# Asset Substitution, Debt Covenants and Conservatism

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## 1 Introduction

In this paper we develop an analytical model to examine how debt covenants are optimally set when borrowers can engage in asset substitution. We also identify the conditions under which conservative accounting enhances the efficiency of debt contracts.

After borrowing to finance investment in a project, a borrower is effectively the holder of a call option and therefore benefits from increasing the risk of the project's cash flows at the expense of the lender. This increase in risk, called asset substitution, causes a transfer of wealth from the lender to the borrower. Asset substitution is value destroying when the cost it imposes on the lender exceeds its benefit to the borrower. At the time debt contracts are signed, lenders can anticipate the borrower's incentive for asset substitution. Therefore value destroying asset substitution can be easily precluded if it is verifiable. In this paper, we analyze a setting in which asset substitution is both value destroying and unverifiable.

Debt covenants written on accounting measures are common components of corporate debt contracts. Debt covenants are meaningful only when they allocate control rights to some verifiable decision that the firm can make after the accounting report is made public and before final repayment of debt occurs. The decision that we examine is an option to either expand or not expand the project. The accounting report informs this decision and in conjunction with the debt covenant determines whether the borrower or the lender will obtain the control rights over this decision.

A decision to expand the project increases the risk of the project. However expansion differs from value destroying unverifiable asset substitution in two important ways. First, the expansion decision is contractible whereas the lack of verifiability makes asset substitution non-contractible. Second, expansion can be value enhancing if the future prospects of the project, assessed after the accounting report is made public, are favorable whereas the asset substitution that we study always destroys value. Thus the contractible increase in risk from expanding the project may be conditionally good, whereas the non-contractible asset substitution risk is unconditionally socially bad. The borrower has a derived preference for more of both types of risk, while the lender has a derived preference for less of both. The debt covenant mediates this conflict of interest.

It has been documented empirically that covenants are initially set tight and violations often subsequently waived by the lender in lieu of concessions made by the borrower. A priori, it is not clear why covenants should be set tight in the first place if violations are only to be waived in the subsequent renegotiation process, when the covenant can be set appropriately at inception to minimize the waiver of violations and the possibility of renegotiation. We provide an explanation for this puzzle.

In addition to regulating the expansion decision, in the presence of the asset substitution problem, the debt covenant can acquire a second role of disciplining the borrower's incentive to engage in asset substitution, if the borrower's appetite for asset substitution depends on whether the project is expanded or not expanded. In such a setting the increase in risk that results from the contractible expansion decision can be used to control the borrower's incentive to increase non-contractible risk via asset substitution. We examine the role of the debt covenant in disciplining the borrower's incentive for asset substitution and in regulating the expansion decision. For the debt covenant to have a chance to discipline asset substitution, it is necessary that the distribution of the accounting report be affected by the asset substitution chosen by the borrower. Therefore, in our model, the asset substitution chosen by the borrower affects the distribution of cash flows from the project, which in turn affects the distribution of the public accounting report that the debt covenant is written on.

Tasked with informing the expansion decision as well as disciplining asset substitution, the optimal debt covenant makes a tradeoff between inducing an efficient interim expansion decision and mitigating value destroying asset substitution. We find that unverifiable asset substitution causes optimal debt covenants to be tighter than when asset substitution is verifiable. A tighter debt covenant implies that the project is not expanded in some circumstances even though the project's future prospects, as assessed from the accounting report, are favorable enough that it ought to be expanded from a pure interim efficiency perspective. Thus the optimal covenant calls for underinvestment in the contractible expansion risk to alleviate the problem of non-contractible asset substitution risk.

We then examine the role of conservative accounting, which involves studying how the properties of the measurement system that produces the accounting report affect the optimal debt covenant, which in turn affects the efficiency of the interim expansion decision and the level of asset substitution. First, we find that even when asset substitution is verifiable and hence can be precluded, so that the only efficiency that matters is that of the interim expansion decision, conservative accounting may be still be optimal. This result is in contrast to the result in Gigler et al (2009) who show that in a world of symmetric information and full verifiability, efficiency considerations call for an accounting system that is always liberal. Second, and more importantly, we also find that debt contract efficiency induced demand for conservative accounting is greater when asset substitution is unverifiable than when it is verifiable.

Garleanu and Zwiebel (2009) also study the design of debt contracts and find that optimal debt covenants are tighter when borrowers are better informed than lenders about their capacity to engage in unverifiable asset substitution. In their model of adverse selection, asset substitution is an exogenous wealth transfer from the lender to the borrower that the borrower is better informed about than the lender, whereas in our model asset substitution is an unverifiable choice made by the borrower. Further, in their model the debt covenant is not written on any public accounting report and assigns rights ex-ante to either the lender or the borrower. In contrast, in our paper asset substitution occurs before the accounting report is released and affects the distribution of the accounting report that the debt covenant is written on. Gorton and Kahn (2000) examine asset substitution that occurs after the accounting report has been realized, while our paper studies asset substitution that affects the accounting report. Neither Garleanu and Zwibel (2009) nor Gorton and Kahn (2000) examines the impact of conservative accounting on the efficiency of debt contracts. Gigler et al (2009) examines the role of conservatism on the efficiency of debt contracts in a world of symmetric information and finds that in the context of a liquidation decision that the debt covenant assigns control rights to, conservative accounting detracts from the efficiency of debt contracts.

The rest of the paper is organized as follows. Section 2 introduces the model, characterizes the first best debt contract and examines the incentives for asset substitution when there is no debt covenant. Section 3 examines how a debt covenant mitigates asset substitution when the borrower and lender can commit to not renegotiating the initial debt contract. Proposition 1 shows that asset substitution increases with debt and decreases as the debt covenant is made stricter. Proposition 2 shows that the optimal full commitment debt covenant is stricter than in the first best case, involves underinvestment and trades off inefficient asset substitution against inefficient expansion.

In Section 4 we allow the lender and borrower to renegotiate the initial debt contract. Proposition 3 shows that renegotiation improves the efficiency of the interim expansion decision as well as permits implementation of a lower level of asset substitution than under full commitment and that the optimal debt covenant is stricter than under first best. That renegotiation improves welfare is consistent with the result in Hermalin and Katz (1991).

Section 5 examines the role of conservatism. Proposition 4 shows that there can be a demand for conservative accounting even when asset substitution is precluded, which is in contrast to the result to the result in Gigler et al (2009). It also shows that unverifiable asset substitution increases the demand for conservatism and that this increase in demand is greater under full commitment than under costless renegotiation.

## 2 The Model

Consider a firm with exclusive rights to a project that needs investment of **K** at Date 0. The cash flows $\tilde{x}$ from the project are uncertain and realized at Date 2. We assume that the entire investment for the project has to be obtained via debt raised at Date 0, to be repaid with interest at Date 2 when the cash flows from the project are realized. The firm can borrow from a competitive debt market where the risk free interest rate is **R**. Lenders as well as the residual claimants (henceforth, borrower) to the firm's cash flows are risk-neutral. So lenders will lend K to the firm if their expected repayment is at least K(1+R).

After borrowing has occurred at Date 0, the borrower may engage in value destroying asset substitution that can be observed by the lender[1], but cannot be verified by a third party and hence is not contractible. That asset substitution is unverifiable implies that it does not shift the support of the distribution of cash flows $\tilde{\boldsymbol{x}}$ from the project. Asset substitution, represented by the continuous variable $a$, increases the risk of the project cash flows and is assumed to be value destroying.

At an interim Date 1, the firm has the option to either expand the project or continue the project without expanding it. For simplicity, we assume that the expansion does not require any cash outlay. Project expansion shifts the support of the distribution of cash flows $\tilde{\boldsymbol{x}}$ from the project and is therefore contractible. Before the expand/do not expand decision is made, the firm's accounting system provides a public report $z$ that is informative about $\tilde{\boldsymbol{x}}$, the Date 2 cash flows of the project.

The debt contract signed at Date 0 is the triplet **{K, D,** $\boldsymbol{Y}$**}**, where **K** is the amount borrowed at Date 0, **D** is the face value of debt to be paid to the lender at Date 2 and **Y** is the debt covenant. Violation of the debt covenant by the accounting report at Date 1 transfers control of the expansion decision to the lender. Else, the decision right vests with the borrower. At the interim Date 1, after the accounting report is made public, the lender and borrower may renegotiate the terms of the initial debt contract. If renegotiation occurs, the party that has the control rights to the expansion decision may surrender its right in return for a change in the face value of debt. Let $\mathbf{D_N}$ denote the renegotiated face value of the debt to be repaid at Date 2. If no renegotiation occurs, then $\mathbf{D_N} = \mathbf{D}$.

Finally, if the cash flows $\tilde{\boldsymbol{x}}$ realized at Date 2 exceed $\mathbf{D_N}$ then full repayment of the face value of debt $\mathbf{D_N}$ occurs and the borrower gets the excess. Else, the borrower gets nothing and the lender takes the entire final cash flows.

The cash flows from the project depend on the state of the world and on whether the project has been expanded or not. The probability of the state of the world depends on asset substitution. As unverifiable asset substitution is an increase in risk that does not shift the support of the distribution of cash flows, we require a minimum of three states of the world to model the increase in risk from asset substitution. So we assume that the state of the world $S$

[1] When the possibility of renegotiation is closed, it does not matter whether asset substitution is observable or not observable by the lender.

can take on three values - Good (G), Medium (M) or Bad (B). Asset substitution changes the probabilities of these three states without affecting the cash flows conditional on each state. In contrast, as the expansion decision is verifiable, we assume that expansion of the project changes the cash flows from the project, conditional on states G and B.

