# Asset Prices in General Equilibrium with Transactions Costs and Recursive Utility\*

Adrian Buss§

Raman Uppal¶

Grigory Vilkov§

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#### Abstract

In this paper, we study the effect of proportional transactions costs on asset prices and liquidity premia in a general equilibrium economy with multiple agents who are heterogeneous. The agents in our model have Epstein-Zin-Weil utility functions and can be heterogeneous with respect to endowments and all three characteristics of their utility functions - time preference, risk aversion, and elasticity of intertemporal substitution. The securities traded in the financial market include a one-period bond and multiple risky stocks. We show how the problem of identifying the equilibrium can be characterized in a recursive fashion even in the presence of transactions costs, which make markets incomplete. We find that transactions costs on stocks or the bond lead investors to reduce the magnitude of their positions in the two financial assets. The holding of each stock is very sensitive to its own transactions cost, but relatively insensitive to the transaction cost for the other stock. Transactions costs also reduce the frequency of trading of the stock; however, the effect on the frequency of trading the bond is much smaller. Our main finding is that even in the presence of non-tradable labor income, the effect of transactions costs on the liquidity premium and expected returns is significantly smaller in general equilibrium than in partial equilibrium: for a proportional transactions cost of 2\%, the difference in the expected return on a stock that incurs this cost and one that does not is at most 0.20% in general equilibrium.

**Keywords:** General equilibrium, incomplete markets, transaction costs, heterogeneous agents

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<sup>§</sup>Goethe University Frankfurt, Finance Department, Grüneburgplatz 1 / Uni-Pf H 25, D-60323 Frankfurt am Main, Germany; Email: buss@finance.uni-frankfurt.de and vilkov@vilkov.net.

<sup>¶</sup>CEPR and Edhec Business School, 10 Fleet Place, Ludgate, London, United Kingdom EC4M 7RB; Email: raman.uppal@edhec.edu.

## 1 Introduction

In a recent article, Lynch and Tan (2011) find that if asset returns are allowed to be predictable and agents have wealth shocks calibrated to labor income, then transactions costs lead to liquidity premia that are of the same order of magnitude as transactions costs. However, their analysis is carried out in a partial equilibrium setting, and the conclusion (p. 36) of their article states:

One important limitation of our analysis is that it is a partial equilibrium analysis. Therefore, it says nothing about how transaction costs affect equilibrium prices by limiting the ability of agents to share risk. More work is needed to understand how transaction costs affect prices and returns in a general equilibrium setting.

Our objective is to fill this gap by studying the effect of proportional transactions costs on asset prices in a general equilibrium economy with multiple agents who are heterogeneous.

In the general equilibrium model we consider, agents have Epstein-Zin-Weil utility functions and can be heterogeneous with respect to endowments (nontradable labor income) and all three characteristics of their utility functions – time preference, risk aversion, and elasticity of intertemporal substitution. We consider a financial market in which the traded securities consist of a one-period bond and multiple risky stocks, and these securities, even in the absence of transactions costs, may not be sufficient to span the market – for instance, because agents have nontraded labor income. We show how the problem of identifying equilibrium in this incomplete-markets economy can be characterized in a recursive fashion even in the presence of costs for transacting in stocks and bonds. We then study the effect of transactions costs on the interest rate, the stock price, the expected return, risk premium, and liquidity premium on the stock, and the volatility of stock and bond returns in the general equilibrium model, and compare it to the partial equilibrium version of the model.

We find that transactions costs on either stocks or the bond lead investors to reduce the magnitude of their positions in these financial assets. The holding of each stock is very sensitive to its own transactions cost, but relatively insensitive to the transaction cost for the other stock. And, as one increases the transactions costs on either stock, the magnitude of bond holding declines because of the decrease in the holding of the stock whose transaction cost

has increased. As one would expect, agents use the stock with the lower transactions cost to share risk and smooth consumption over time. Transactions costs also reduce the frequency of trading of the stock. However, the effect on the frequency of trading the bond is much smaller; for even relatively large transactions costs in the bond market, the investors continue to trade the bond.

Transactions costs make it less attractive to hold financial assets, and therefore, require an increase in expected returns (that is, a decrease in prices). Our key finding is that, while the increase in expected returns as a consequence of introducing transactions costs is large in a partial equilibrium setting in which there are shocks to labor income, the effect is much smaller in general equilibrium. For example, for two identical stocks where one can be traded without cost and the other incurs a 2% transaction cost, the difference in returns is of the same order of magnitude as the transactions cost in partial equilibrium, but only about 0.20% in general equilibrium. The reason why the result in our general-equilibrium model is different from that in the partial-equilibrium model of Lynch and Tan (2011) is that in our setting prices for the bond and stock are allowed to change, and thus, this absorbs some of the effect of the transactions costs so that the effect on stock returns is smaller. The second reason for the smaller effect is that in general equilibrium risk sharing between the heterogeneous agents reduces the impact of the transactions costs.

Our paper makes two major contributions. One, our paper contributes to the asset-pricing literature by extending the results of existing models examining the effects of transactions costs to a general equilibrium setting with heterogeneous investors: in particular, the model we study allows for an endogenous interest rate, recursive utility functions, and agents who are heterogeneous with respect to their endowments and/or preferences. And, as discussed above, the extension to a general equilibrium setting leads to economic insights that are very different from those in a partial equilibrium setting, such as the one considered in Lynch and Tan (2011), where there is a single agent with power utility function, prices are given exogenously, and the interest rate is constant.

Two, we demonstrate how to identify the equilibrium in an economy where there are heterogeneous agents who have recursive utility, even in the presence of transactions costs and incomplete financial markets. There are two problems that arise in identifying the equilibrium when markets are incomplete. The first is that one can no longer use a "central planner" to identify the equilibrium, which can be done conveniently, in two steps: first, allocate consumption optimally across agents, and then, determine the asset prices and portfolio policy for each investor that supports this allocation. The reason why one cannot use the central-planner's approach in markets that are incomplete is that the consumption allocation that one chooses must lie in the span of the traded assets. Thus, when markets are incomplete one must solve for the consumption and portfolio policies *simultaneously*. This makes it difficult to implement a recursive scheme, because the portfolio chosen at the current date depends on asset prices in the future, but these asset prices depend on consumption at the next date, which is already fixed when solving the model backwards.

The second problem is that in the presence of transactions costs, the problem of each investor becomes path dependent: whether or not to trade depends not just on exogenous state variables, but also on the current portfolio of the investor. We show how both these problems can be resolved, and hence, our solution method can be applied to study other problems in general equilibrium with incomplete markets. For example, it allows us to extend to incomplete markets the complete-markets analysis of Dumas, Uppal, and Wang (2000), who show how to characterize equilibrium in a setting with multiple heterogeneous agents with recursive utility, and the analysis of Bhamra and Uppal (2010), who identify asset prices in a model where agents are heterogeneous with respect to their time-additive utility functions and their beliefs.

Our work is related to five strands of the literature, and in the rest of this section we describe how our paper extends existing work. The first strand of the literature consists of partial equilibrium models that study the effects of transactions costs on asset prices. Amihud and Mendelson (1986) consider a single-period model in which agents are risk-neutral and must exit the market at which time they sell stock to newly arriving agents; they find that the excess return on a stock equals the product of the asset's turnover and the proportional transaction cost. Constantinides (1986) shows that because the agent chooses when to trade optimally, the effect of transactions costs on asset prices is much smaller than suggested by Amihud and Mendelson. Vayanos (1998) considers an overlapping-generations model with multiple stocks and also finds that transactions costs have a small impact on prices. One of the strengths of

his paper is that the model has a closed-form solution, which can be used to obtain several interesting insights. However, to obtain a closed-form solution, several restrictive assumptions need to be made: the interest rate is assumed to be exogenous and constant, which can have an important bearing on results, as shown by Loewenstein and Willard (2006) and as we also find in our model; agents are assumed to have exponential utility functions, which do not allow for the study of wealth effects; dividends follow an Ornstein-Uhlenbeck, so they are normal, instead of being lognormal; transactions costs are proportional to the number of shares rather than the value of shares; the model is one of overlapping-generations, with risk aversion increasing with age; and, it is assumed (in his Section 5) that the shortsale constraint is binding.

Lo, Mamaysky, and Wang (2004) consider a setting with fixed transaction costs and high-frequency transaction needs; they find that the effect of transactions costs in such a setting is larger, and of the same order as the transaction costs. Just like in Vayanos (1998), they also assume a constant (exogenous) interest rate and exponential utility, but in contrast to Vayanos (1998), they consider fixed transactions costs for stocks and no transactions costs for bonds. The motivation for trading in the model is heterogeneous nontraded (labor) income, which in aggregate sums to zero; that is, there is no aggregate risk. Moreover, it is assumed that the risk in the nontraded asset is perfectly correlated with the stock, which implies that the non-traded income is marketed. These assumptions allow one to get a closed-form solution for the special case where agents can trade at only the first date or for the case where transactions costs are small. As in Lo, Mamaysky, and Wang (2004), Lynch and Tan (2011) also find that the effect of transactions costs is large: liquidity premia are of the same order of magnitude as transactions costs in their model. They obtain this result by considering a model where asset returns are specified exogenously to be predictable and agents have nontradable wealth shocks.<sup>1</sup>

A second strand of the literature consists of partial-equilibrium models that focus on the effect of transactions costs on portfolio policies.<sup>2</sup> In particular, these papers identify the "region"

<sup>&</sup>lt;sup>1</sup>There is also a large literature studying the effect of liquidity on asset prices, but in contrast to that literature we choose to focus on the effect of trading costs on asset prices. See, for example, Gârleanu (2009) and the references therein.

<sup>&</sup>lt;sup>2</sup>This includes the work in Davis and Norman (1990), Duffie and Sun (1990), Dumas and Luciano (1991), Gennotte and Jung (1994), Atkinson and Wilmott (1995), Morton and Pliska (1995), Korn (1998), Bertsimas and Lo (1998), Schroder (1998), Balduzzi and Lynch (1999), Lynch and Balduzzi (2000), Akian, Sulem, and Taksar (2001), Liu and Loewenstein (2002), Liu (2004), Muthuraman and Kumar (2006), and Garleânu and Pedersen (2009). There is also the literature that uses partial-equilibrium models to study how life-cycle considerations influence portfolio selection; see, for example, Campbell, Cocco, Gomes, Maenhout, and Viceira (2001), Gomes

of no-trade" where, because of transactions costs, an investor finds it optimal not to rebalance her portfolio even though asset prices have changed. We, too, identify the region of no trade, but in contrast to these papers, our agents have recursive utility, asset prices are endogenous, and we allow for transactions costs on not just risky assets but also the bond.

The third strand of the literature to which our analysis contributes consists of general equilibrium models with heterogeneous investors but complete financial markets. This includes models with time-additive preferences, such as Dumas (1989), Dumas, Kurshev, and Uppal (2009), and Bhamra and Uppal (2010), and models where agents have recursive utility, as in Dumas, Uppal, and Wang (2000) and Dumas and Uppal (2001). We extend these models to the setting where markets are incomplete.

The fourth strand of the literature consists of general-equilibrium models with incomplete markets and transactions costs. One set of models in this strand introduces transactions costs and time-additive utility but no idiosyncratic labor income; see, for example, Vayanos and Vila (1999), who study the effects of transactions costs in an overlapping generations model with two assets, both of which are risk-free, but where one asset has transactions costs but the other does not. A second set of models in this strand, such as Heaton and Lucas (1996), allows for both idiosyncratic labor income and transactions costs, but with time-additive utility functions. In their model, heterogeneity across agents arises because of idiosyncratic labor income shocks and there is a quadratic transaction cost for trading the stock.<sup>3</sup> They find that the model can produce a sizable equity premium only if transactions costs are large or the assumed quantity of tradable assets is limited.

