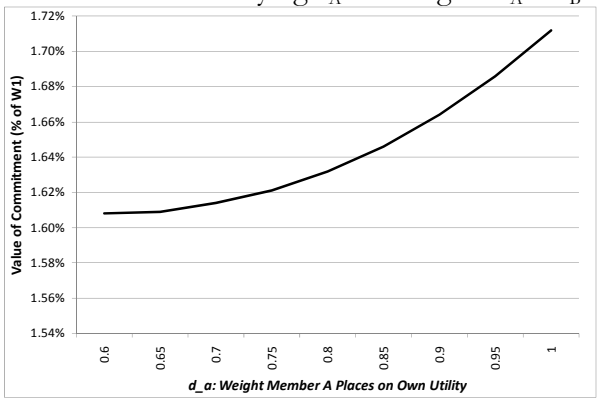
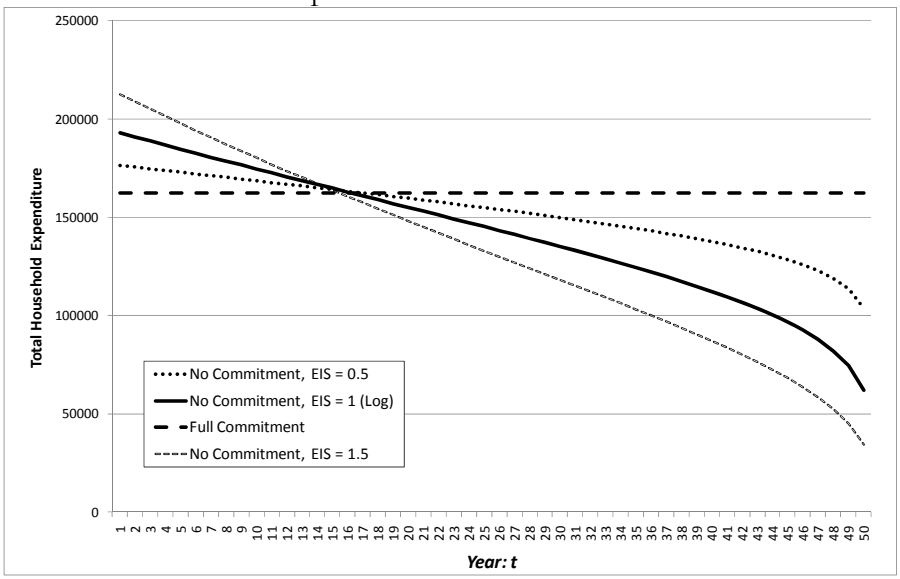


Figure 3
Household with CRRA Preferences

These plots study how changing the EIS varies the consumption path and value of commitment for the household. Each panel is drawn using the following parameters: Initial household wealth is W_1=3,000,000, $\delta_A=\delta_B$=0.6, exponential discount factor is β=0.95, the gross interest rate is R=1/0.95, the household exists for T=50 years and there are N=1 period within each year. Panel A shows the equilibrium level of total household expenditure in every period without commitment X^*_t for values of EIS of 0.5, 1, and 1.5 as well as the optimal full commitment consumption path X^{**}_t.(it is the same for all three parameters). Panel B shows how the value of commitment varies with the EIS.

Panel A: Consumption Paths for Different Values of EIS



Panel B: Value of Commitment for Different Values of EIS

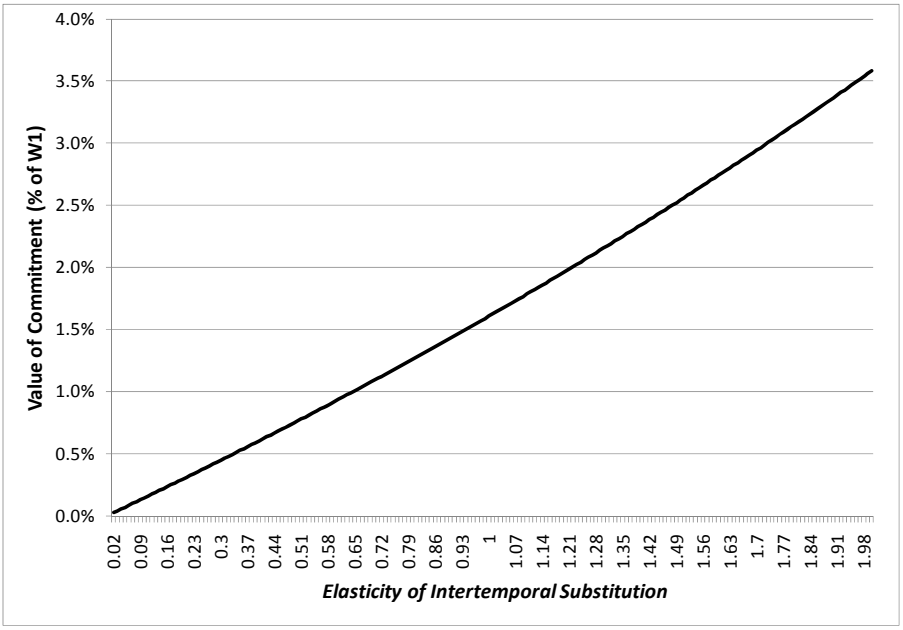


Figure 4
The Value of Commitment with Public Consumption

This plot shows the fraction of W_1 that the household would be willing to pay at t=1 to achieve the full commitment consumption path for different values of μ. Due to log additive utility functions this fraction is invariant to the choice of W_1. It is drawn using the following parameters: the weight both household members place on their own utility is δ=0.6, their exponential discount factor is β=0.95, the gross interest rate is R=1/0.95 and the household exists for T=50 years with N=1 periods per year.

Figure 5
Comparative Statics: The Value of Commitment with Hyperbolic Individuals

These plots show the amount the household would be willing to pay at t=1 (as a fraction of W_1) to achieve the full commitment consumption path. Due to log additive utility functions this fraction will be invariant to the choice of W_1. The figure shows how the value of commitment varies with $\delta=\delta_A=\delta_B$(i.e varying both symmetrically). It is drawn for individual hyperbolic discount factors of Ψ=1 and Ψ=0.85 holding other parameters constant at, β=0.95, R=1/0.95, T=50, and N=1.