If there is no asset substitution, the probability of state M is $p$ and that of states G and B is $\frac{1-p}{2}$ each. Asset substitution $a$ increases risk by moving probability mass $a$ from the center of the distribution(state M) of cash flows to the two extremes (states G and B), without changing the support of the distribution of cash flows from the project. This increase in risk is consistent with that developed in Rothschild and Stiglitz (1970). The probability mass moved to the Good state is $\alpha a - \frac{ca^2}{2}$ while the probability mass moved to the Bad state is $(1-\alpha)\, a + \frac{ca^2}{2}$.

$\alpha > 0$ $and$ $c > 0$ are parameters that reflect the cost of asset substitution. A sufficiently low value of the parameter $\alpha$ ensures that asset substitution reduces the value of the project for any level of asset substitution. As the extent of asset substitution increases, the probability mass moved to the Good state increases at a decreasing rate, while the probability moved to the Bad state increases at an increasing rate. It can be verified that the probabilities of the three states sum to 1 for all values of $a$.

The cash flow conditional on state M is $X_M$, regardless of whether the project is expanded or not. If the project is not expanded, the cash flow conditional on state G is $X_H$ while the cash flow conditional on state B is $X_L$. We assume that $X_H > X_M > X_L > 0$ are known constants, and that $X_M$ is the mid-point of $[X_L, X_H]$. Expansion increases the cash flow conditional on the state G by β > 0 and decreases that of the state B by T > 0, where $X_L$ - T > 0. Thus expansion increases the risk of the project, and it does so by moving the support of the distribution of cash flows.

The table on the next page and figures 1 through 4 describe how the distribution of the cash flows from the project is affected by asset substitution and by the expansion decision.

| | Probability of Good State G | Probability of Medium State M | Probability of Bad State B | Cash Flows from Project in State | | |
|---|---|---|---|---|---|---|
| | | | | G | M | B |
| No asset substitution and No expansion | $\frac{1-p}{2}$ | $p$ | $\frac{1-p}{2}$ | $X_H$ | $X_M$ | $X_L$ |
| No asset substitution with expansion | $\frac{1-p}{2}$ | $p$ | $\frac{1-p}{2}$ | $X_H$+β | $X_M$ | $X_L$-T |
| With asset substitution $a$ and No expansion | $\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}$ | $p-a$ | $\frac{1-p}{2} + (1-\alpha)\, a + \frac{ca^2}{2}$ | $X_H$ | $X_M$ | $X_L$ |
| With asset substitution $a$ and with expansion | $\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}$ | $p-a$ | $\frac{1-p}{2} + (1-\alpha)\, a + \frac{ca^2}{2}$ | $X_H$+β | $X_M$ | $X_L$-T |

Let $f(z|S,\delta)$ and $F(z|S,\delta)$ denote the conditional density and conditional cumulative distribution functions respectively, of the accounting signal z, given state S, where δ is a parameter that represents how liberal the accounting system is. We will elaborate on the role of δ in Section 5 when we examine the role of conservatism. We assume that the support of $\boldsymbol{z}$ is $[\underline{Z}, \overline{Z}]$ and that it has the monotone likelihood ratio property (MLRP).

**Assumption 1**: For any given δ and $S < \hat{S}$, the likelihood ratio $\frac{f(z|S,\delta)}{f(z|\hat{S},\delta)}$ is decreasing in z (MLRP) and attains zero in a neighborhood near $\overline{Z}$.

Assumption 1 guarantees that higher values of the accounting report move the posterior distribution of cash flows to the right, for every non-degenerate prior distribution of cash flows so that higher values of the accounting report constitute good news.

## 2.1 Value Destroying Asset Substitution

The option to expand the project at the interim date has positive value because the expansion decision is made after observing the accounting report that is informative about the cash flows. This option value increases with the risk of the project, which in turn increases with asset substitution. So even if asset substitution decreases the expected value[2] of the cash flows, it is possible that the value of the project, that includes the option value of expansion, does not decrease with asset substitution. We now identify a sufficient condition that ensures that asset substitution is value destroying.

Let $V(a, Y)$denote the Date 0 expected value of the project when the decision rule is that the project is expanded if and only if $z > Y$.

$$V(a, Y) = Prob(G; a)X_H + Prob(M; a)X_M + Prob(B; a)X_L + \beta Prob(G; a)[1 - F(Y|G)]$$

$$-TProb(B; a)[1 - F(Y|B)]$$

For asset substitution to be value destroying, the derivative $V_a(a, Y)$ has to be negative for all values of $\alpha$ and Y. Differentiation the above equation with respect to $\alpha$ after substituting for the probabilities of the three states, we get

$$V_a(a, Y) = (\alpha - ac)X_H - X_M + (1 - \alpha + ac)X_L + \beta[1 - F(Y|G)](\alpha - ac)$$

$$-T[1 - F(Y|B)](1 - \alpha + ac)$$

[2] $\alpha < \frac{1}{2}$ ensures that the expected value of cash flows decreases with asset substitution when the project is not expanded, while

$$\alpha < \frac{X_H - X_L + 2T}{2(X_H - X_L + \beta + T)}$$

ensures that the expected value of cash flows decreases with asset substitution when the project is expanded. For asset substitution to be value destroying when the value of the option to expand is also taken into account, it is necessary that α be lower than that required to decrease the expected value of cash flows, which is ensured by Assumption 2.

Note that α < 1. We will show later that $\alpha - ac > 0$. So $V_a(a,Y)$ attains its maximum when the accounting signal and the threshold Y are such that they distinguish states G and B perfectly i.e. when $F(Y|G) = 0$ and $F(Y|B) = 1$. Substituting these values for $F(Y|G)$ and $F(Y|B)$, we get

$$V_a(a,Y) < (\alpha - ac)X_H - X_M + (1 - \alpha + ac)X_L \; + \; \beta(\alpha - ac)$$

$$\leftrightarrow V_a(a,Y) < \alpha(X_H + \beta - X_L) + X_L - X_M - ac(X_H + \beta - X_L)$$

As $X_M$ is the mid-point of $[X_L, X_H]$ the above inequality reduces to

$$V_a(a,Y) < \alpha(X_H + \beta - X_L) + \frac{X_L}{2} - \frac{X_H}{2} - ac(X_H + \beta - X_L)$$

A sufficient condition to ensure that the right-hand-side in the inequality above is negative is

$2\alpha(X_H + \beta - X_L) + X_L - X_H < 0$ that motivates the following assumption.

**Assumption 2:** $\alpha < \frac{X_H - X_L}{2(X_H - X_L + \beta)} < \frac{1}{2}$

## 2.2 Asset Substitution with No Debt Covenant

We first examine the incentives of the borrower to engage in asset substitution and the conflict of interest between the lender and the borrower when there is no debt covenant. We begin by characterizing the Date 0 expected payoffs of the borrower and the lender when there is no accounting signal at the interim date. This characterization is done separately for the unexpanded project and the expanded project. Let $B(a,D)$ and $L(a,D)$ denote the expected payoffs to the borrower and lender respectively at Date 0, given asset substitution $a$ and face value of debt D.

**Unexpanded Project: Expected Payoffs of Borrower and Lender**

If $\boldsymbol{D > X_M}$ then $B(a, D, no\ expansion)$ = $(X_H - D)Prob(G;a) = \; (X_H - D)\left\{\frac{1-p}{2} + \; \alpha a - \; \frac{ca^2}{2}\right\}$. The borrower will choose asset substitution to maximize his expected payoff which attains its maximum at $a = \frac{\alpha}{c}$, which is independent of D. To ensure that at the maximum level of asset substitution the prior probability of the medium state M remains non-negative, we make the following assumption.

**Assumption 3**: The cost parameter c is sufficiently high to ensure that $\frac{\alpha}{c} \leq p$.

If $\boldsymbol{D < X_M}$ then $B(a, D, no\ expansion)$= $(X_H - D)Prob(G;a) + (X_M - D)Prob(M;a)$

$$= \left\{\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}\right\}(X_H - D) + (X_M - D)\{p - a\}$$

The first order condition to the borrower's asset substitution choice problem is

$$B_a(a, D, no\ expansion) = (X_H - D)(\alpha - ac) - (X_M - D)$$

which is linear in $a$ and yields the unique solution

$$a(D) = \frac{1}{c}\left\{\alpha - \frac{X_M - D}{X_H - D}\right\}$$

It can be verified that $a(D) > 0$ if and only if D is sufficiently high i.e. if

$$D > \frac{X_H(1-2\alpha) + X_L}{2(1-\alpha)} \equiv D_{NE}$$

and that $a(D) = 0$ if $D < D_{NE}$ and that $X_L < D_{NE} < X_M$.

We now examine the lender's expected payoff.