A fifth strand of the literature to which our paper is related is the work studying general-equilibrium models with incomplete markets but without transactions costs. One set of models in this strand studies investors with time-additive utility who are constrained or prohibited from holding some of the financial assets; see, for example, Basak and Cuoco (1998), Garleânu and Pedersen (2011), and especially Dumas and Lyasoff (2010), who propose an elegant solution method that is recursive; we will use many of the insights in this paper for solving our model. A second set of models considers a setting where the source of market incomplete-

and Michaelides (2003), Gomes and Michaelides (2005), Cocco, Gomes, and Maenhout (2005). In our work, we do not focus on life-cycle issues.

<sup>&</sup>lt;sup>3</sup>Heaton and Lucas (1996, Equation (19)) also consider a specification where the transaction cost function is quadratic for small transactions and linear for larger transactions.

ness is idiosyncratic labor income. For example, Lucas (1994) and Telmer (1993) examine asset prices in a model with agents who have time-additive utility functions and transitory idiosyncratic income shocks, Constantinides and Duffie (1996) look at the case of permanent idiosyncratic shocks. Gomes and Michaelides (2005) extends these models, in which agents have time-additive utility functions, to allow for recursive utility functions.

The rest of the paper is organized as follows. In Section 2, we describe the general model. In Section 3, we characterize the equilibrium and explain how it can be described by a system of path-independent backward-only (recursive) equations instead of a system of backward-forward equations. We analyze the effect of transactions costs on asset prices in Section 4, and conclude in Section 5.

## 2 The General Model

In this section, we describe the features of the model we study. In our model, there is a single consumption good. Time is assumed to be discrete. We denote time by t, with the first date being t=0 and the terminal date being t=T. In our model we will allow for K=2 agents, who are indexed by k and who have recursive utility functions. We assume that there are multiple sources of uncertainty, with the number of sources of uncertainty denoted by M. There are N+1 risky assets that are indexed by  $n=\{0,1,\ldots,N\}$ , where the first asset, n=0, is assumed to be a one-period bond; the remaining N assets are assumed to be stocks. We allow for the possibility that the number of risky assets traded in financial markets is strictly less than the number of sources of uncertainty, that is, N < M. The main feature of our model is that there is a proportional transactions cost for trading financial assets. We allow for transactions costs on both the bond and the N stocks, with the possibility that these transactions costs are different for different assets.<sup>4</sup> We are interested in examining the effect of the transactions costs on the trading of financial assets by the two agents, and the effect of this on asset prices. In the rest of this section, we give the details of the model.

<sup>&</sup>lt;sup>4</sup>The transactions costs could differ also across agents.

## 2.1 Uncertainty

Time is assumed to be discrete, with  $t = \{0, 1, ..., T\}$ . Uncertainty is represented by a  $\sigma$ algebra  $\mathcal{F}$  on the set of states  $\Omega$ . The filtration  $\mathbb{F}$  denotes the collection of  $\sigma$ -algebras  $\mathcal{F}_t$  such
that  $\mathcal{F}_t \in \mathcal{F}_s, \forall s > t$ , with the standard assumptions that  $\mathcal{F}_0 = \{\emptyset, \Omega\}$  and  $\mathcal{F}_T = \mathcal{F}$ . In
addition to time being discrete, we will also assume that the set of states is finite, and so the
filtration can be represented by a tree, with each node on the tree representing a particular
state of nature,  $\omega(t, s)$ . The probability measure on this space is represented by  $P : \mathcal{F} \to [0, 1]$ with the usual properties that  $P(\emptyset) = 0, P(\Omega) = 1$ , and for a set of disjoint events  $A_i \in \mathcal{F}$  we
have that  $P(\cup_i A_i) = \sum_i P(A_i)$ .

In our implementation of the model, we will assume that uncertainty is generated by a M-dimensional multinomial process, as described in He (1990), which is an extension of the binomial process that is often used for pricing options in a discrete-time and discrete-state framework (see Cox, Ross, and Rubinstein (1979)).<sup>5</sup>

## 2.2 Financial assets

We assume that there are N+1 assets that are traded in financial markets. The first asset is a one-period discount bond in zero net supply. The other N stocks are assumed to be in unit supply and have a dividend d(n,t), which is assumed to be  $\mathcal{F}_t$  measurable. Aggregate dividends at any node are then given by  $\sum_{n=1}^{N} d(n,t)$ . The ex-dividend price of each asset n as perceived by agent k at date t, S(n,k,t), is determined in equilibrium; note that in the presence of transactions costs, agents may choose not to trade a particular asset at a particular date, in which case agents will not agree on the price of this asset:  $S(n,1,t) \neq S(n,2,t)$ . The exdivided price on the terminal date for these assets is zero. The number of units of a particular asset n held by investor k at date t is denoted by  $\theta(n,k,t)$ .

In the special case where one assumes M = N, then each component of the multinomial process could be interpreted as the exogenous dividend from the  $n^{\text{th}}$  "tree".<sup>6</sup> In the general case where N < M, one could interpret N components of the multinomial process as the

<sup>&</sup>lt;sup>5</sup>Given that we allow for incomplete financial markets, the exact process used to generate uncertainty could be more general; for instance, we could allow for jumps.

<sup>&</sup>lt;sup>6</sup>Note that, because of the presence of transactions costs, financial markets are incomplete even for the case in which M = N.

exogenous dividends for the N trees, and the remaining M-N processes as nontradable labor income received by the agents.

## 2.3 Labor Income

The labor income of Agent k is denoted by Y(k,t). We adopt the same process for labor income as in Lynch and Tan (2011), who, following Carroll (1996, 1997), specify the logarithmic of labor income,  $\log Y(k,t) = y(k,t)$ , to have both permanent and temporary components:

$$y(k,t) = y^{P}(k,t) + \varepsilon(k,t) \tag{1}$$

$$y^{P}(k,t) = y^{P}(k,t-1) + \bar{g}(k) + b_{q}b(t) + u(k,t),$$
(2)

where  $\varepsilon_t$  and  $u_t$  are uncorrelated i.i.d. processes that have normal distributions,  $\bar{g}(k)$  is a constant representing the average growth rate for the labor income of Agent k,  $b_g$  is a constant, and b(t) is a predictive variable that generates predictability in labor income; for most of our analysis, we will set this term to zero. Just as in Lynch and Tan (2011), we also turn off the temporary component because, when calibrated to data, the temporary component has a negligible impact on liquidity premia. Thus, throughout our analysis, we consider the case in which  $y(k,t) = y^P(k,t)$  and  $\varepsilon(k,t) = 0$  for all t.

## 2.4 Preferences

We assume that the preferences of agents are of the Kreps and Porteus (1978) type. These utility functions nest the more standard time-separable utility functions, and in particular, the constant relative risk aversion power utility function, but have the well-known advantage that the risk aversion parameter, which drives the desire to smooth consumption across states of nature, is distinct from the elasticity of intertemporal substitution parameter, which drives the desire to smooth consumption over time. We adopt the Epstein and Zin (1989) and Weil (1990) specification of this utility function, in which lifetime utility V(k,t) is defined recursively:

$$V(k,t) = \left[ (1 - \beta_k) c(k,t)^{1 - \frac{1}{\psi_k}} + \beta_k E_t \left[ V(k,t+1)^{1 - \gamma_k} \right]^{\frac{1}{\phi_k}} \right]^{\frac{\phi_k}{1 - \gamma_k}}.$$
 (3)

In the above specification,  $E_t$  denotes the conditional expectation at t, c(k,t) > 0 is the consumption of agent k at date t in state  $\omega(t,s)$ ,  $^7\beta_k$  is the subjective rate of time preference,  $\gamma_k > 0$  is the coefficient of relative risk aversion,  $\psi_k > 0$  is the elasticity of intertemporal substitution, and  $\phi_k = \frac{1-\gamma_k}{1-1/\psi_k}$ . The above specification reduces to the case of constant relative risk aversion if  $\phi_k = 1$ , which occurs when  $\psi_k = 1/\gamma_k$ . The index k for the parameters  $\beta_k$ ,  $\gamma_k$ , and  $\psi_k$  indicates that the agents may differ along all three dimensions of their utility functions.

### 2.5 Transactions costs

We assume that agents pay a proportional cost for trading financial assets. The transaction cost at t depends on the value of assets being traded.<sup>8</sup> We denote this transaction cost by  $\tau(\theta(n,k,t),\theta(n,k,t-1))$ . We assume that this is a deadweight cost for making a transaction, and hence, this amount does not flow to any agent.

## 3 Characterization of Equilibrium

In this section, we first describe the optimization problem of each agent. We then impose market clearing to obtain a characterization of equilibrium, which is given in terms of a backward-forward system of equations. Finally, we show how this backward-forward system of equations can be transformed into a recursive (backward-only) system of equations.

## 3.1 The optimization problem of each agent

The objective of each investor k is to maximize lifetime utility given in (3) by choosing consumption, c(k,t) and the portfolio positions in each of the financial assets,  $\theta(n,k,t)$ ,  $n = \{0,1,\ldots,N\}$ . This optimization is subject to a dynamic budget constraint:

$$c(k,t) + \sum_{n=0}^{N} \theta(n,k,t)S(n,k,t) + \sum_{n=0}^{N} \tau(\theta(n,k,t),\theta(n,k,t-1)) \le$$
(4)

$$Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1) \Big( S(n,k,t) + d(n,t) \Big),$$

<sup>&</sup>lt;sup>7</sup>To simplify notation, we do not write explicitly the dependence on the state  $\omega(t,s)$ .

<sup>&</sup>lt;sup>8</sup>We could also consider the case where the transaction cost depends on the number of shares being traded, which is the specification studied in Vayanos (1998).

where the left-hand side of the above equation is the amount of wealth allocated to consumption and the purchase of assets at date t, and the right-hand is the sum of labor income and the value of shares purchased at date t-1 using the prices prevailing at date t and the "dividends" received from these assets; this sum can be interpreted as the investor's wealth at t. We assume that each agent is endowed with some shares of the risky assets at the start of time. Note that in the above formulation we have not imposed constraints on short selling or borrowing; if one wished, constraints on portfolio positions could be imposed on the trading strategy of the agent.

Thus, the Lagrangian for the utility function in (3) that is to be maximized subject to the (4) is:

$$\mathcal{L}(k,t) = \sup_{c(k,t),\theta(n,k,t)} \inf_{\lambda(k,t)} \left[ (1-\beta_k) c(k,t)^{1-\frac{1}{\psi_k}} + \beta_k E_t \left[ V(k,t+1)^{1-\gamma_k} \right]^{\frac{1}{\phi_k}} \right]^{\frac{\phi_k}{1-\gamma_k}}$$

$$+ \lambda(k,t) \left[ Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1) \left( S(n,k,t) + d(n,t) \right) \right]$$

$$-c(k,t) - \sum_{n=0}^{N} \theta(n,k,t) S(n,k,t) - \sum_{n=0}^{N} \tau \left( \theta(n,k,t), \theta(n,k,t-1) \right) \right],$$
(5)

where  $\lambda(k,t)$  is the Lagrange multiplier for the dynamic budget constraint.

Based on the above Lagrangian, the first-order conditions with respect to c(k,t),  $\theta(n,k,t)$ , and  $\lambda(k,t)$  are:

$$0 = \frac{\partial V(k,t)}{\partial c(k,t)} - \lambda(k,t), \tag{6}$$

$$0 = \lambda(k,t) \left[ S(n,k,t) + \frac{\partial \tau \Big( \theta(n,k,t), \theta(n,k,t-1) \Big)}{\partial \theta(n,k,t)} \right]$$

$$\left[ \frac{\partial V(k,t+1)}{\partial t} \Big( \frac{\partial \tau \Big( \theta(n,k,t+1), \theta(n,k,t) \Big)}{\partial t} \Big) \right]$$

$$\left[ \frac{\partial V(k,t+1)}{\partial t} \Big( \frac{\partial \tau \Big( \theta(n,k,t+1), \theta(n,k,t) \Big)}{\partial t} \Big) \right]$$

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$$\left[ \frac{\partial V(k,t+1)}{\partial t} \Big] \Big]$$

$$-E_t \left[ \frac{\partial V(k,t+1)}{\partial c(k,t+1)} \left( S(n,k,t+1) + d(n,t+1) - \frac{\partial \tau \Big( \theta(n,k,t+1), \theta(n,k,t) \Big)}{\partial \theta(n,k,t)} \right) \right],$$

$$0 = Y(k,t) + \sum_{n=0}^{N} \theta(n,k,t-1) \Big( S(n,k,t) + d(n,t) \Big)$$

$$-c(k,t) - \sum_{n=0}^{N} \theta(n,k,t)S(n,k,t) - \sum_{n=0}^{N} \tau \Big(\theta(n,k,t), \theta(n,k,t-1)\Big).$$
 (8)

Equation (6) is the first order condition for consumption and it equates the marginal utility of consumption to  $\lambda(k,t)$ , the shadow price for relaxing the budget constraint. Equation (7) equates the benefit from holding the stock versus selling the stock, net of transactions costs. Equation (8) is the budget constraint that the optimal consumption and portfolio policies must satisfy. One can substitute for  $\lambda(k,t)$  in Equation (7) using Equation (6).

$$0 = \frac{\partial V(k,t)}{\partial c(k,t)} \left[ S(n,k,t) + \frac{\partial \tau \Big( \theta(n,k,t), \theta(n,k,t-1) \Big)}{\partial \theta(n,k,t)} \right]$$

$$- E_t \left[ \frac{\partial V(k,t+1)}{\partial c(k,t+1)} \left( S(n,k,t+1) + d(n,t+1) - \frac{\partial \tau \Big( \theta(n,k,t+1), \theta(n,k,t) \Big)}{\partial \theta(n,k,t)} \right) \right].$$
(9)

After this substitution, we need to solve only for optimal consumption and the optimal portfolio at each node in order to identify the optimal policies for each agent. Thus, the solution of the problem of maximizing the lifetime utility in (3) subject to the budget constraint in (4) is characterized by the system of equations given in (8) and (9), which must hold for each date and state on the tree.

## 3.2 Market-Clearing Conditions

In the economy we are considering, there are financial markets for the risk-free asset and the N risky securities, and a commodity market for the consumption good. The market-clearing condition for the bond is that the aggregate demand for bonds must net to zero:

$$0 = \sum_{k=1}^{K} \theta(0, k, t). \tag{10}$$

The market-clearing condition for equity is that the aggregate demand for each stock must add up to the number of shares outstanding, which we normalize to one:

$$1 = \sum_{k=1}^{K} \theta(n, k, t), \quad \forall n = \{1, 2, \dots, N\}.$$
 (11)

Finally, aggregate dividends and labor income should be equal to aggregate consumption and transactions costs:<sup>9</sup>

$$0 = \left(\sum_{k=1}^{K} Y(k,t) + \sum_{n=0}^{N} d(n,t)\right) - \left(\sum_{k=1}^{K} c(k,t) + \sum_{k=1}^{K} \sum_{n=0}^{N} \tau \left(\theta(n,k,t), \theta(n,k,t-1)\right)\right).$$
(12)

## 3.3 Equilibrium in the Economy

Equilibrium in this economy is defined as a set of consumption policies, c(k,t), and portfolio policies,  $\theta(n,k,t)$ , along with the resulting price processes for the financial assets, S(n,k,t), such that the consumption policy of each agent maximizes her lifetime utility; that this consumption policy is financed by the optimal portfolio policy; financial markets clear, and the market for the consumption good clears.

## 3.4 Solving for the Equilibrium in Markets that are Incomplete

When financial markets are complete, one can divide the task of identifying the equilibrium into two distinct steps by exploiting the condition that in complete markets agents can achieve perfect risk sharing. Consequently, at each date and state, the marginal utility of consumption must be the same across all agents. This condition can be used to identify the optimal allocation of aggregate consumption across agents, which is often referred to as the solution to the "central planner's problem." Once we know the allocation of consumption across agents, we can use this to determine asset prices and also the portfolio policy of each investor that supports this allocation.

However, when financial markets are incomplete, one cannot divide the task of identifying the equilibrium into two steps. The reason why one cannot use the central-planner's approach is that the consumption allocation one chooses must lie in the span of traded assets. Thus, when markets are incomplete one must solve for the consumption and portfolio policies *simultaneously*.

In principle, one can identify the equilibrium by solving simultaneously the set of nonlinear first-order conditions for the two agents in (8) and (9), along with the market-clearing conditions in (10), (11), (12), for all the states across all dates. Dumas and Lyasoff (2010), who

<sup>&</sup>lt;sup>9</sup>Note that we use the consumption good as the numeraire, and therefore, its price is then equal to unity.

consider the problem of identifying the equilibrium in an economy with incomplete markets but no transactions costs, call this the "global method." The problem in implementing this approach is that the number of equations grows exponentially with the number of periods. In the presence of transactions costs, the number of equations grows even faster because the optimal policies of the agents are path dependent and so the decision tree is not recombining. For example, even for a problem with only ten dates, the number of equations to be solved is in the millions. An additional complication that arises in the presence of transactions costs is that whether a particular security is traded or not at a given node is determined endogenously; this makes it more difficult to large solve the system of nonlinear equations, because the system of equations to be solved changes depending on whether one is inside or outside the no-trade region.

In order to simplify the task of identifying the equilibrium in markets that are incomplete, Dumas and Lyasoff (2010) propose a "recursive method." In this method, one determines the equilibrium at each date in a recursive fashion; that is, at date t one solves for the equilibrium having already solved for the equilibrium at date t + 1. Thus, at each node in the tree one needs to solve only a small number of equations.

There are two problems in solving the system of equations in (8)–(12) recursively in a general-equilibrium setting. The first problem is that the current consumption and portfolio choices depend on the prices of assets, which from Equation (9) we see depend on future consumption. But, in a general-equilibrium setting, when the agent attempts to solve for the optimal consumption and portfolio policies at date t, asset prices need to adjust in order for markets to clear; but these prices cannot adjust because they depend on future consumption that has already been determined when one is solving the system of equations backward. Thus, to solve these equations, one would need to iterate backwards and forwards until the equations for all the nodes on the tree are satisfied. Dumas and Lyasoff (2010) address this problem by proposing a "time-shift" whereby at date t one solves for the optimal portfolio for date t and the optimal consumption for date t + 1, instead of the optimal consumption for t. Using this insight allows one to write the system of equations so that it is recursive.

In partial equilibrium, the recursive approach for determining the optimal portfolio policy of an investor requires one to introduce the agent's wealth as an additional endogenous state variable, in addition to other exogenous state variables that characterize the investment opportunity set. Dumas and Lyasoff (2010) show that in a general-equilibrium setting with incomplete markets, using the distribution of consumption across agents as the additional state variable, instead of individual wealth, has two advantages: one, it is a variable that is bounded, and two, the recursive problem becomes path-independent, so that the decision tree is recombining. In the presence of transactions costs, we show that it is not sufficient to include only consumption as an additional state variable; one needs to include also the portfolio composition of the investor as an endogenous state variable. The reason for this is that, because of transactions costs, the choice of the optimal portfolio at date t will depend also on the composition of the portfolio at date t-1.

The second problem in solving the system of equations in (8) and (9) recursively arises because of transactions costs. If agents choose to trade all assets, then they will agree on the prices of these assets. However, if agents find it optimal not to trade some of the assets, then agents will disagree on the prices of the assets that are not traded at that node. Consequently, the number of unknowns to be solved for, and the system of equations characterizing the solution, depends on whether or not agents choose to trade all assets or only some of the assets. We explain below how this problem can be addressed.

Note that the past portfolio holdings enter the system of equations only though the condition (9), as a first partial derivative of the transaction cost function  $\tau(\cdot)$  with respect to the current portfolio investment. Under the assumption that the transaction costs are a constant proportion  $\kappa_n$  of the value of an asset n being traded, we observe that there are only three possibilities for the form of this derivative. It is equal to zero when an agent decides not to trade; it is equal to  $\kappa_n \times S(n,k,t)$  when the agent decides to increase the position in the asset; or, it is equal to  $-\kappa_n \times S(n,k,t)$  when the agent sells the asset. Consequently, all the  $\theta(n,k,t-1)$  values for which the agent decides to buy an asset at time t result in the same solution  $\theta(n,k,t)$  for a given value of current consumption c(k,t). Similarly, all  $\theta(n,k,t-1)$  values for which the agent decides to sell an asset at t result in the same solution  $\theta(n,k,t)$  for a given value of current consumption. And all other values of past portfolio holdings will result in no trading at t. In other words, instead of solving the problem over the wide grid of portfolio holdings at t-1 that is difficult to determine, we can solve it first for the two trading decisions—sell or

buy—at time t; that is, over the two values of the derivative of the transaction cost function. The solution to this provides us with the bounds of the no-trade region, for which the portfolio investment from t-1 to t does not change. Knowing the bounds of the no-trade region, we solve the system of equations for future consumption c(k, t+1) only, explicitly restricting current portfolio holdings within the no-trade region to be equal to the past portfolio holdings  $\theta(n, k, t-1)$ . It is important to recognize that within the no-trade bounds the agents can disagree on the prices of the traded assets, and hence we lose the "kernel" condition that requires the agents to agree on asset prices.

In this way we are able to solve the system recursively in a backward fashion, knowing for each set of values of state variables if we are currently in the no-trade region with a smaller number of equations to be solved for consumption only, or the full set of equations to be solved for consumption and the investment portfolio. After we solve the dynamic program recursively up to time t = 0, we undertake the "forward step" to determine the equilibrium quantities for each state of nature that satisfy the initial conditions.

We show in the appendix that the principle of the dynamic programming applies to the problem with transaction costs, that is, the maximization goal of Agent k at time 0 is achieved if and only if the value function of the recursive problem is maximized at all times and states. In particular, we show that the first-order conditions of the dynamic program are equivalent to the first order conditions (8) and (9); moreover, one can also show that the value function is concave, and thus, satisfying the first-order conditions of the dynamic program is necessary and sufficient for optimality.

# 4 Implications of Transaction Costs for Asset Prices

To study the quantitative implications of our model, we use the following parameter values. We assume that the economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods; that is, t ranges from 0 to 5. For the dividend dynamics of the stocks we assume that the expected return  $\mu = 0.08$  and the volatility  $\sigma = 0.15$ . In case of two available risky assets we assume a dividend correlation of 0.25. For the stocks, we vary the transaction costs from 10 basis points to 200 basis points whereas the transaction costs on

the bond are either set to zero or to 10 basis points. We consider three setups in terms of the preferences of the agents: Setup 1 has two agents with power utility with relative risk aversion (RRA) coefficients  $\gamma_1 = 2$  and  $\gamma_2 = 5$ ; Setup 2 has two agents with Epstein-Zin preferences, each having the same RRA,  $\gamma_1 = \gamma_2 = 5$ , and elasticity of intertemporal substitution (EIS) equal to  $\psi = 0.50$ ; and, Setup 3 has two agents with Epstein-Zin preferences, each having the same RRA,  $\gamma_1 = \gamma_2 = 5$ , and EIS equal to  $\psi = 1.50$ .