For $\boldsymbol{X_L < D < X_M}$

$$L(a, D, no\ expansion) = \{Prob(G;a) + Prob(M;a)\}D + Prob(B;a)X_L$$

$$= \left\{\frac{1+p}{2} - (1-\alpha)a - \frac{ca^2}{2}\right\}D + \left\{\frac{1-p}{2} + (1-\alpha)\,a + \frac{ca^2}{2}\right\}X_L$$

To examine how the lender's expected payoff varies with asset substitution we take the derivative of the above expression with respect to $a$ below

$$L_a(a, D, no\ expansion) == \{(1-\alpha) + ca\}\{X_L - D\}$$

and note that it is negative which implies that the lender is hurt by asset substitution.

**Expanded Project: Expected Payoffs of Borrower and Lender**

If $\boldsymbol{D < X_M}$

$$B(a, D, expansion) = (X_H + \beta - D)Prob(G;a) + (X_M - D)Prob(M;a)$$

$$= \left\{\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}\right\}(X_H + \beta - D) + (X_M - D)\{p - a\}$$

The first order condition to the borrower's asset substitution choice problem is

$$B_a(a, D, expansion) = (X_H + \beta - D)(\alpha - ac) - (X_M - D)$$

which is linear in $a$ and yields the unique solution

$$a(D) = \frac{1}{c}\left\{\alpha - \frac{X_M - D}{X_H + \beta - D}\right\}$$

It can be verified that $a(D) > 0$ if and only if D is sufficiently high i.e. if

$$D > \frac{X_H(1 - 2\alpha) + X_L - 2\alpha\beta}{2(1 - \alpha)} \equiv D_E$$

and that $a(D) = 0$ if $D < D_E$. Assumption 2 ensures that $X_L < D_E$ and it can be verified that $D_E < D_{NE} < X_M$

Turning to the lender, if $\boldsymbol{X_L < D < X_M}$ then

$$L(a, D, expansion) =\{Prob(G; a) + Prob(M; a)\}D + Prob(B; a)[X_L - T]$$

$$= \left\{\frac{1+p}{2} - (1-\alpha)a - \frac{ca^2}{2}\right\}D + \left\{\frac{1-p}{2} + (1-\alpha)\,a + \frac{ca^2}{2}\right\}[X_L - T]$$

and

$$L_a(a, D, expansion) = \{-(1-\alpha) - ca\}D + \{(1-\alpha) + ca\}[X_L - T]$$
$$= \{(1-\alpha) + ca\}\{X_L - T - D\} < 0$$

which confirms that the lender is hurt by asset substitution.

**Comparison of Expansion and No Expansion Expected Payoffs**

Comparing the expansion and no expansion payoffs of the borrower, we find that the borrower's expected payoff is always higher when the project is expanded, whereas the lender's expected payoff is always higher when the project is expanded. Also, it can be verified by comparing the expressions for $B_a(a, D, no\ expansion)$ and $B_a(a, D, expansion)$ that the borrower's incentive to engage in asset substitution is greater when the project is expanded than when it is not expanded. In other words, the borrower's appetite for non-contractible asset substitution risk is increasing in the risk from the contractible expansion decision.

The participation constraint for the lender requires that the lender's expected payoff is at least $K(1+R)$. The lender is hurt by expansion and by asset substitution. So the face value of debt D will be at its highest when the lender anticipates that the project will be expanded and when asset substitution is at its maximum value of $\frac{\alpha}{c}$. Therefore a sufficient condition to ensure $D < X_M$ is that K(1+R) be small enough to ensure that

$$L\left(a=\frac{\alpha}{c}\ ,D=X_M, expansion\right) > K(1+R)$$

$$\leftrightarrow \left\{\frac{1+p}{2}-\frac{\alpha}{c}+\ \frac{\alpha^2}{2c}\right\}X_M+\left\{\frac{1-p}{2}+\frac{\alpha}{c}-\ \frac{\alpha^2}{2c}\right\}[X_L-T] > K(1+R)$$

which motivates the following assumption.

**Assumption 4**: $2K(1+R) < \left\{1+p-\frac{2\alpha}{c}+\frac{\alpha^2}{c}\right\}X_M+\left\{1-p+\frac{2\alpha}{c}-\frac{\alpha^2}{c}\right\}[X_L-T]$

We have shown that the borrower wants to expand the project even if the future looks dismal, whereas the lender does not want to expand the project, even if the future looks bright. This conflict can be resolved by appropriate assignment of control rights to the expansion decision. A debt-covenant, which takes the form of a threshold signal, is such a contract. The covenant assigns the decision right to expand or not expand the project to the lender when the accounting signal is such that the debt covenant is violated and leaves the decision right with the borrower whenever the accounting signal satisfies the debt covenant. We next characterize the optimal covenant when asset substitution is verifiable and hence can be precluded.

## 2.3 First Best Benchmark

We now characterize the first best debt contract, when asset substitution is contractible. Clearly, given that asset substitution is value destroying, the first best value of $a$ is zero. With asset substitution precluded, when the debt covenant is Y, the borrower's expected payoff $B(D,Y)$ is

$$B(D,Y)=(X_H+\beta-D)Prob(G)\{1-F(Y|G)\}+(X_M-D)Prob(M)\{1-F(Y|M)\}$$
$$+(X_H-D)Prob(G)F(Y|G)+(X_M-D)Prob(M)F(Y|M)$$

which simplifies to

$$B(D,Y)=(X_H-D)Prob(G)+(X_M-D)Prob(M)+\ \beta Prob(G)\{1-F(Y|G)\}$$

The lender's expected payoff $L(D,Y)$ is

$$L(D,Y)\equiv\{Prob(G)+Prob(M)\}D+Prob(B)F(Y|B)X_L\ +Prob(B)[1-F(Y|B)][X_L-T]$$

which simplifies to

$$L(D,Y)=\{1-Prob(B)\}D+Prob(B)X_L-TProb(B)[1-F(Y|B)]$$

The value of the project which is the sum of the expected payoffs to the borrower is independent of the face value of debt as it is a mere transfer between the two parties and is given by

$$V(Y) = X_H Prob(G) + X_M Prob(M) + X_L Prob(B) + \beta Prob(G)\{1 - F(Y|G)\} - T Prob(B)[1 - F(Y|B)]$$

We will derive the optimal debt contract by maximizing the expected payoff of the borrower, subject to the lender's participation constraint. The first debt contract is then the solution to the following program

$$\underset{D,Y}{Max} \quad B(D,Y)$$

subject to

PC: $L(D,Y) = K(1+R)$

The above program is equivalent to

$$\underset{D,Y,\mu}{Max} \quad B(D,Y) \, + \, \mu \,[L(D,Y) - K(1+R)]$$

where μ is the Lagrange multiplier associated with the IR constraint.

The first order conditions (in addition to the IR) are:

$$B_D(D,Y) = -\mu L_D(D,Y)$$

$$B_Y(D,Y) = -\mu L_Y(D,Y)$$

As $B_D + L_D = 0$ $[because\ B_D = -Prob(M) - Prob(G)\ and\ L_D = 1 - Prob(B)]$ , from the first FOC above, the multiplier μ is equal to 1 which implies that a marginal increase in $K(1+R)$decreases the maximized value of the objective function by the same amount.

Substituting $\mu = 1$ in the second FOC above, and using $B_Y(D,Y) = -\beta Prob(G) f(Y|G)$ and $L_Y(D,Y) = T Prob(B)\, f(Y|B)$ yields

$$\beta Prob(G) f(Y|G) = \, T Prob(B)\, f(Y|B)$$

So the first best debt covenant $Z^*$ is given by

$$\frac{f(Z^*|G)}{f(Z^*|B)} = \frac{T Prob(B)}{\beta Prob(G)}$$

The right hand side of the above equation is independent of Z*, and by MLRP the left hand side is increasing in Z*. So there exists a unique Z* that satisfies the above equation.

It can be verified that

$$\beta Post(G|z) - TPost(B|z) = 0$$

where $Post(S|z)$ is the posterior probability of state S conditional on accounting signal z yields the same equation for Z*.

Given Z*, the first best D can be obtained by solving the IR.

***Lemma 1:*** *The first best contract ensures that there is no asset substitution, that the efficient expansion decision rule equates the expected marginal payoffs of the borrower and the lender from an increase in the debt covenant and that the efficient debt covenant Z* is given by*

$$\frac{f(Z^*|G)}{f(Z^*|B)} = \frac{TProb(B)}{\beta Prob(G)}$$

When the borrower can engage in asset substitution, in addition to inducing an efficient expansion decision, the debt covenant can also be used to discipline asset substitution. We now examine these two roles of the debt covenant and the tradeoff between them when asset substitution is unverifiable.

## 3 Full Commitment Optimal Debt Covenant

We first derive the optimal debt contract when the borrower and lender can commit to not renegotiating the initial debt contract. Renegotiation is likely to not occur with public debt where the costs of coordination among multiple lenders are too high. The debt contract signed at Date 0 is the triplet {K, D, $Y$}, where **K** is the amount borrowed at Date 0, **D** is the face value of debt to be paid to the lender at Date 2 and **Y** is the debt covenant. Violation of the debt covenant by the accounting report at Date 1 transfers rights to the expansion decision to the lender. Else, the decision right vests with the borrower. As lenders prefer to not expand the project while the borrower prefers expansion, the project is not expanded if the debt covenant is violated and expanded if it is satisfied.