## 4.1 One Risky Asset

We first consider the case where the asset menu available to the two agents consists of a bond and a single stock; one can imagine this to be the case where the investor has to allocate assets between a risk-free bond, and the market portfolio.

## 4.1.1 Portfolio Holdings and Trading Behavior

In Tables 1 and 2, and in Figures 1 and 2, we present the portfolio holdings of the first agent at the initial node of tree. As expected, the agents' holdings in the stock are a decreasing function of the stock's transaction costs. That is, higher transaction costs imply a less extreme position in the stock. Consequently, the first, less risk-averse, agent also borrows less from the second, more risk-averse, agent and so the bond position is also less extreme. For instance for Setup 3, when we increase the transaction cost from 10 basis points to 200 basis points, the stock holding decreases from 0.71 to 0.65, a decrease of about 9%, and similarly the bond holding changes from -0.90 to -0.66. Similarly, for a given level of transactions cost for the stock, the presence of transactions cost on the bond causes the agents to take less extreme positions in financial assets.

To understand the trading behavior of the two agents over the full horizon of the economy, we present in Tables 3 and 4, and in Figures 3 and 4 the number of nodes at which the agents trade. In total there are 15 possible node where the agents may trade. The agents always trade the bond – independent of the level of transaction costs on the bond and stock. However, for the stock we see a rather strong deviation from the optimal behavior in the absence of transaction costs where agents would always trade. That is, for transaction costs of 100 basis points on the stock the agents only trade at 5–6 nodes in the tree; that is, at about 1/3 of the available

nodes. This also implies that the agents will often not agree on the price of the stock but have different private valuations. In contrast to this, the agents will always agree on the price of the bond.

#### 4.1.2 Asset Prices

The prices of the two available assets are presented in Tables 5 and 6, and in Figures 5 and 6. Note from the results in the preceding section that the agents always trade the bond and the stock at the initial node. Therefore, at t = 0 the agents agree on the prices of both assets and so we report their common valuations.

We observe a similar pattern for the prices of both assets, across the different preference setups as well as for the various transaction costs combinations: the prices of the assets are decreasing in the level of transaction costs for both the bond and the stock transaction. Increasing transaction costs for the stock from 10 basis points to 200 basis points decreases the price of the bond and the stock by about 2% and 3%, respectively. These effects are a bit stronger in the presence of transaction costs on the bond.

These price effects are driven by three key determinants: First, the agents have to pay transaction costs which reduces their consumption levels and therefore alters the pricing kernels. Second, due to the presence of the transaction costs, agents portfolio holdings differ from the zero transaction costs holdings (the "Merton line") and accordingly the consumption levels differ and also the pricing kernels. Third, the fact that the agents hold less extreme positions, reduces the demand for the assets.

## 4.1.3 Return Characteristics

The changes in the assets' prices also have an impact on the return characteristics of the two assets, shown in Tables 7 to 14, and in Figures 7 to 14. Focusing first on the bond, we find that the one-period expected return on the bond increases in the level of transaction costs.<sup>10</sup> Given the bond price effects presented above, this is what we would expect.

<sup>&</sup>lt;sup>10</sup>As described in Section 4.1.1, the two agents trade the bond for all combinations of transaction costs on all nodes in the tree such that the two agents always agree on bond prices and accordingly on bond returns.

To study the magnitude of the effects on the bond return one has to carefully distinguish between the different preference setups. For example, for the CRRA case an increase of stock transaction costs from 10 to 200 basis points yields a sizable increase in bond returns from 12.3% to 18.2%. However, for the Epstein-Zin setup with IES of 1.5 the bond return only increase from 1.2% to 2.8%. That is, the use of CRRA preferences strongly overstates the effects of transaction costs on bond returns.

Recall, the bond available to the two agents is a long-lived bond, i.e., it only guarantees a unit payoff at time T and bond returns are therefore also volatile – on a low absolute level. The effects on transaction costs on bond return volatility are presented in Table 8 and in Figure 8. The CRRA and the Epstein-Zin setups yield different results. While the results under CRRA preferences indicate an increase of bond return volatility in the presence of transaction costs, we observe a decrease in volatility for Epstein-Zin preferences. However, the effects are relatively small.

Focusing on the stock, recall that the agents will often not trade the stock; therefore, they will not agree on the price of the stock and instead will have their private valuations. In Tables 9 and 10, and in Figures 9 and 10 we therefore present the equity premium from the views of the two agents separately. While we observe a clear pattern from the view of the first agent, i.e. an increase in the equity premium in the presence of transaction costs, the relation between transaction costs and the equity premium for the second agent is less clear. Specifically, the equity premium increases for low levels of transaction costs but decreases for higher level of transaction costs. Overall, the equity premium from the perspective of the second agent is always smaller or equal than the equity premium as perceived by the first agent. This can be explained by the fact that the second, more risk-averse agent who is more unwilling to hold the risky asset, will have a lower private valuation than the first, less risk-averse, agent.

Similarly, the two agents also have a different stock return volatilities in mind. Specifically, the effect of transaction costs on the stock return volatility is inversely for the two agents. For one agent the volatility is an increasing function of transaction costs whereas the second one perceives volatility as a decreasing function of transaction costs. Which agent has the lower or higher volatility perception depends on the preference setup. Moreover, similar to the effect

on bond returns, the impact of transaction costs on stock volatility are stronger in the CRRA setup.

In Tables 13 and 14, and in Figures 13 and 14 we also present the Sharpe Ratio, i.e., the ratio of the equity premium and the stock return volatility. While the Sharpe Ratio is an increasing function of transaction costs for the first agent, the Sharpe Ratio of the second agent increases slightly for low levels of transaction costs and the starts to decrease. Specifically, for the EZ setup with IES of 1.5, the first agent's Sharpe Ratio increases from 0.414 to 0.470 while the second agent's Sharpe Ratio decreases from 0.414 to 0.388 if we increase stock transaction costs from 10 to 200 basis points.

## 4.2 Two Risky Assets

We now study the effect of transaction costs in a setup with two risky assets. Specifically, we want to understand how the transaction costs of one stock affect the holdings, prices and returns of the other stock.

## 4.2.1 Portfolio Holdings and Trading Behavior

Focusing first on the portfolio holdings of the first, less risk-averse, agent, presented in Tables 15 to 17, we observe that the investment into each stock decreases strongly with the transaction costs on the stock. For example, if we increase the transaction costs on the first stock from 75 basis points to 200, while keeping the transaction costs of the second stock fixed at 100 basis points, the investment into the first stock decreases from 0.68 to 0.59, and similarly for the second stock.

In contrast to this, the investment into one stock are relatively insensitive to changes of the other stock's transaction costs. For instance, if we fix the transaction costs on the first stock at 100 basis points and vary the second stock's transaction costs between 50 and 150 basis points, the holdings in the first stock increase by about 0.002.

The results for the bond investment are unambiguous: The bond holdings of the first agent increases with the transaction costs on both stocks. The explanation for this is that higher transaction costs in one of the stocks decreases the agent's holdings in this specific stock strongly, while the holdings in the other stock are virtually unaffected, such that the first agent has to borrow less capital to finance the stock purchases.

The corresponding trading behavior of the agents are shown in Tables 18 to 20. Due to the zero transaction costs on the bond, the agents always trade the bond, whereas the stocks are traded infrequently. Within the stock universe, the agents try to smooth their consumption using the stock with the lower level of transaction costs.

#### 4.2.2 Asset Prices

The bond price in our economy is, as shown in Table 21, decreasing in both stocks' transaction costs. The lower demand for the bond in the presence of transaction costs on the stocks, as described in the preceding section, causes this price effect.

Similarly, the prices of the two stocks,<sup>11</sup> presented in Tables 22 and 23, decrease in the transaction costs of *both* stocks. Similar to the results in the preceding section, the effects of the transaction costs of one stock on the price of the other stock are much very small compared to the effect on the specific stock where we change the transaction costs. For instance, when varying the second stock's transaction costs between 50 and 150 basis points, the first stock price decreases by only 0.02.

#### 4.2.3 Return Characteristics

Obviously, the fact that the bond price is a decreasing function of the stocks' transaction costs, makes the bond return (shown in Table 24) an increasing function of the stocks' transaction costs. For the third preference setup the increase in bond return is about 0.9% when going from stock transaction costs of 10 basis points for each to stock transaction costs of 200 and 150 basis points for the first and the second stock, respectively. As transaction costs in the stock increase and bond returns increase, the volatility of the bond returns decrease, making the risk-return trade-off on the bond more favorable. While the volatility is low in absolute terms, it almost halves when going from the smallest to the highest transaction costs setup (see Table 25).

<sup>&</sup>lt;sup>11</sup>As the agents trade both stocks at the initial node, they agree on the prices of both assets such that we only show the price from the view of agent 1.

Coming to the equity premium, the stock return volatilities and the Sharpe ratios of the two stocks as presented in Tables 26 to 37, recall that with higher transaction costs the agents will trade less such that they will have different numerical values for these quantities in mind, due to their private valuation of the risky assets. Moreover, the second agent will typically have a lower private valuation of the stocks and accordingly lower expected returns and Sharpe ratios.

Accordingly, for both stocks the equity premium is an increasing function of transaction costs for the first agent and a decreasing function for the second agent. Again, a change in one stock's transaction costs only have small effects on the other stock's expected return. Based on 'mid-quotes', i.e., the mean value of the agents' private valuations the expected returns for both stocks increase by about 0.4-0.5% if we go from the small transaction costs setup (10/10 basis points) to the highest transaction costs setup (200/150 basis points).

Similarly to the on stock results the volatilities of the two stocks are only marginally affected, such that the changes in the Sharpe ratio are mainly drive the effects on the equity premium. Accordingly, the first agents Sharpe ratios are an increasing function of transaction costs whereas the second agents' Sharpe ratios are a decreasing function. Based on mid-quotes we see a small increase in Sharpe ratios in the presence of stock transaction costs.

## 5 Conclusion

In this paper, we develop a method that allows us to obtain asset prices in a general equilibrium economy with multiple agents who are heterogeneous when there are proportional costs for trading financial assets. The agents in our model have Epstein-Zin-Weil utility functions and can be heterogeneous with respect to endowments and all three characteristics of their utility functions – time preference, risk aversion, and elasticity of intertemporal substitution. The securities traded in the financial market include a long maturity discount bond and multiple risky stocks. Our method allows us to identify the equilibrium in a recursive fashion even in the presence of transactions costs, which make markets incomplete. This is in contrast to the usual approach for identifying the equilibrium in a general equilibrium model with incomplete financial markets, where one needs to iterate backwards and forwards until the

system converges. We use our model to study the effect of transactions costs on the interest rate, prices of stocks, the expected return and risk premium on stocks, and the volatility of stock and bond returns.

When there is only a single risky asset, we find that transactions costs on the stock or the bond lead investors to reduce the magnitude of their positions in the two financial assets. Transactions costs also reduce the frequency of trading equity. However, the effect on the frequency of trading the bond is much smaller; for even large transactions costs in the bond market, the investors continue to trade the bond. Moreover, because transactions costs make it less attractive to hold financial assets, there is a reduction in the prices of assets. The expected return on the bond increases with transactions costs. The effect on the volatility of bond returns, however, depends on preferences; as we increase transactions costs on the stock, the volatility of bond returns increases for the case of power utility but decreases for the case of Epstein-Zin preferences. We find that for moderate levels of transactions costs, the equity risk premium and Sharpe ratio increase with transactions costs for both investors.