The efficient expansion decision rule is to expand the project at the interim Date 1 if and only if the accounting report is favorable enough that conditional on the accounting report, the expected net gain from expansion is positive i.e.

$$\beta Post(G; a|z) - TPost(B; a|z) > 0$$

where $Post(S; a|z)$ is the posterior probability, given accounting signal z, of state S. Given MLRP, the left hand side of the inequality above is increasing in z. Let Z*(a) be the threshold value of the accounting signal z such that the left hand side is exactly zero.

Let $B(a, D, Y)$ denote the expected payoff to the borrower at Date 0, given asset substitution $a$, face value of debt D and the debt covenant Y. We want to characterize the optimal $a(D, Y)$ chosen by the borrower and examine how it varies with D and Y.

$$B(a, D, Y) = (X_H + \beta - D)Prob(G; a)\{1 - F(Y|G)\} + (X_M - D)Prob(M; a)\{1 - F(Y|M)\}$$

$$+(X_H - D)Prob(G; a)F(Y|G) + (X_M - D)Prob(M; a)F(Y|M)$$

which simplifies to

$$B(a, D, Y) = (X_M - D)Prob(M; a) + (X_H - D)Prob(G; a) + \beta Prob(G; a)\{1 - F(Y|G)\}$$

The first two terms on the right hand side of the above equation are the same as the borrower's expected payoff absent any debt covenant and expansion. The last term represents the expected payoff from expansion and decreases as the debt covenant is made tighter. Differentiate the borrower's expected payoff with respect to $a$ to get

$$B_a(a, D, Y) = (X_H - D)(\alpha - ac) - (X_M - D) + \beta\{1 - F(Y|G)\}(\alpha - ac)$$

The first order condition to the borrower's asset substitution choice problem is linear in $a$ and yields a unique solution

$$a(D, Y) = \frac{1}{c}\left\{\alpha - \frac{(X_M - D)}{(X_H - D + \beta\{1 - F(Y|G)\})}\right\}$$

Note that

$$B_{aa}(a, D, Y) = -c\{X_H - D + \beta[1 - F(Y|G)]\} < 0$$

$$B_{aD}(a, D, Y) = -(\alpha - ac) + 1 = 1 - \alpha + ac > 0$$

So $a_D(D, Y) = \frac{-B_{aD}(a,D,Y)}{B_{aa}(a,D,Y)}$ is strictly positive i.e. asset substitution increases with the face value of debt D. Also

$$B_{aY}(a, D, Y) = -\beta f(Y|G)(\alpha - ac) < 0$$

because $(\alpha - ac) > 0$ . So

$a_Y(D, Y) = \frac{-B_{aY}(a,D,Y)}{B_{aa}(a,D,Y)}$ is strictly negative i.e. asset substitution decreases as the debt covenant becomes stricter, which leads us to the following proposition.

***Proposition 1***

*Under full commitment, given a debt contract {D,Y}:*

*(i) there exists a unique* $a(D, Y)$ *that solves the borrower's asset substitution choice problem*

*(ii)asset substitution increases with the face value of debt and decreases as the debt covenant is made stricter i.e.* $a(D, Y)$ *is increasing in the face value of debt D and decreasing in Y.*

Per Proposition 1 the borrower's incentive to engage in asset substitution increases with the face value of debt - this result is consistent with the result in Green and Talmor (1986) who show that asset substitution increases with leverage. We also find that a stricter debt covenant mitigates asset substitution. A stricter debt covenant implies that the project is less likely to be expanded. The borrower's incentive to engage in asset substitution is stronger when the project is expanded than when it is not expanded. As a stricter debt covenant decreases the probability of expansion, it decreases the borrower's incentive for asset substitution. So Proposition 1 opens up the possibility that the debt covenant may be used as a control variable to mitigate asset substitution. To focus on the role of the debt covenant in mitigating asset substitution, we assume that $K(1+R)$is large enough to ensure that the borrower has an incentive to engage in asset substitution even when the project is not expanded. Recall that if $D > D_{NE}$ then the borrower has an incentive to engage in asset substitution. The lender is hurt by expansion and by asset substitution. To satisfy the lender's participation constraint, D will be at its lowest when the lender anticipates that the project will not be expanded and when there is no asset substitution. So a sufficient condition to ensure $D > D_{NE}$ is that $K(1+R)$ be large enough to ensure that

$$L(a = 0\,, D = D_{NE}, no\ expansion) = \left\{\frac{1+p}{2}\right\} D_{NE} + \left\{\frac{1-p}{2}\right\} X_L < K(1+R)$$

which yields the following assumption upon substituting for $D_{NE}$ in the above inequality.

**Assumption 5**[3]: $2K(1+R) > (1+p)\frac{X_H(1-2\alpha)+X_L}{2(1-\alpha)} + (1-p)X_L$

We derive the optimal debt contract by maximizing the expected payoff of the borrower, subject to the lender's participation constraint and the incentive compatibility constraint of the borrower that ensures that the borrower chooses asset substitution to maximize his expected payoff. So the optimal full commitment debt contract is the solution to

$$\underset{a,D,Y}{Max} \quad B(a, D, Y)$$

[3] It can be verified that Assumptions 2 through 5 are compatible i.e. the set of parameter values that satisfy all conditions is non-empty.

subject to

IR: $L(a, D, Y) = K(1 + R)$

IC: $B_a(a, D, Y) = 0$ i.e. the borrower chooses asset substitution to maximize his expected payoff, given D and Y[4].

The lender's participation constraint, given asset substitution, and given a debt contract {D,Y} is

$$\begin{aligned} L(a, D, Y) \equiv & \{Prob(G; a) + Prob(M; a)\}D \\ & + Prob(B; a)F(Y|B)X_L + Prob(B; a)[1 - F(Y|B)][X_L - T] = K(1 + R) \end{aligned}$$

which simplifies to

$$\begin{aligned} L(a, D, Y) \equiv & \{Prob(G; a) + Prob(M; a)\}D + Prob(B; a)X_L - TProb(B; a)[1 - F(Y|B)] \\ & = \{1 - Prob(B; a)\}D + Prob(B; a)X_L - TProb(B; a)[1 - F(Y|B)] \\ & = K(1 + R) \end{aligned}$$

We use the method of Lagrange multipliers to solve for the optimal debt covenant. The full commitment debt contract is the solution to

$$\underset{a,D,Y,\lambda,q}{Max} \quad B(a, D, Y) + \lambda\,[L(a, D, Y) - K(1 + R)] + qB_a(a, D, Y)$$

where λ and q are Lagrange multipliers.

The first order conditions (in addition to the IR and IC constraints) are:

$$B_D(a, D, Y) = -\lambda L_D(a, D, Y) - qB_{aD}(a, D, Y) \qquad FOC\ (1)$$

$$B_Y(a, D, Y) = -\lambda L_Y(a, D, Y) - qB_{aY}(a, D, Y) \qquad FOC\ (2)$$

$$B_a(a, D, Y) = -\lambda L_a(a, D, Y) - qB_{aa}(a, D, Y) \qquad FOC\ (3)$$

The IC requires that $B_a(a, D, Y) = 0$. So FOC (3) reduces to

$$-\lambda = q\frac{B_{aa}(a, D, Y)}{L_a(a, D, Y)}$$

Both the numerator and denominator of the fraction on the right hand side of the equation above are negative, which implies that λ and $q$ have opposite signs. Substituting for λ in FOC (2) and solving for $q$ yields

[4] The first order approach of replacing the IC by the first order condition is valid as the borrower's asset substitution choice problem has been shown to have a unique solution.

$$q = \frac{B_Y(a, D, Y)}{\frac{B_{aa}(a, D, Y)}{L_a(a, D, Y)} L_Y(a, D, Y) - B_{aY}(a, D, Y)}$$

In the above expression, the numerator is negative. In the denominator, the first term is positive while the second term is negative. Therefore $q$ is negative. As λ and $q$ have been shown to be of opposite signs, a negative $q$ implies that λ is positive, which makes sense because we expect the maximized value of the objective function decreases when K(1+R) increases. Recall that in the first best case, the Lagrange multiplier associated with the IR constraint was equal to 1. We now proceed to show that in the second best full commitment contract, the Lagrange multiplier associated with the IR constraint is greater than 1.

Using the fact that $B_D(a, D, Y) = -L_D(a, D, Y)$ in FOC (1) and dividing it by $B_D(a, D, Y)$ , we reduce it to the simpler

$$\lambda = 1 + qB_{aD}(a, D, Y)$$

As both $q$ and $B_{aD}(a, D, Y)$ are negative, the above equation implies that λ is greater than 1, which implies that a marginal increase in K(1+R) decreases the maximized value of the objective function by more than the increase in K(1+R). That λ is greater than 1 means that in the presence of asset substitution, debt imposes a social cost. Proposition 2 shows that this social cost arises because the optimal debt covenant trades off the cost of inefficient asset substitution against the cost of inefficient expansion and that this tradeoff causes underinvestment i.e. the optimal debt covenant is stricter than the one that induces an efficient expansion decision.

***Proposition 2***

*When the borrower can engage in asset substitution, the optimal full commitment second best debt covenant Y* trades off inefficient asset substitution against inefficient expansion and is such that Y* > Z*(a) i.e. the optimal covenant is stricter than the first best covenant associated with an efficient expansion decision. The optimal debt contract tolerates some underinvestment in the sense that expansion is not undertaken when the accounting signal is in the region [Z*(a), Y*] to mitigate asset substitution.*

## 4 Optimal Debt Covenant Under Costless Renegotiation

The full commitment optimal debt covenant is not renegotiation proof. So we now allow renegotiation after the accounting signal is observed at Date 1. We assume that renegotiation is costless. Let $\{D, Y\}$ be the initially debt contract and let $a(D, Y)$ be the asset substitution that it induces the borrower to choose. With costless renegotiation, given the induced asset substitution, the expansion decision will be efficient i.e. the project will be expanded when the accounting signal exceeds Z*(a) and not expanded when it falls below Z*(a).

We assume that the entire bargaining power at the interim date vests with the lender. This means that the lender can make a take it or leave it offer to the borrower. The borrower will accept the offer as long as his Date 1 expected payoff, given the accounting report, from accepting the offer is at least as great as the expected payoff from rejection or status quo. Therefore the lender will make an offer that makes the borrower indifferent between acceptance and rejection of the offer. Thus the lender will appropriate the entire expected gains from renegotiation[5].

The contracted debt covenant Y may lie either above Z*(a) or below it. In both cases, gains from renegotiation arise when the accounting signal lies between the debt covenant and Z*(a). We examine each case below. Let $Post(S; a|z)$ be the posterior probability, given accounting signal z, of state S.

(i) Z*(a) < z < Y: In this case, the realized accounting signal is such that the debt covenant has been violated. So the lender has the control rights. From an efficiency perspective, the project should be expanded. Renegotiation will occur if the expected gain from expansion is positive i.e.

$$\beta Post(G; a|z) - TPost(B; a|z) > 0$$

(ii) Y < z < Z*(a): In this case, the realized accounting signal satisfies the debt covenant. So the borrower has the control rights to the expansion decision. From an efficiency perspective, the project should not be expanded. Renegotiation will occur if the expected gain from not expanding the project is positive i.e.

$$-\beta Post(G; a|z) + TPost(B; a|z) > 0$$

In the first case above, to allow expansion, the lender will extract the expected gains from expansion via an increase in the face value of debt. In the second case, the borrower will have to be compensated via a reduction in the face value of debt for allowing expansion to not occur. We will analyze the first case below and show that the optimal debt covenant will be chosen such that the second case will not occur.

**<u>Case: Z*(a) < z < Y</u>**

Given the accounting signal z and the already chosen level of asset substitution, the expected Date 1 payoff to the borrower is as follows.

[5] The qualitative nature of the results does not depend on assignment of bargaining power.

If there is no renegotiation, the borrower's expected payoff is:

$$B(D, a|z, no\ expansion) = (X_H - D)Post(G; a|z) + (X_M - D)Post(M; a|z)$$

If there is renegotiation that allows for expansion and the renegotiated face value of debt is $D_N$, then the borrower's expected payoff is:

$$B(D_N, a|z, expansion) = (X_H + \beta - D_N)Post(G; a|z) + (X_M - D_N)Post(M; a|z)$$

The take it or leave it offer from the lender will be such that

$$B(D, a|z, no\ expansion) = B(D_N, a|z, expansion)$$

Let $\xi \equiv D_N - D$

Equating the expressions for the expected payoffs to the borrower for the expansion and no expansion cases, we get

$$(\beta - \xi)Post(G; a|z) - \xi Post(M; a|z) = 0$$

which yields

$$\xi(\mathrm{a}, \mathrm{z}) = \frac{\beta}{1 + \frac{Post(M; a|z)}{Post(G; a|z)}} > 0$$

A positive $\xi(\mathrm{a}, \mathrm{z})$ implies that the renegotiated face value of debt is higher than the originally contracted face value of debt. Further, from MLRP, $\xi(\mathrm{a}, \mathrm{z})$ is increasing in the accounting signal. The renegotiated face value of debt is increasing in the accounting signal because we have assumed that the lender has all the bargaining power when the debt covenant is violated. So the lender sets the renegotiated face value of debt such that he extracts all the expected gains from expansion, which in turn is increasing in z.

This increase in the face value of debt will be anticipated and priced in the initial debt contract to satisfy the lender's participation constraint. So when Z*(a) < Y, the initially contracted value of D will reflect the anticipated increase in the face value of debt upon renegotiation and will be lower than it would be without renegotiation. This decrease in the initially contracted face value of debt will decrease the borrower's incentive to engage in asset substitution.

We are now in a position to characterize the borrower's Date 0 expected payoff that anticipates the possibility of renegotiation. The renegotiated value of debt is such that the borrower's interim expected payoff, conditional on accounting signal z, is $B(D, a|z, no\ expansion)$ whenever the debt covenant is violated, which in turn implies that the borrower's expected Date 0 payoff $B(a, D, Y)$ is

$$B(a,D,Y) = \int_{\underline{Z}}^{Y} B(D,a|z, no\ expansion)h(z,a)dz \ + \int_{Y}^{\overline{Z}} B(D,a|z, expansion)\, h(z,a)dz$$

where $h(z,a)$ is the probability density function of the accounting signal. In the above characterization of the borrower's Date 0 expected payoff, the increase in face value of debt when renegotiation occurs in the region $z \in [Z^*(a), Y]$ has been taken into account in the first term. Substituting for $B(D,a|z, no\ expansion)$ and for $B(D,a|z, expansion)$ in the above equation, we get

$$B(a,D,Y) = \int_{\underline{Z}}^{Y} \{(X_H - D)Post(G;a|z) + (X_M - D)Post(M;a|z)\}h(z,a)dz$$

$$+ \int_{Y}^{\overline{Z}} \{(X_H + \beta - D)Post(G;a|z) + (X_M - D)Post(M;a|z)\}h(z,a)dz$$

Substituting for the posterior probabilities in the above equation yields

$$B(a,D,Y) = \int_{\underline{Z}}^{Y} \{(X_H - D)Prob(G,z;a) + (X_M - D)Prob(M,z;a)\}dz$$

$$+ \int_{Y}^{\overline{Z}} \{(X_H + \beta - D)Prob(G,z;a) + (X_M - D)Prob(M,z;a)\}dz$$

where $Prob(S,z;a)$ is the joint probability of state S and accounting signal z when asset substitution is $a$.

Substituting for the joint probabilities reduces $B(s,D,Y)$ to the same characterization as the borrower's Date 0 expected payoff in the full commitment setting i.e.

$$B(a,D,Y) = (X_H + \beta - D)Prob(G;a)\{1 - F(Y|G)\} + (X_M - D)Prob(M;a)\{1 - F(Y|M)\}$$

$$+ (X_H - D)Prob(G;a)F(Y|G) + (X_M - D)Prob(M;a)F(Y|M)$$

which simplifies to

$$B(a,D,Y) = (X_M - D)Prob(M;a) + (X_H - D)Prob(G;a) + \beta Prob(G;a)\{1 - F(Y|G)\}$$

The borrower's expected payoff takes the same form above even when the debt covenant is set below Z*(a). Therefore what follows applies also to the case when the debt covenant is set below Z*(a). We now turn to examine the lender's Date 0 expected payoff. The Date 0 value of the project is the sum of the Date 0 expected payoffs of the borrower and the lender. The value of the project incorporates the anticipated efficient interim expansion decision threshold

$Z^*(a)$ and the borrower's asset substitution choice $a(D, Y)$ that maximizes the borrower's Date 0 expected payoff. So the lender's Date 0 expected payoff, given initial debt contract $\{D, Y\}$ is

$$L(a(D,Y), D, Y) = V\big(a(D,Y), Z^*(a(D,Y))\big) - B(a(D,Y), D, Y)$$

With costless renegotiation, the expansion decision is always efficient. Therefore economic surplus is maximized when the initial face value of debt D and the debt covenant Y are chosen to meet the lender's participation constraint such that asset substitution is minimized.

Differentiate the lender's Date 0 expected payoff above with respect to $Y$ and invoke the envelope theorem to get

$$V_a \frac{\partial a}{\partial Y} - B_Y$$

From Proposition 1, the asset substitution choice $a(D, Y)$ of the borrower is decreasing in the strictness of the debt covenant. Further, $V_a < 0$ and $B_Y < 0$. So the lender's Date 0 expected payoff increases as the debt covenant Y is made stricter. In contrast to the full commitment case, a stricter debt covenant does not detract from the efficiency of the expansion decision because the covenant is renegotiated at the interim date. Yet a stricter debt covenant gives more control rights over the expansion decision to the lender. This increase in control rights leads to higher expected gains from renegotiation that accrues to the lender. At Date 0, these anticipated gains will be priced to determine the initial face value of debt that meets the lender's participation constraint. The anticipated gains allow the initial face value of debt that meets the lender's participation constraint to be lower than in the full commitment case. The asset substitution induced by the combination of a stricter debt covenant and lower initial face is therefore lower than in the full commitment case. These anticipate gains will arise even when the borrower has all the bargaining power at the interim renegotiation stage.

***Proposition 3***

*When the borrower can engage in unverifiable asset substitution and renegotiation is costless, the*

- *(i)* *expansion decision is efficient.*
- *(ii)* *optimal debt covenant is stricter than under first best.*
- *(iii)* *optimal initially contracted face value of debt is lower than under full commitment*
- *(iv)* *asset substitution induced by the optimal debt contract is lower than under full commitment.*

## 5 Role of Conservative Accounting

We now turn to examine how the efficiency of the debt contract is affected by the degree of conservatism of the accounting system. Recall that $f(z|S,\delta)$ and $F(z|S,\delta)$ denote the conditional density and cumulative distribution functions respectively, of the accounting signal z, given state S, where δ is a parameter that represents how liberal the accounting system is. We now impose specify three conditions on the measurement and reporting process that ensure that as δ decreases, the distribution and information content of the accounting report changes in a way that is consistent with the accounting system becoming more conservative.