We find that the holding of each stock is very sensitive to its own transactions cost, but relative insensitive to the transaction cost for the other stock. As intuition would suggest, when there are two stocks, agents use the stock with the lower transactions cost to share risk and smooth consumption over time. Asset prices respond to the changes in demands described above. Hence, an increase in the transaction cost of a particular stock leads to a decrease in the price of that stock and the bond, but has only a small effect on the price of the other stock.

## A The Proofs for Dynamic Programming

## A.1 The Derivations of the First-Order Conditions

Note, for ease of exposition we omit the subscript k for agent k and concentrate on the case for one asset with transaction costs. The case with several assets subject to transaction costs follows accordingly. Below we show that the first-order conditions derived from the recursive utility function and from the indirect utility function in the dynamic programming formulation are the same.

#### A.1.1 Global Solution

We have to solve at each time t simultaneously the following problem:

$$\sup_{c_t, \theta_t} V_t = \sup_{c_t, \theta_t} \left[ (1 - \beta) c_t^{1 - \frac{1}{\psi}} + \beta E_t \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma}}$$
(A1)

subject to for all points in time t:

$$c_t + \theta_t \cdot S_t + \tau \left(\theta_t, \theta_{t-1}\right) = \theta_{t-1} \cdot \left(S_t + d_t\right) \tag{A2}$$

Form the Lagrangian with  $\xi_t$  as the multiplier for the budget constraint:

$$\mathcal{L}_{t} = \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma}}$$

$$+ \xi_{t} \cdot (\theta_{t-1} \cdot (S_{t} + d_{t}) - c_{t} - \theta_{t} \cdot S_{t} - \tau \left( \theta_{t}, \theta_{t-1} \right) ), \tag{A3}$$

and take the first-order conditions:

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = \frac{\phi}{1 - \gamma} \cdot \left[ (1 - \beta) c_t^{1 - \frac{1}{\psi}} + \beta E_t \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma} - 1}$$

$$\cdot (1 - \beta) \cdot \left( 1 - \frac{1}{\psi} \right) \cdot c_t^{-\frac{1}{\psi}} - \xi_t$$

$$= V_t^{\frac{1}{\psi}} \cdot c_t^{-\frac{1}{\psi}} \cdot (1 - \beta) - \xi_t = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial \xi_t} = \theta_{t-1} \cdot (S_t + d_t) - c_t - \theta_t \cdot S_t - \tau \left( \theta_t, \theta_{t-1} \right) = 0$$

$$\frac{\partial \mathcal{L}_{t}}{\partial \theta_{t}} = \frac{\phi}{1 - \gamma} \cdot \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma} - 1} \cdot \frac{\beta}{\phi} \cdot E_{t} \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi} - 1} \\
\cdot (1 - \gamma) \cdot E_{t} \left[ V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial \theta_{t}} \right] - \xi_{t} \cdot \left( S_{t} + \frac{\tau \left( \theta_{t}, \theta_{t-1} \right)}{\partial \theta_{t}} \right) \\
= V_{t}^{\frac{1}{\psi}} \cdot \beta \cdot E_{t} \left[ V_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi} - 1} \cdot E_{t} \left[ V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial \theta_{t}} \right] - \xi_{t} \cdot \left( S_{t} + \frac{\tau \left( \theta_{t}, \theta_{t-1} \right)}{\partial \theta_{t}} \right) = 0 \quad (A4)$$

Now use that:

$$\frac{\partial V_{t+1}}{\partial \theta_t} = \frac{\partial V_{t+1}}{\partial c_{t+1}} \cdot \frac{\partial c_{t+1}}{\partial \theta_t}$$

$$= \left[ V_{t+1}^{\frac{1}{\psi}} \cdot c_{t+1}^{-\frac{1}{\psi}} \cdot (1 - \beta) \right] \cdot \left[ S_{t+1} + d_{t+1} - \frac{\partial \tau \left( \theta_{t+1}, \theta_t \right)}{\partial \theta_t} \right]$$

and plug into (A4), we get:

$$V_{t}^{\frac{1}{\psi}} \cdot \beta \cdot E_{t} \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{\phi}-1} \cdot E_{t} \left[ V_{t+1}^{-\gamma+\frac{1}{\psi}} c_{t+1}^{-\frac{1}{\psi}} \cdot (1-\beta) \cdot \left[ S_{t+1} + d_{t+1} - \frac{\partial \tau \left( \theta_{t+1}, \theta_{t} \right)}{\partial \theta_{t}} \right] \right]$$

$$= \xi_{t} \cdot \left( S_{t} + \frac{\tau \left( \theta_{t}, \theta_{t-1} \right)}{\partial \theta_{t}} \right)$$

## A.1.2 Dynamic Programming Solution

Define the value function recursively as:

$$J_T = \sup_{\{c_T\}} (1 - \beta) c_T$$

$$J_t = \sup_{\{c_t\}} \left[ (1 - \beta) c_t^{1 - \frac{1}{\psi}} + \beta E_t \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma}}$$

and write the problem as follows

$$J_{t} = \sup_{c_{t}, \theta_{t}} \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\varphi}{1 - \gamma}}$$
(A5)

subject to (A2). The Lagrangian:

$$\mathcal{L}_{t} = \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ J_{t+1}^{1-\gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1-\gamma}} + \xi_{t} \cdot (\theta_{t-1} \cdot (S_{t} + d_{t}) - c_{t} - \theta_{t} \cdot S_{t} - \tau (\theta_{t}, \theta_{t-1}))$$

with first-order conditions

$$\frac{\partial \mathcal{L}_{t}}{\partial c_{t}} = \frac{\phi}{1 - \gamma} \cdot \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma} - 1} \\
\cdot (1 - \beta) \cdot \left( 1 - \frac{1}{\psi} \right) \cdot c_{t}^{-\frac{1}{\psi}} - \xi_{t} \\
= J_{t}^{\frac{1}{\psi}} \cdot c_{t}^{-\frac{1}{\psi}} \cdot (1 - \beta) - \xi_{t} \\
= 0. \tag{A6}$$

$$\frac{\partial \mathcal{L}_{t}}{\partial \xi_{t}} = \theta_{t-1} \cdot (S_{t} + d_{t}) - c_{t} - \theta_{t} \cdot S_{t} \\
= 0. \tag{A7}$$

$$\frac{\partial \mathcal{L}_{t}}{\partial \theta_{t}} = \frac{\phi}{1 - \gamma} \cdot \left[ (1 - \beta) c_{t}^{1 - \frac{1}{\psi}} + \beta E_{t} \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi}} \right]^{\frac{\phi}{1 - \gamma} - 1} \cdot \frac{\beta}{\phi} \cdot E_{t} \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi} - 1} \\
\cdot (1 - \gamma) \cdot E_{t} \left[ J_{t+1}^{-\gamma} \frac{\partial J_{t+1}}{\partial \theta_{t}} \right] - \xi_{t} \cdot \left( S_{t} + \frac{\tau (\theta_{t}, \theta_{t-1})}{\partial \theta_{t}} \right) \\
= J_{t}^{\frac{1}{\psi}} \cdot \beta \cdot E_{t} \left[ J_{t+1}^{1 - \gamma} \right]^{\frac{1}{\phi} - 1} \cdot E_{t} \left[ J_{t+1}^{-\gamma} \frac{\partial J_{t+1}}{\partial \theta_{t}} \right] - \xi_{t} \cdot \left( S_{t} + \frac{\tau (\theta_{t}, \theta_{t-1})}{\partial \theta_{t}} \right) \\
= 0. \tag{A8}$$

The envelope theorem gives us:

$$\frac{\partial J_t}{\partial \theta_{t-1}} = \frac{\partial \mathcal{L}_t}{\partial \theta_{t-1}} = \xi_t \cdot \left( S_t + d_t - \frac{\partial \tau \left( \theta_t, \theta_{t-1} \right)}{\partial \theta_{t-1}} \right),$$

which we can plug into (A8) to get:

$$J_{t}^{\frac{1}{\psi}} \cdot \beta \cdot E_{t} \left[ J_{t+1}^{1-\gamma} \right]^{\frac{1}{\phi}-1} \cdot E_{t} \left[ J_{t+1}^{-\gamma+\frac{1}{\psi}} c_{t+1}^{-\frac{1}{\psi}} \cdot (1-\beta) \cdot \left[ S_{t+1} + d_{t+1} - \frac{\partial \tau \left( \theta_{t+1}, \theta_{t} \right)}{\partial \theta_{t}} \right] \right]$$

$$= \xi_{t} \cdot \left( S_{t} + \frac{\tau \left( \theta_{t}, \theta_{t-1} \right)}{\partial \theta_{t}} \right)$$
(A9)

The first-order conditions for the global case and the recursive case are the same, except that in the global formulation the utility function  $V_t$  shows up, and in the recursive formulation the value function  $J_t$  shows up. However, in optimum, when we solve the global case for all t, the utility function and the value function are the same.

## Table 1: One Stock Case: Bond Investment Agent 1

TC.	, bp		Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	-1.058471	-0.978076	-0.894973
25	0	-1.023104	-0.942110	-0.866667
50	0	-0.984684	-0.905634	-0.836945
75	0	-0.951160	-0.875699	-0.809846
100	0	-0.917405	-0.845596	-0.782566
125	0	-0.883071	-0.813337	-0.753546
150	0	-0.846523	-0.780285	-0.723517
175	0	-0.809557	-0.746861	-0.693251
200	0	-0.772184	-0.713023	-0.662697
10	10	-1.011661	-0.931007	-0.854464
25	10	-0.983671	-0.903751	-0.832621
50	10	-0.950544	-0.873543	-0.806503
75	10	-0.916866	-0.843227	-0.779503
100	10	-0.882284	-0.812501	-0.751937
125	10	-0.847526	-0.782077	-0.723980
150	10	-0.812909	-0.749277	-0.695735
175	10	-0.776083	-0.715473	-0.664327
200	10	-0.737816	-0.681369	-0.633745

Table 2: One Stock Case: Stock Investment Agent 1

TC, bp			Preferences		
Stock	Bond	Setup 1	Setup 2	Setup 3	
10	0	0.715747	0.706621	0.711539	
25	0	0.708826	0.698997	0.704844	
50	0	0.701265	0.691260	0.697749	
75	0	0.694640	0.684891	0.691249	
100	0	0.687978	0.678495	0.684697	
125	0	0.681205	0.671623	0.677714	
150	0	0.673968	0.664577	0.670476	
175	0	0.666642	0.657449	0.663168	
200	0	0.659224	0.650226	0.655776	
10	10	0.706304	0.696597	0.702160	
25	10	0.700806	0.690816	0.696964	
50	10	0.694240	0.684388	0.690698	
75	10	0.687584	0.677937	0.684213	
100	10	0.680753	0.671402	0.677581	
125	10	0.673891	0.664941	0.670844	
150	10	0.667064	0.657951	0.664028	
175	10	0.659773	0.650739	0.656435	
200	10	0.652168	0.643456	0.649026	