**Condition C1**: $F_{\delta}\,(z;\delta|S) < 0$ i.e. $F(z;\delta|S)$ is decreasing in δ for all $z, S, \delta$.

**Condition C2**: For any given $z$ and any $S > \hat{S}$ $\frac{f(z;\delta|S)}{f(z;\delta|\hat{S})}$ is decreasing in δ.

**Condition C3**: $F_{\delta}\,(z;\delta|S) = F_{\delta}\,(z;\delta|\hat{S})$ for all $z, S, \hat{S}, \delta$.

Condition C1 is the same as condition A2 of Gigler et al (2009) and ensures that as the degree of conservatism increases, the distribution of accounting signals shifts to the left, conditional on each state of the world. It is consistent with the notion that conservatism imparts a downward bias to accounting reports. Condition C2, which is the same as condition A3 of Gigler et al (2009), ensures that as the degree of conservatism increases, the assessed distribution of cash flows given a fixed accounting signal, becomes more favorable. Condition C3 is the same as condition A4 of Gigler et al (2009) and ensures that δ is an index of unconditional conservatism in the sense that the downward shift in the distribution of accounting signals from decreases in δ that is ensured by Condition C1, is independent of the events being measured and therefore independent of the future cash flow of the firm.

Having specified how conservatism affects the distribution and informational properties of the accounting system, we turn to the problem of debt contracting and analyze how changes in the degree of accounting conservatism affect the efficiency of optimal debt contracts. We first examine the first best case, where asset substitution is verifiable and hence is entirely precluded, so that the only efficiency that matters is that of the interim expansion decision. When asset substitution is precluded, the Date 0 value of the project $V(Y)$ is the sum of the expected payoff of the borrower and the lender when the decision rule is that the project is expanded if and only if the accounting signal exceeds the threshold $Y$. The value of the project depends on the threshold Y and on the degree of conservatism and is given by

$$V(Y,\delta) = X_H Prob(G) + X_M Prob(M) + X_L Prob(B) + \beta Prob(G)\{1 - F(Y;\delta|G)\} \\ - T Prob(B)[1 - F(Y;\delta|B)]$$

We want to examine how changes in the degree of conservatism impact the maximized value of the project. The optimal threshold $Z^*$ is chosen to maximize the value of the project and depends on the degree of conservatism. As the value of the project is the sum of the expected payoff of the borrower and the lender,

$$V(Z^*, \delta) = B(D, Z^*) + \ L(D, Z^*)$$

The envelope theorem allows us to ignore how the optimal threshold varies with the degree of conservatism so that

$$\frac{dV}{d\delta} = V_{\delta}(Z^*, \delta) = \ \beta Prob(G)\{1 - F_{\delta}(Z^*; \delta|G)\} - TProb(B)[1 - F_{\delta}(Z^*; \delta|B)]$$

Using Conditions C1 and C3, the sign of the left hand side of the above equation is the same as the sign of

$$\beta Prob(G) - TProb(B)$$

which implies that unconditional conservative accounting increases the value of the project by promoting debt contract efficiency if and only if the above expression is negative i.e.

$$\beta Prob(G) - TProb(B) < 0$$

The above inequality is met when the ex-ante belief at the time the project is initiated is that the expected gain from expansion is negative, so that it is optimal to expand the project at the interim date only if the accounting report is favorable enough that beliefs are sufficiently upgraded. This implies that the likelihood ratio at the optimal threshold $Z^*$ must be greater than 1. Condition C2 implies that any increase in conservatism results in a gain of information content at signal values where the likelihood ratio is greater than 1. Therefore conservative accounting dominates liberal accounting when the optimal threshold of the accounting signal is such that the likelihood ratio at the threshold is greater than 1. If the ex-ante belief is reversed i.e. expected returns from expansion are positive at the time of project inception, then liberal accounting is optimal.

This result that in even in the first best case, conservative accounting may be optimal is in contrast to the result in Gigler et al (2009) that liberal accounting enhances the efficiency of debt contracts in a world of symmetric information and full verifiability. The two results can be reconciled by focusing on ex-ante beliefs and noting that in Gigler et al (2009), the interim date decision is a liquidation decision, whereas we study an expansion decision. In the context of a liquidation decision, it is reasonable to assume that ex-ante beliefs are such that liquidation is not optimal so that a deterioration in initial beliefs is required for liquidation to be optimal, which in turn causes liberal accounting to be optimal. However in the context of an expansion decision, it is not necessary that ex-ante beliefs be such that expansion is optimal. When ex-ante beliefs are such that expansion is optimal, liberal accounting is optimal whereas when the ex-ante beliefs are such that expansion is not optimal, it is conservative accounting that is optimal.

When asset substitution is unverifiable, if costless renegotiation is possible, we have established in Section 4 that the expansion decision is efficient. The only difference between the first best case and the setting analyzed in Section 4 is that asset substitution is not entirely precluded under costless renegotiation when asset substitution is unverifiable so that the inequality that needs to be satisfied for conservative accounting to be optimal is

$$\beta Prob(G; a) - TProb(B; a) < 0$$

where in contrast to the first best case $a > 0$. As value destroying asset substitution increases the probability of the Bad state more than that of the Good state,

$$\beta Prob(G; a) - TProb(B; a) < \beta Prob(G) - TProb(B)$$

which implies that it is possible for conservative accounting to be optimal when asset substitution is unverifiable and renegotiation is costless, even if it is not optimal in the first best scenario.

We now turn to examine the impact of conservative accounting on debt contract efficiency when asset substitution is unverifiable and the parties cannot renegotiate the initial contract. In this scenario, as shown in Section 3, the optimal debt covenant trades off inefficient expansion against value destroying asset substitution. So changes in the degree of conservatism can impact the expansion decision as well as asset substitution. Recall that the objective function in this scenario was

$$\underset{a,D,Y,\lambda,q}{Max} \quad B(a, D, Y) \;+\; \lambda\, [L(a, D, Y) - K(1 + R)] + qB_a(a, D, Y)$$

where λ and q are Lagrange multipliers. We want to examine how the maximized value of this objective function changes with the degree of conservatism. By the general envelope theorem, the derivative of the maximized value of the objective function with respect to $\delta$ is independent of how the choice variables in the maximization problem vary with $\delta$ so that conservative accounting is optimal if and only if

$$B_\delta(a,D,Y) + \lambda L_\delta(a,D,Y) + qB_{a\delta}(a,D,Y) < 0$$

We will examine the sign of the three terms on the left hand side of the above inequality. The derivative $B_\delta(a,D,Y)$ in the first term is the marginal change in the borrower's expected payoff as accounting is made more liberal. The borrower's expected payoff increases as accounting is made more liberal because *ceteris paribus* Condition C1 implies that the debt covenant is more likely to be met when accounting is more liberal. The derivative $L_\delta(a,D,Y)$ in the second term is the marginal change in the lender's expected payoff as accounting is made more liberal. The lender's expected payoff decreases as accounting is made more liberal because *ceteris paribus* Condition C1 implies that the debt covenant is more likely to be met when accounting is more liberal.

Turning to the cross-partial derivative in the third term of the above inequality, note that as accounting is made more liberal, the debt covenant is more likely to be met and therefore the project is more likely to be expanded. As the marginal change in the borrower's expected payoff from asset substitution increases with the likelihood of project expansion, the cross-partial derivative is positive. Substitution of the expressions for the derivatives in the above inequality reduces it to

$$-\beta Prob(G;a)F_\delta(Y|G) + \lambda T Prob(B;a)F_\delta(Y|B) - q\beta(\alpha - ac)F_\delta(Y|G) < 0$$

By Conditions C1 and C3, under unconditional conservatism, the above inequality is satisfied whenever

$$\beta Prob(G;a) - \lambda T Prob(B;a) + q\beta(\alpha - ac) < 0$$

The level of asset substitution under full commitment is higher than under costless renegotiation case, which implies that under full commitment $Prob(G;a)$ is lower and $Prob(B;a)$ is higher than under costless renegotiation. Therefore *ceteris paribus* the above inequality is more likely to be met under full commitment than under costless renegotiation. Further as $\alpha - ac > 0$ and as established in Section 3, $\lambda > 1$ and $q < 0$. Therefore, the above inequality is satisfied for a larger set of parameter values under full commitment than when renegotiation is costless. That $\lambda$ is greater than 1 captures the fact that the optimal debt covenant under full commitment is higher than the threshold for efficient expansion. A higher $\lambda$ implies that from a Date 0 perspective the project is more likely to be not expanded which creates a greater demand for conservative accounting. A negative $q$ captures the fact that if the borrower's incentive to engage in asset substitution is lowered, then efficiency is enhanced. And as accounting is made more conservative, the borrower's incentive to engage in asset substitution decreases. So we have established the following result.

***Proposition 4***

*When asset substitution is verifiable:*

(a) *conservative accounting enhances the efficiency of debt contracting whenever the ex-ante belief at the time the project is initiated is that the project is more likely to be not expanded.*
(b) *Unverifiable asset substitution increases the demand for conservative accounting.*
(c) *The demand for conservative accounting is higher under full commitment than under costless renegotiation.*

**To be Concluded.**

**Figure 1 : The Base Case of No Expansion and No Asset Substitution**

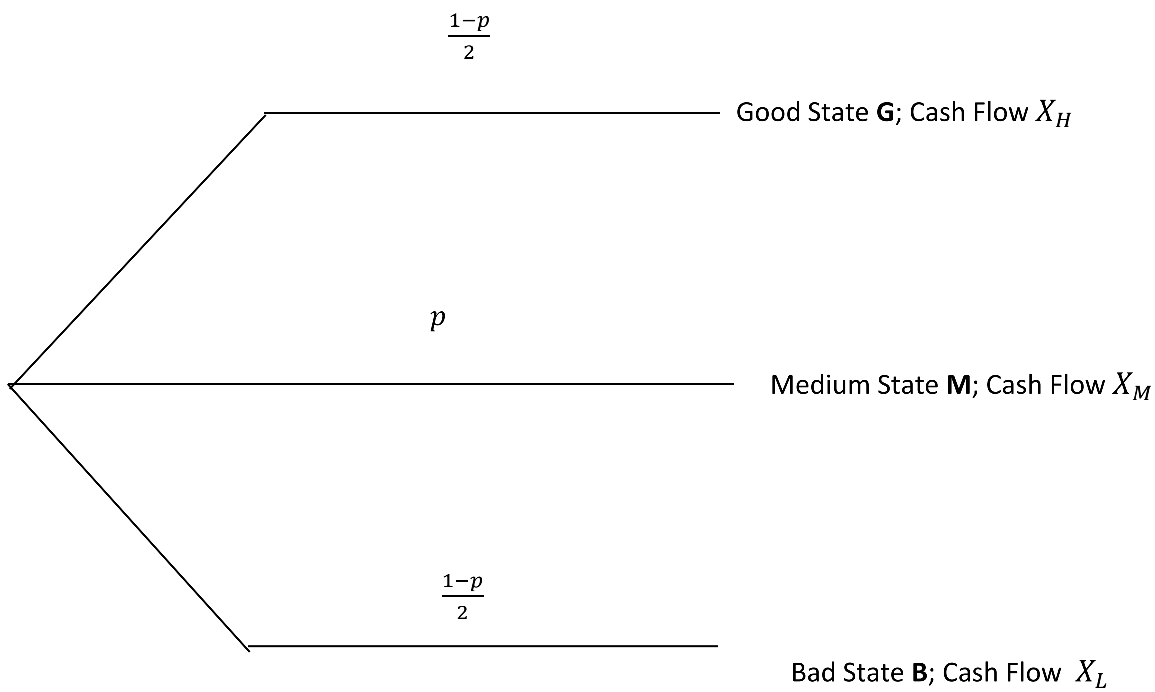
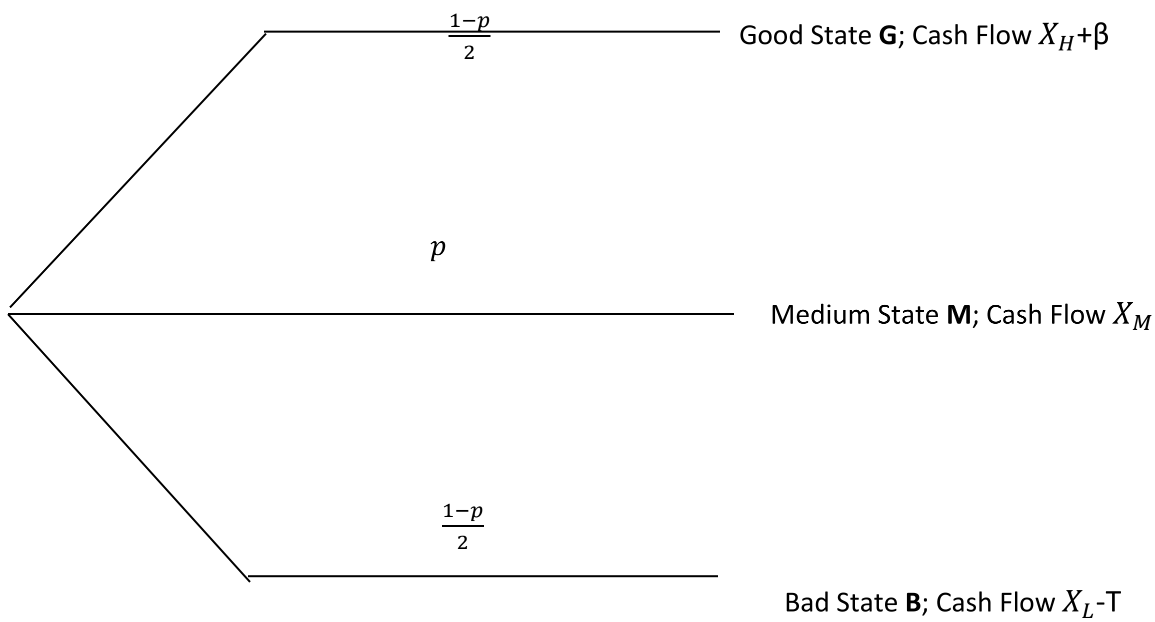


**Figure 2: The Case of Expansion with No Asset Substitution**



**<u>Figure 3: The Case of No Expansion with Asset Substitution</u>**

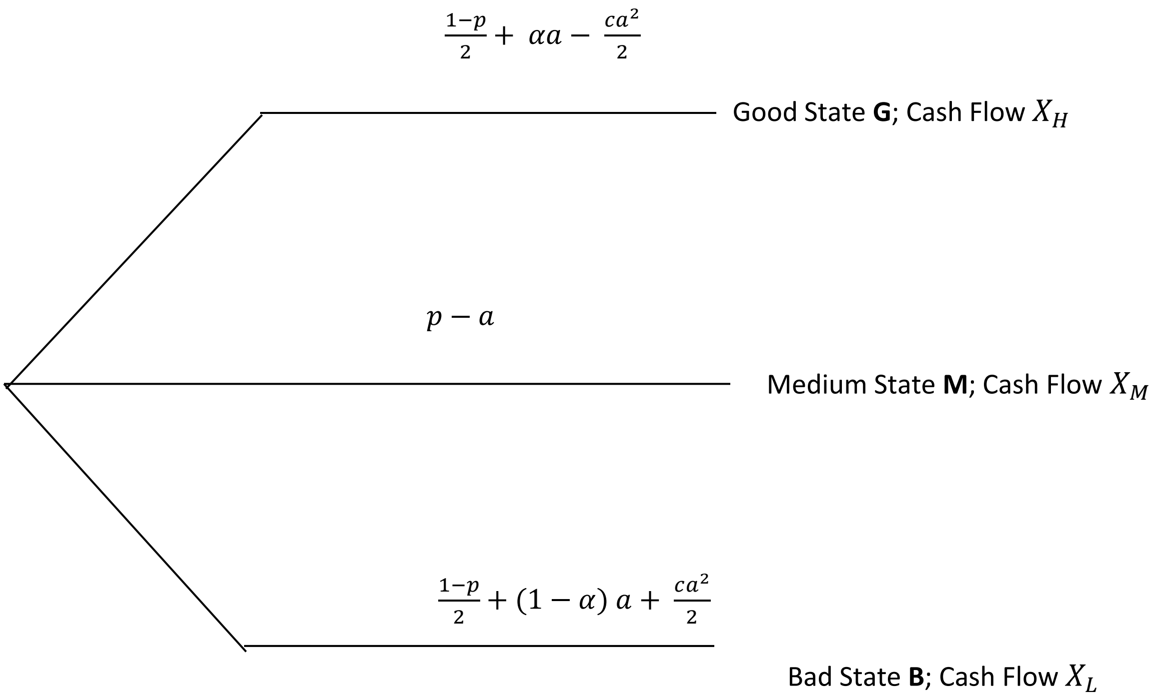
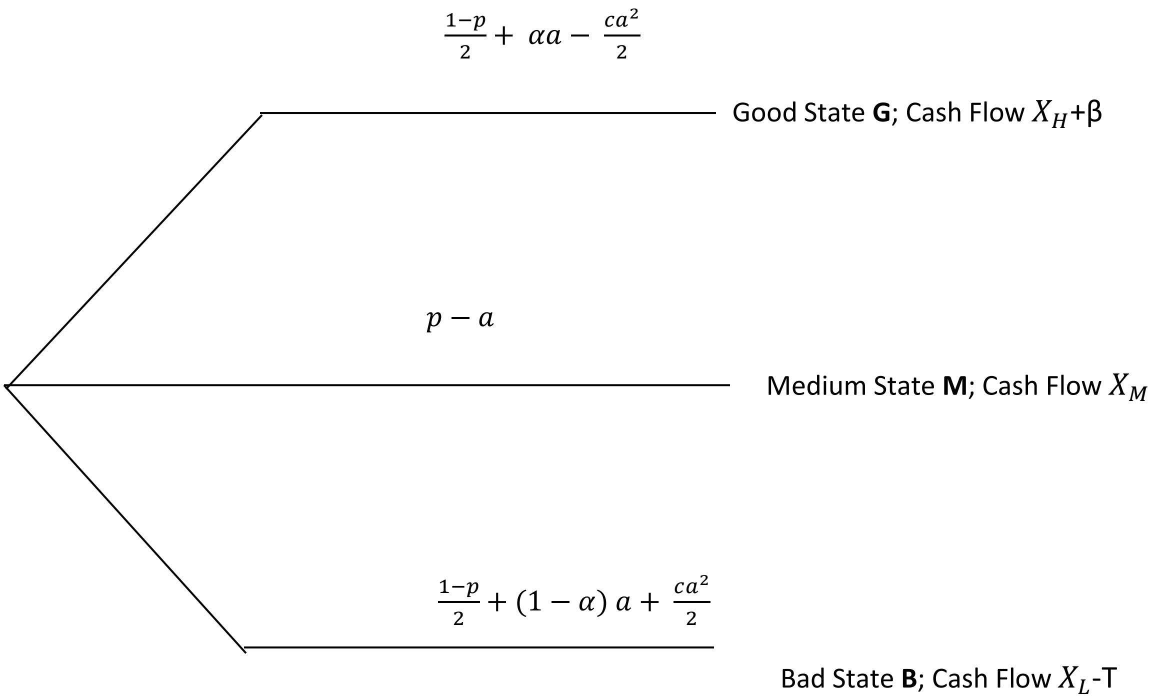