## Table 3: One Stock Case: Number of Bond Trades

TC, bp			Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	15.000000	15.000000	15.000000
25	0	15.000000	15.000000	15.000000
50	0	15.000000	15.000000	15.000000
75	0	15.000000	15.000000	15.000000
100	0	15.000000	15.000000	15.000000
125	0	15.000000	15.000000	15.000000
150	0	15.000000	15.000000	15.000000
175	0	15.000000	15.000000	15.000000
200	0	15.000000	15.000000	15.000000
10	10	15.000000	15.000000	15.000000
25	10	15.000000	15.000000	15.000000
50	10	15.000000	15.000000	15.000000
75	10	15.000000	15.000000	15.000000
100	10	15.000000	15.000000	15.000000
125	10	15.000000	15.000000	15.000000
150	10	15.000000	15.000000	15.000000
175	10	15.000000	15.000000	15.000000
200	10	15.000000	15.000000	15.000000

Table 4: One Stock Case: Number of Stock Trades

TC	, bp		Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	12.000000	15.000000	13.000000
25	0	10.000000	10.000000	10.000000
50	0	7.000000	7.000000	7.000000
75	0	6.000000	6.000000	6.000000
100	0	6.000000	5.000000	5.000000
125	0	3.000000	3.000000	3.000000
150	0	3.000000	3.000000	3.000000
175	0	3.000000	3.000000	3.000000
200	0	2.000000	2.000000	2.000000
10	10	15.000000	15.000000	15.000000
25	10	11.000000	11.000000	11.000000
50	10	8.000000	8.000000	8.000000
75	10	7.000000	6.000000	6.000000
100	10	6.000000	6.000000	6.000000
125	10	5.000000	4.000000	4.000000
150	10	4.000000	3.000000	3.000000
175	10	3.000000	3.000000	3.000000
200	10	3.000000	2.000000	2.000000

## Table 5: One Stock Case: Bond Price

TC, bp			Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	0.635147	0.736180	0.951567
25	0	0.631521	0.733403	0.950164
50	0	0.625512	0.729043	0.947931
75	0	0.619943	0.725029	0.945899
100	0	0.615020	0.721533	0.944127
125	0	0.610714	0.718525	0.942593
150	0	0.607030	0.716025	0.941319
175	0	0.603940	0.714033	0.940316
200	0	0.601431	0.712532	0.939575
10	10	0.632983	0.734345	0.950651
25	10	0.629434	0.731747	0.949319
50	10	0.623563	0.727569	0.947211
75	10	0.618379	0.723878	0.945358
100	10	0.613826	0.720692	0.943750
125	10	0.609882	0.718015	0.942402
150	10	0.606560	0.715848	0.941312
175	10	0.603828	0.714171	0.940479
200	10	0.601692	0.712995	0.939918

Table 6: One Stock Case: Stock Price

TC, bp			Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	3.164098	3.453686	4.054217
25	0	3.143729	3.438105	4.045177
50	0	3.111997	3.413858	4.030918
75	0	3.082959	3.391547	4.017632
100	0	3.056662	3.371321	4.005355
125	0	3.033035	3.353431	3.994207
150	0	3.012354	3.337746	3.984187
175	0	2.994287	3.324176	3.975244
200	0	2.978767	3.312708	3.967390
10	10	3.154368	3.444581	4.048038
25	10	3.135170	3.429990	4.039468
50	10	3.105094	3.407002	4.025850
75	10	3.077821	3.386119	4.013253
100	10	3.053272	3.367331	4.001683
125	10	3.031308	3.350498	3.991141
150	10	3.011891	3.336108	3.981633
175	10	2.995266	3.323878	3.973481
200	10	2.981441	3.313732	3.966304

## Table 7: One Stock Case: Expected Interest Rate

TC	, bp		Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	0.123928	0.079595	0.011973
25	0	0.129889	0.083679	0.013517
50	0	0.139235	0.090091	0.015994
75	0	0.148069	0.096169	0.018340
100	0	0.156246	0.101824	0.020529
125	0	0.163759	0.106993	0.022543
150	0	0.170486	0.111664	0.024372
175	0	0.176522	0.115873	0.026026
200	0	0.181898	0.119675	0.027522
10	10	0.126583	0.081773	0.012765
25	10	0.131817	0.085517	0.014241
50	10	0.140607	0.091511	0.016550
75	10	0.148711	0.097095	0.018703
100	10	0.156145	0.102234	0.020693
125	10	0.162923	0.106937	0.022528
150	10	0.168998	0.111111	0.024168
175	10	0.174321	0.114784	0.025626
200	10	0.178886	0.118020	0.026910

Table 8: One Stock Case: Volatility of Bond Return

TC, bp		Preferences	
Stock	Bond	Setup 1 Setup 2 Setup	3
10	0	0.000310 0.007583 0.0036	312
25	0	0.000430  0.007659  0.0036	312
50	0	0.001185 0.007837 0.0036	321
75	0	0.001505  0.007784  0.0035	68
100	0	0.001696 0.007613 0.0034	170
125	0	0.001748  0.007259  0.0032	292
150	0	0.001634 0.006722 0.0030	)42
175	0	0.001347  0.006030  0.0027	′33
200	0	0.000855  0.005139  0.0023	554
10	10	0.000659 0.006988 0.0033	372
25	10	0.000190 0.007198 0.0034	69
50	10	0.000468 0.007479 0.0035	41
75	10	0.001053 0.007610 0.0035	555
100	10	0.001518	22
125	10	0.001854  0.007524  0.0034	44
150	10	0.002032  0.007231  0.0032	279
175	10	0.002042 0.006740 0.0030	052
200	10	0.001887 0.006076 0.0027	'50_

## Table 9: One Stock Case: Equity Premium Agent 1

TC	, bp		Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	0.073002	0.071724	0.066527
25	0	0.073626	0.072045	0.066717
50	0	0.075385	0.073390	0.067786
75	0	0.077048	0.074803	0.068880
100	0	0.078690	0.076183	0.069969
125	0	0.080290	0.077464	0.071009
150	0	0.081783	0.078691	0.072015
175	0	0.083216	0.079871	0.073005
200	0	0.084595	0.081041	0.073991
10	10	0.074085	0.072777	0.067509
25	10	0.075077	0.073359	0.067961
50	10	0.076763	0.074782	0.069073
75	10	0.078409	0.076145	0.070161
100	10	0.080013	0.077491	0.071239
125	10	0.081578	0.078836	0.072300
150	10	0.083145	0.080065	0.073347
175	10	0.084593	0.081226	0.074301
200	10	0.085915	0.082367	0.075277

Table 10: One Stock Case: Equity Premium Agent 2

TC, bp			Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	0.073002	0.071724	0.066527
25	0	0.073957	0.072599	0.067071
50	0	0.074267	0.072753	0.066863
75	0	0.074320	0.072602	0.066530
100	0	0.074326	0.072441	0.066186
125	0	0.074276	0.072220	0.065845
150	0	0.074206	0.071986	0.065502
175	0	0.074065	0.071688	0.065136
200	0	0.073860	0.071367	0.064762
10	10	0.074085	0.072777	0.067509
25	10	0.074150	0.072847	0.067185
50	10	0.074196	0.072712	0.066828
75	10	0.074193	0.072521	0.066468
100	10	0.074154	0.072302	0.066106
125	10	0.074055	0.072084	0.065711
150	10	0.073954	0.071766	0.065392
175	10	0.073810	0.071454	0.064956
200	10	0.073533	0.071106	0.064576

## Table 11: One Stock Case: Volatility of Stock Return Agent 1

TC	, bp		Preferences	
Stock	Bond	Setup 1	Setup 2	Setup 3
10	0	0.180129	0.169639	0.160591
25	0	0.180637	0.170199	0.160766
50	0	0.180814	0.170251	0.160281
75	0	0.180982	0.170226	0.159769
100	0	0.181148	0.170237	0.159274
125	0	0.181302	0.170255	0.158799
150	0	0.181402	0.170295	0.158342
175	0	0.181489	0.170351	0.157898
200	0	0.181608	0.170526	0.157507
10	10	0.180827	0.170362	0.160898
25	10	0.181079	0.170506	0.160703
50	10	0.180991	0.170261	0.160088
75	10	0.180857	0.170020	0.159486
100	10	0.180704	0.169801	0.158904
125	10	0.180545	0.169616	0.158330
150	10	0.180402	0.169426	0.157804
175	10	0.180209	0.169265	0.157254
200	10	0.179964	0.169181	0.156767

Table 12: One Stock Case: Volatility of Stock Return Agent 2

TC, bp			Preferences		
Stock	Bond	Setup 1	Setup 2	Setup 3	
10	0	0.180129	0.169639	0.160591	
25	0	0.180306	0.169644	0.160412	
50	0	0.181931	0.170887	0.161204	
75	0	0.183710	0.172427	0.162120	
100	0	0.185512	0.173979	0.163057	
125	0	0.187316	0.175498	0.163963	
150	0	0.188979	0.177000	0.164855	
175	0	0.190640	0.178534	0.165767	
200	0	0.192343	0.180200	0.166736	
10	10	0.180827	0.170362	0.160898	
25	10	0.182006	0.171018	0.161479	
50	10	0.183558	0.172331	0.162333	
75	10	0.185073	0.173644	0.163180	
100	10	0.186563	0.174991	0.164037	
125	10	0.188067	0.176369	0.164920	
150	10	0.189593	0.177725	0.165758	
175	10	0.190992	0.179037	0.166599	
200	10	0.192346	0.180442	0.167469	

## Table 13: One Stock Case: Sharpe Ratio Agent 1

TC, bp			Preferences			
Stock	Bond	Setup 1	Setup 2	Setup 3		
10	0	0.405273	0.422804	0.414263		
25	0	0.407591	0.423298	0.414995		
50	0	0.416919	0.431068	0.422921		
75	0	0.425722	0.439431	0.431123		
100	0	0.434397	0.447509	0.439299		
125	0	0.442854	0.454986	0.447164		
150	0	0.450840	0.462087	0.454809		
175	0	0.458518	0.468864	0.462353		
200	0	0.465813	0.475238	0.469767		
10	10	0.409702	0.427189	0.419577		
25	10	0.414610	0.430244	0.422895		
50	10	0.424122	0.439219	0.431466		
75	10	0.433543	0.447859	0.439920		
100	10	0.442782	0.456366	0.448318		
125	10	0.451842	0.464794	0.456644		
150	10	0.460887	0.472567	0.464797		
175	10	0.469418	0.479875	0.472488		
200	10	0.477400	0.486859	0.480188		

Table 14: One Stock Case: Sharpe Ratio Agent 2

TC, bp			Preferences			
Stock	Bond	Setup 1	Setup 2	Setup 3		
10	0	0.405273	0.422804	0.414263		
25	0	0.410177	0.427950	0.418116		
50	0	0.408214	0.425738	0.414773		
75	0	0.404548	0.421060	0.410373		
100	0	0.400655	0.416376	0.405906		
125	0	0.396527	0.411514	0.401587		
150	0	0.392666	0.406703	0.397330		
175	0	0.388504	0.401537	0.392941		
200	0	0.384003	0.396045	0.388413		
10	10	0.409702	0.427189	0.419577		
25	10	0.407404	0.425959	0.416061		
50	10	0.404207	0.421933	0.411676		
75	10	0.400887	0.417638	0.407329		
100	10	0.397472	0.413174	0.402996		
125	10	0.393771	0.408709	0.398441		
150	10	0.390065	0.403806	0.394505		
175	10	0.386458	0.399103	0.389896		
200	10	0.382293	0.394064	0.385598		

## Table 15: Two Stocks Case: Bond Investment Agent 1

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	-2.483133	-2.215405	-1.983465		
25	25	-2.383364	-2.127751	-1.909834		
50	50	-2.215070	-1.984444	-1.786391		
50	150	-1.897178	-1.708955	-1.548524		
75	100	-1.961877	-1.765112	-1.596869		
100	50	-2.025830	-1.818936	-1.643268		
100	100	-1.864082	-1.679581	-1.522975		
100	150	-1.699694	-1.536528	-1.399746		
125	100	-1.765123	-1.592948	-1.448268		
125	150	-1.599044	-1.447426	-1.323166		
150	100	-1.665323	-1.504973	-1.372445		
200	100	-1.459461	-1.318663	-1.212531		
200	150	-1.283828	-1.163554	-1.079012		

## Table 16: Two Stocks Case: Stock 1 Investment Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp		Preferences			
Stock 1	Stock 2		Setup 1	Setup 2	Setup 3
	_				
10	10		0.725276	0.710975	0.715110
25	25		0.715866	0.701992	0.706414
50	50		0.699904	0.687094	0.691797
50	150		0.702780	0.690215	0.694120
75	100		0.684095	0.672137	0.676857
100	50		0.665431	0.654066	0.659558
100	100		0.666437	0.655248	0.660364
100	150		0.667265	0.656233	0.661009
125	100		0.648668	0.638251	0.643780
125	150		0.649282	0.639040	0.644268
150	100		0.630857	0.621104	0.627012
200	100		0.594295	0.585045	0.591822
200	150		0.593873	0.584896	0.591430

## Table 17: Two Stocks Case: Stock 2 Investment Agent 1

TC, bp		Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3	
10	10	0.723865	0.708129	0.711162	
25	25	0.715943	0.700521	0.703978	
50	50	0.702657	0.688280	0.691920	
50	150	0.644213	0.632765	0.637784	
75	100	0.674347	0.661606	0.665748	
100	50	0.704196	0.689993	0.693245	
100	100	0.674952	0.662281	0.666250	
100	150	0.645300	0.634029	0.638697	
125	100	0.675418	0.662841	0.666638	
125	150	0.645665	0.634284	0.638820	
150	100	0.675731	0.663274	0.666936	
200	100	0.675996	0.663826	0.667292	
200	150	0.645497	0.634277	0.638485	

## Table 18: Two Stocks Case: Number of Bond Trades

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	40.000000	40.000000	40.000000		
25	25	40.000000	40.000000	40.000000		
50	50	40.000000	40.000000	40.000000		
50	150	40.000000	40.000000	40.000000		
75	100	40.000000	40.000000	40.000000		
100	50	40.000000	40.000000	40.000000		
100	100	40.000000	40.000000	40.000000		
100	150	40.000000	40.000000	40.000000		
125	100	40.000000	40.000000	40.000000		
125	150	40.000000	40.000000	40.000000		
150	100	40.000000	40.000000	40.000000		
200	100	40.000000	40.000000	40.000000		
200	150	40.000000	40.000000	40.000000		

#### Table 19: Two Stocks Case: Number of Stock 1 Trades

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp			Preferences	
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	15.000000	19.000000	19.000000
25	25	14.000000	10.000000	10.000000
50	50	8.000000	6.000000	6.000000
50	150	4.000000	4.000000	4.000000
75	100	4.000000	3.000000	3.000000
100	50	3.000000	3.000000	2.000000
100	100	3.000000	2.000000	2.000000
100	150	2.000000	2.000000	2.000000
125	100	2.000000	2.000000	2.000000
125	150	2.000000	2.000000	2.000000
150	100	2.000000	1.000000	1.000000
200	100	1.000000	1.000000	1.000000
200	150	1.000000	1.000000	1.000000

# Table 20: Two Stocks Case: Number of Stock 2 Trades

TC, bp			Preferences	
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	20.000000	22.000000	20.000000
25	25	13.000000	11.000000	11.000000
50	50	10.000000	5.000000	5.000000
50	150	2.000000	2.000000	2.000000
75	100	4.000000	2.000000	2.000000
100	50	6.000000	5.000000	5.000000
100	100	2.000000	2.000000	2.000000
100	150	2.000000	2.000000	2.000000
125	100	2.000000	2.000000	2.000000
125	150	2.000000	1.000000	1.000000
150	100	2.000000	2.000000	2.000000
200	100	2.000000	2.000000	2.000000
200	150	1.000000	1.000000	1.000000

#### Table 21: Two Stocks Case: Bond Price

TC	, bp			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	0.528779	0.640437	0.875863
25	25	0.525619	0.638015	0.874537
50	50	0.521168	0.634635	0.872676
50	150	0.516652	0.631819	0.871209
75	100	0.516355	0.631184	0.870763
100	50	0.518426	0.632767	0.871654
100	100	0.515240	0.630458	0.870348
100	150	0.513849	0.629807	0.870017
125	100	0.514654	0.630226	0.870219
125	150	0.513257	0.629561	0.869858
150	100	0.514595	0.630493	0.870378
200	100	0.516108	0.632619	0.871612
200	150	0.514773	0.631972	0.871205

#### Table 22: Two Stocks Case: Stock 1 Price

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp			Preferences	
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	3.009326	3.375958	4.098999
25	25	2.990166	3.360828	4.089783
50	50	2.962395	3.338872	4.076069
50	150	2.935569	3.320746	4.067025
75	100	2.931950	3.315638	4.061757
100	50	2.940023	3.320659	4.061733
100	100	2.922207	3.307688	4.055206
100	150	2.913368	3.302403	4.052670
125	100	2.915181	3.302076	4.049894
125	150	2.906405	3.296887	4.047418
150	100	2.910828	3.298819	4.045836
200	100	2.910557	3.300130	4.041812
200	150	2.902598	3.295635	4.039759

# Table 23: Two Stocks Case: Stock 2 Price

TC, bp			Preferences	
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	2.996443	3.360787	4.080045
25	25	2.977439	3.345794	4.070951
50	50	2.949879	3.324033	4.057406
50	150	2.910540	3.292434	4.031631
75	100	2.916435	3.297546	4.038991
100	50	2.934110	3.312887	4.051760
100	100	2.909929	3.293067	4.036716
100	150	2.894787	3.280991	4.025766
125	100	2.906110	3.290900	4.035649
125	150	2.890989	3.278867	4.024701
150	100	2.904942	3.291066	4.035810
200	100	2.911016	3.299231	4.040201
200	150	2.896557	3.287714	4.029495

#### Table 24: Two Stocks Case: Expected Interest Rate

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	0.179929	0.120002	0.034445		
25	25	0.186209	0.124256	0.036074		
50	50	0.195438	0.130568	0.038506		
50	150	0.206721	0.138210	0.041523		
75	100	0.206332	0.138028	0.041419		
100	50	0.201852	0.134957	0.040238		
100	100	0.209153	0.139986	0.042199		
100	150	0.213463	0.142929	0.043383		
125	100	0.211082	0.141321	0.042747		
125	150	0.215462	0.144240	0.043927		
150	100	0.212115	0.142009	0.043047		
200	100	0.210952	0.140637	0.042629		
200	150	0.214855	0.143103	0.043641		

#### Table 25: Two Stocks Case: Volatility of Bond Return

TC, bp			Prefere	nces		
Stock 1	Stock 2	S	etup 1	Setup	2	Setup 3
10	10	0.	005948	0.0042	24	0.002127
25	25	0.	005520	0.0043	03	0.002143
50	50	0.	004975	0.0042	55	0.002105
50	150	0.	004421	0.0035	33	0.001844
75	100	0.	004436	0.0037	94	0.001919
100	50	0.	004904	0.0037	25	0.001860
100	100	0.	004475	0.0034	10	0.001760
100	150	0.	004471	0.0028	320	0.001567
125	100	0.	004623	0.0028	91	0.001552
125	150	0.	004675	0.0023	888	0.001398
150	100	0.	004862	0.0022	55	0.001312
200	100	0.	005125	0.0015	75	0.001022
200	150	0.	004790	0.0015	90	0.001063

#### Table 26: Two Stocks Case: Equity Premium Stock 1 Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp		Preferences	
Stock 1	Stock 2	Setup 1 Setup 2 Setup 3	
10	10	0.040069  0.039111  0.035967	,
25	25	0.041250  0.040124  0.036831	
50	50	0.043180  0.041786  0.038258	,
50	150	0.043630  0.041898  0.038282	,
75	100	0.045176  0.043420  0.039652	,
100	50	0.046546  0.044818  0.040980	)
100	100	0.046834  0.044920  0.041000	)
100	150	0.046992  0.044925  0.040985	)
125	100	0.048460  0.046394  0.042338	š
125	150	0.048616  0.046394  0.042315	)
150	100	0.050046  0.052757  0.048335	)
200	100	0.059801  0.057017  0.052334	Ŀ
200	150	0.059902  0.056945  0.052229	)

#### Table 27: Two Stocks Case: Equity Premium Stock 1 Agent 2

TC, bp			Preference	s
Stock 1	Stock 2	Setup	1 Setup 2	Setup 3
10	10	0.03970	0.039012	0.035815
25	25	0.0395	15 0.038767	0.035480
50	50	0.03914	48 0.038264	0.034888
50	150	0.03948	89 0.038329	0.034899
75	100	0.0388	0.037763	0.034309
100	50	0.0380	51 0.037116	0.033693
100	100	0.03820	68 0.037178	0.033707
100	150	0.0383'	78 0.037156	0.033687
125	100	0.0376'	76 0.036558	0.033087
125	150	0.0377'	75 0.036523	0.033058
150	100	0.03704	47 0.030994	0.027793
200	100	0.0291	17 0.028404	0.025352
200	150	0.02929	93 0.028445	0.025413

#### Table 28: Two Stocks Case: Equity Premium Stock 2 Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC	, bp	Preference		
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3
10	10	0.042381	0.041306	0.037941
25	25	0.043598	0.042343	0.038825
50	50	0.045584	0.044044	0.040283
50	150	0.052393	0.050174	0.045799
75	100	0.049231	0.047231	0.043090
100	50	0.045868	0.044141	0.040313
100	100	0.049352	0.047264	0.043097
100	150	0.052703	0.050286	0.045829
125	100	0.049427	0.047262	0.043089
125	150	0.052781	0.055034	0.050334
150	100	0.049453	0.047223	0.043065
200	100	0.049360	0.047029	0.042955
200	150	0.057585	0.054681	0.050080

#### Table 29: Two Stocks Case: Equity Premium Stock 2 Agent 2

TC, bp			I	Preferences	S
Stock 1	Stock 2	Setup	1	Setup 2	Setup 3
10	10	0.042	124	0.041325	0.037877
25	25	0.0419	912	0.041060	0.037509
50	50	0.0418	500	0.040519	0.036864
50	150	0.0390	049	0.037953	0.034253
75	100	0.0404	426	0.039300	0.035563
100	50	0.0416	678	0.040544	0.036853
100	100	0.0404	490	0.039294	0.035550
100	150	0.0392	211	0.037959	0.034228
125	100	0.0405	517	0.039256	0.035523
125	150	0.0392	230	0.033217	0.029725
150	100	0.0405	503	0.039186	0.035481
200	100	0.0403	352	0.038947	0.035352
200	150	0.0342	299	0.033196	0.029824

# Table 30: Two Stocks Case: Volatility of Stock 1 Return Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp			Preferences		
Stock 1	Stock 2		Setup 1	Setup 2	Setup 3
10	10		0.182198	0.169334	0.157108
25	25		0.182423	0.169475	0.156923
50	50		0.182712	0.169712	0.156634
50	150		0.184504	0.171290	0.157296
75	100		0.183414	0.170380	0.156546
100	50		0.181961	0.169293	0.155744
100	100		0.183052	0.170223	0.156127
100	150		0.183994	0.171108	0.156504
125	100		0.182681	0.170075	0.155719
125	150		0.183697	0.170930	0.156087
150	100		0.182286	0.176276	0.161353
200	100		0.189612	0.176951	0.161938
200	150		0.190186	0.177294	0.162042