**Figure 4 : The Case of Expansion with Asset Substitution**




Given that $X_M$ is the mid-point of $[X_L,\ X_H]$ , asset substitution decreases the expected value of cash flows from the expanded project if

$$\alpha < \frac{X_H - X_L + 2T}{2(X_H - X_L + \beta + T)}$$

## References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# Appendix

**List of Some Useful Expressions**

$$B_D(a, D, Y) = -Prob(M; a) - Prob(G; a) = -\{p - a\} - \left\{\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}\right\} < 0$$

$$B_Y(a, D, Y) = -\beta Prob(G; a) f(Y|G) = -\beta \left\{\frac{1-p}{2} + \alpha a - \frac{ca^2}{2}\right\} f(Y|G) < 0$$

$$B_{aa}(a, D, Y) = -c\{X_H - D + \beta[1 - F(Y|G)]\} < 0$$

$$B_{aD}(a, D, Y) = -(\alpha - ac) + 1 = 1 - \alpha + ac > 0$$

$$B_{aY}(a, D, Y) = -\beta f(Y|G)(\alpha - ac) < 0$$

$$B_{aaY}(a, D, Y) = c\beta f(Y|G) > 0$$

The lender's IR, given asset substitution, and given the debt covenant Y is

$$\begin{aligned} L(a, D, Y) \equiv &\{Prob(G; a) + Prob(M; a)\}D \\ &+ Prob(B; a)F(Y|B)X_L + Prob(B; a)[1 - F(Y|B)][X_L - T] = K(1 + R) \end{aligned}$$

which simplifies to

$$\begin{aligned} L(a, D, Y) &\equiv \{Prob(G; a) + Prob(M; a)\}D + Prob(B; a)X_L - TProb(B; a)[1 - F(Y|B)] \\ &= \{1 - Prob(B; a)\}D + Prob(B; a)X_L - TProb(B; a)[1 - F(Y|B)] \\ &= K(1 + R) \end{aligned}$$

$$L_a(a, D, Y) = \frac{\partial Prob(B; a)}{\partial a}\{-D + X_L - T[1 - F(Y|B)]\} < 0$$

$$L_D(a, D, Y) = \{1 - Prob(B; a)\} > 0$$

Note that $L_D(a, D, Y) + U_D(a, D, Y) = 0$ which means that as D is changed, the increase in expected payoff to one party is exactly offset by the decrease in expected payoff to the other party.

$$L_Y(a, D, Y) = TProb(B; a)\, f(Y|B) > 0$$

$$L_{aY}(a, D, Y) = Tf(Y|B)\frac{\partial Prob(B; a)}{\partial a} > 0$$

**Proof of Proposition 2**

Having characterized the Lagrange multipliers, we now proceed to characterize the second best debt covenant Y*.

Use $B_D(a, D, Y) = -L_D(a, D, Y)$, suppress arguments and substitute the reduced form of FOC (3) and the expression for $q$ above into FOC (1), to get

$$\left\{\frac{B_{aa}}{L_a}L_Y - B_{aY}\right\}\frac{B_D}{B_Y} = -B_D\frac{B_{aa}}{L_a} - B_{aD}$$

which can be simplified to

$$\frac{-L_Y}{B_Y} = \frac{L_a}{B_{aa}}\left\{\frac{B_{aa}}{L_a} + \frac{B_{aD}}{B_D} - \frac{B_{aY}}{U_Y}\right\} \qquad (\boldsymbol{FCS})$$

The left hand side of equation (FCS) above is

$$\frac{TProb(B;a)\,f(Y|B)}{\beta Prob(G;a)f(Y|G)}$$

while the right hand side of (FCS) can be rewritten as

$$1 + \frac{L_a}{B_{aa}}\frac{B_{aD}}{B_D} - \frac{L_a}{B_{aa}}\frac{B_{aY}}{B_Y}$$

A unique solution to (FCS) exists at a point above Z*(a) if all of the following claims are established.

(i) The right hand side of (FCS) is less than 1 for all Y.
(ii) The right hand side of (FCS) is increasing in Y when Y > Z*(a).
(iii) The LHS of (FCS) is decreasing in Y, equal to 1 at Z*(a), and equal to zero at $\bar{Y}$.

The last claim above follows from the assumption made on the accounting signal and the characterization of the efficient expansion decision rule. The first claim follows from the fact that the second and third terms of the right hand side of (FCS) are negative and positive, respectively. So it suffices to prove the second claim above.

Note that both $\frac{B_{aD}}{B_D}$ and $\frac{B_{aY}}{B_Y}$ are independent of Y and that

$$\frac{B_{aD}}{B_D} - \frac{B_{aY}}{B_Y} < 0$$

so that it suffices to show that the sign of the derivative of $\frac{L_a}{B_{aa}}$ is negative for Y > Z*(a). The sign of the derivative of $\frac{L_a}{B_{aa}}$ is given by the sign of

$$B_{aa}L_{aY} - B_{aaY}L_a$$

Using the expressions derived earlier for the derivatives above, the above expression reduces to

$$-c\{X_H - D + \beta[1 - F(Y|G)]\}\, Tf(Y|B)\frac{\partial Prob(B;a)}{\partial a} - c\beta f(Y|G)\frac{\partial Prob(B;a)}{\partial a}\{-D + X_L - T[1 - F(Y|B)]\}$$

$$= -c\frac{\partial Prob(B;a)}{\partial a}\{\, Tf(Y|B)(X_H - D + \beta[1 - F(Y|G)]) - \beta f(Y|G)(X_L - D - T[1 - F(Y|B)])\, \}$$

As $\frac{\partial Prob(B;a)}{\partial a}$ is positive, it suffices to show that for Y > Z*(a)

$$Tf(Y|B)(X_H - D + \beta[1 - F(Y|G)]) > \beta f(Y|G)(X_L - D - T[1 - F(Y|B)])$$

To verify that the above inequality holds, we note the following:

MLRP implies MHR which implies that $f(Y|B)[1 - F(Y|G)] > f(Y|G)[1 - F(Y|B)]$

$X_H - D > X_L - D$ and

$\frac{Tf(Y|B)}{\beta f(Y|G)} > 1\ when$ Y > Z*(a)

which completes the proof of Proposition 2.

**Proof of Proposition 3**

The lender's Date 0 expected payoff, given initial debt contract $\{D, Y\}$ is

$$L(a(D,Y), D, Y) = V\big(a(D,Y), Z^*(a(D,Y))\big) - B(a(D,Y), D, Y)$$

We have already established that the lender's Date 0 expected payoff is increasing in the debt covenant Y and that renegotiation allows for an efficient expansion decision.

Evaluate the lender's Date 0 expected payoff at $D = D_E$ and $Y = Y_{FC}$ and consider the following two mutually exclusive and exhaustive cases.

Case 1: $L(a(D, Y), D, Y)|_{D=D_E, Y=Y_{FC}} < K(1 + R)$

Case 2: $L(a(D, Y), D, Y)|_{D=D_E, Y=Y_{FC}} > K(1 + R)$

As the expected gains from renegotiation are positive, the lender's Date 0 expected payoff evaluated at $D = D_{FC}$ and $Y = Y_{FC}$ is greater than $K(1 + R)$. Therefore, in Case 1, continuity ensures that there exists a $D < D_{FC}$ that meets the lender's participation constraint with $Y = Y_{FC}$. At this debt contract, the induced asset substitution is lower than under full commitment.

In Case 2, decrease D below $D_E$ to decrease the lender's expected payoff. Note that when D is below $D_E$, the borrower has no incentive to engage in asset substitution and therefore the lender's expected payoff is decreasing in D. If the lender's expected payoff exceeds $K(1 + R)$ even at $D = 0$ and $Y = Y_{FC}$ then note that Assumption 5 ensures that at $D = 0$ and $Y = Z^*$ the lender's expected payoff is less than $K(1 + R)$. So with at $D = 0$ there exists $Y > Z^*$ such that the participation constraint is satisfied as an equality. At this debt contract, the induced asset substitution is zero.