#### Table 31: Two Stocks Case: Volatility of Stock 1 Return Agent 2

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	0.182541	0.169571	0.157410		
25	25	0.183503	0.170402	0.157878		
50	50	0.185012	0.171766	0.158642		
50	150	0.186713	0.173220	0.159170		
75	100	0.186860	0.173471	0.159513		
100	50	0.186641	0.173545	0.159818		
100	100	0.187682	0.174421	0.160130		
100	150	0.188567	0.175248	0.160438		
125	100	0.188506	0.175385	0.160760		
125	150	0.189466	0.176185	0.161058		
150	100	0.189306	0.169948	0.155328		
200	100	0.181829	0.169709	0.155042		
200	150	0.182640	0.170275	0.155367		

# Table 32: Two Stocks Case: Volatility of Stock 2 Return Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	0.17716	0.164718	0.152551		
25	25	0.17757	0.165013	0.152496		
50	50	0.178143	0.165471	0.152409		
50	150	0.177609	0.165367	0.151367		
75	100	0.17852'	7 0.165911	0.152088		
100	50	0.179050	0.166243	0.152748		
100	100	0.178974	4 0.166323	0.152270		
100	150	0.178719	0.166326	0.151782		
125	100	0.17937	0.166709	0.152444		
125	150	0.179176	0.171662	0.156693		
150	100	0.17970	0.167052	0.152598		
200	100	0.179769	0.166985	0.152614		
200	150	0.18441	0.171440	0.156568		

#### Table 33: Two Stocks Case: Volatility of Stock 2 Return Agent 2

TC, bp			Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3		
10	10	0.176890	0.164398	0.152324		
25	25	0.177762	0.165103	0.152680		
50	50	0.179114	0.166264	0.153262		
50	150	0.181795	0.169137	0.155023		
75	100	0.180984	0.168073	0.154227		
100	50	0.179874	0.166873	0.153438		
100	100	0.181345	0.168410	0.154329		
100	150	0.182741	0.169955	0.155280		
125	100	0.181656	0.168719	0.154423		
125	150	0.183114	0.165232	0.150579		
150	100	0.181902	0.168986	0.154494		
200	100	0.181777	0.168736	0.154323		
200	150	0.177857	0.165311	0.150740		

# Table 34: Two Stocks Case: Sharpe Ratio Stock 1 Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\gamma_2=5$  and EIS agents, having the same RRA  $\gamma_3=5$  and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp		Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3	
10	10	0.219921	0.230970	0.228929	
25	25	0.226125	0.236757	0.234708	
50	50	0.236330	0.246215	0.244253	
50	150	0.236470	0.244602	0.243373	
75	100	0.246304	0.254843	0.253290	
100	50	0.255800	0.264733	0.263125	
100	100	0.255853	0.263887	0.262608	
100	150	0.255401	0.262551	0.261879	
125	100	0.265272	0.272789	0.271885	
125	150	0.264653	0.271419	0.271099	
150	100	0.274545	0.299287	0.299558	
200	100	0.315385	0.322217	0.323175	
200	150	0.314967	0.321188	0.322319	

#### Table 35: Two Stocks Case: Sharpe Ratio Stock 1 Agent 2

TC, bp			Preferences			
Stock 1	Stock 2	Setup	o 1 Se	tup 2	Setup 3	
10	10	0.2175	506  0.2	30060	0.227524	
25	25	0.2153	337  0.2	27501	0.224727	
50	50	0.2113	596 - 0.2	22767	0.219916	
50	150	$0.211^{2}$	498 0.2	21274	0.219255	
75	100	0.2078	807 0.2	17690	0.215089	
100	50	0.2038	873 0.2	13871	0.210820	
100	100	0.2038	895 0.2	13153	0.210495	
100	150	0.203	524 0.2	12022	0.209968	
125	100	0.1998	865 0.2	08446	0.205813	
125	150	0.1993	378 0.2	07300	0.205257	
150	100	0.195'	700 0.1	82371	0.178929	
200	100	0.160	133 0.1	67367	0.163518	
200	150	0.1603	385 0.1	67052	0.163568	

# Table 36: Two Stocks Case: Sharpe Ratio Stock 2 Agent 1

This tables provides selected results from the analysis of a general equilibrium economy with transaction costs proportional to the value of the financial asset traded. The economy has two heterogeneous agents maximizing their lifetime utility of consumption over 5 time periods. We have three setups in terms of preferences of agents: (i) Setup 1 with two CRRA agents having the relative risk aversion (RRA) coefficients  $\gamma_1=2$  and  $\gamma_2=5$ ; (ii) Setup 2 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$ , and elasticity of intertemporal substitution (EIS)  $\psi=0.5$ ; (iii) Setup 3 with EZ agents, having the same RRA  $\gamma_1=2$  and  $\gamma_2=5$  and EIS  $\psi=1.5$ . In all three setups, we have two risky assets (Stock 1 and Stock 2) and one riskless asset (bond). Trading in the risky stocks may incur transaction costs.

TC, bp		Preferences			
Stock 1	Stock 2	Setup 1 Setup 2 Setup	3		
	_				
10	10	0.239217  0.250764  0.2487	14		
25	25	0.245524  0.256605  0.2545	97		
50	50	0.255883  0.266171  0.2643	80		
50	150	0.294991  0.303409  0.3025	67		
75	100	0.275764 $0.284675$ $0.2833$	20		
100	50	0.256172  0.265519  0.2639	17		
100	100	0.275747  0.284172  0.2830	27		
100	150	0.294895  0.302331  0.3019	42		
125	100	0.275557  0.283501  0.2826	52		
125	150	0.294577  0.320594  0.3212	26		
150	100	0.275193  0.282683  0.2822	11		
200	100	0.274577  0.281638  0.2814	59		
200	150	0.312263  0.318951  0.3198	58		

#### Table 37: Two Stocks Case: Sharpe Ratio Stock 2 Agent 2

TC, bp		Preferences			
Stock 1	Stock 2	Setup 1	Setup 2	Setup 3	
10	10	0.238137	0.251372	0.248660	
25	25	0.235775	0.248694	0.245670	
50	50	0.231698	0.243704	0.240530	
50	150	0.214800	0.224390	0.220958	
75	100	0.223368	0.233824	0.230587	
100	50	0.231705	0.242963	0.240182	
100	100	0.223274	0.233326	0.230353	
100	150	0.214570	0.223347	0.220427	
125	100	0.223043	0.232673	0.230034	
125	150	0.214237	0.201033	0.197408	
150	100	0.222665	0.231886	0.229662	
200	100	0.221985	0.230814	0.229076	
200	150	0.192846	0.200808	0.197851	

# Figure 1: One Stock Case: Bond Investment Agent 1

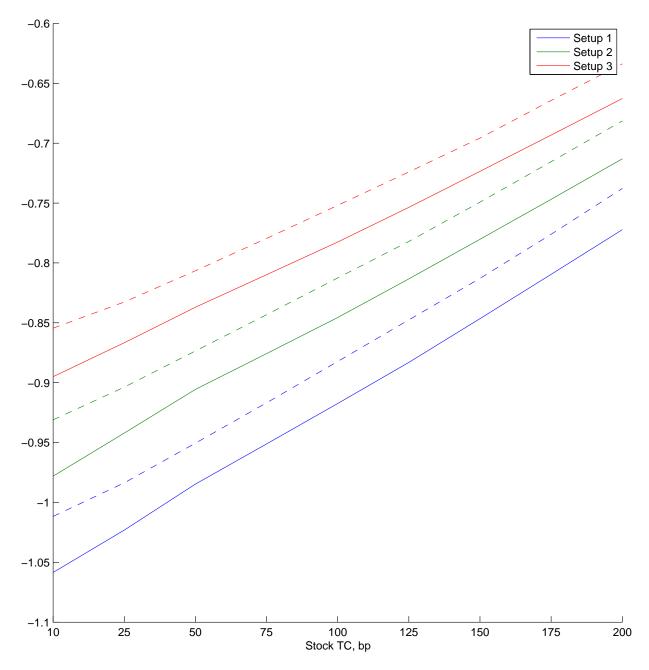


Figure 2: One Stock Case: Stock Investment Agent 1

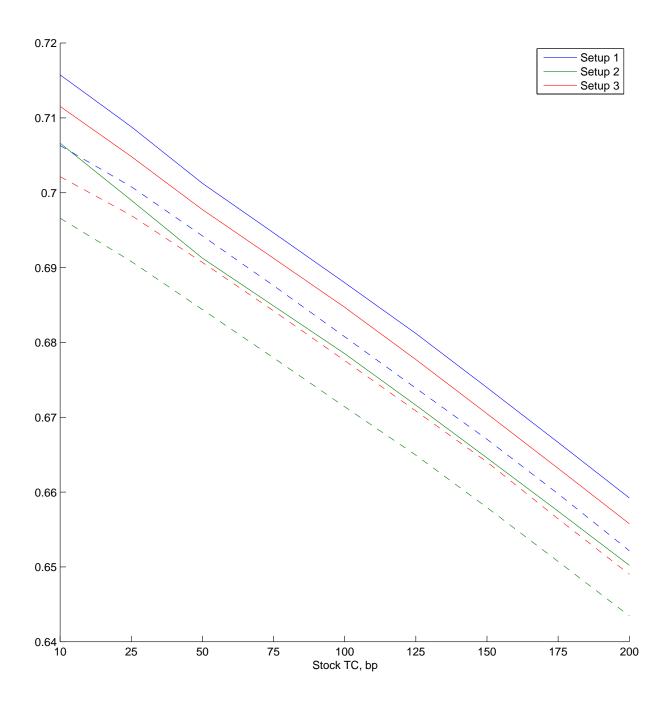


Figure 3: One Stock Case: Number of Bond Trades

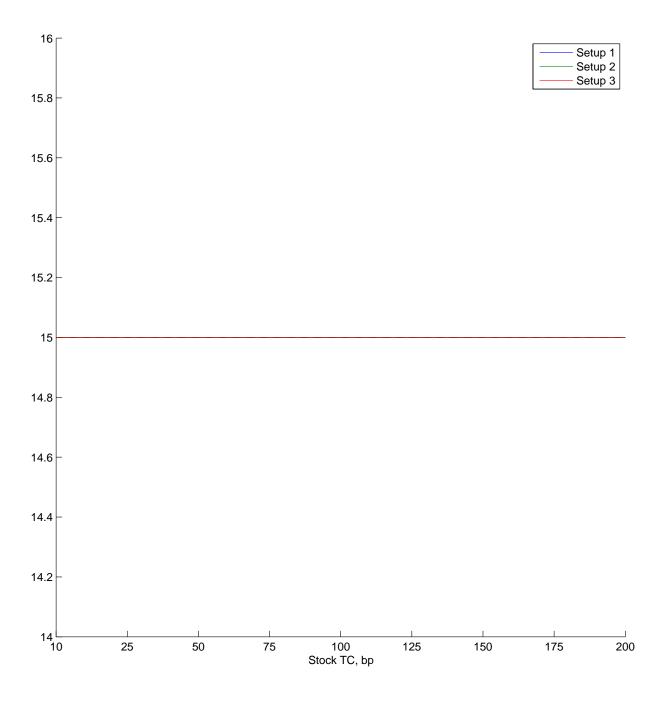


Figure 4: One Stock Case: Number of Stock Trades

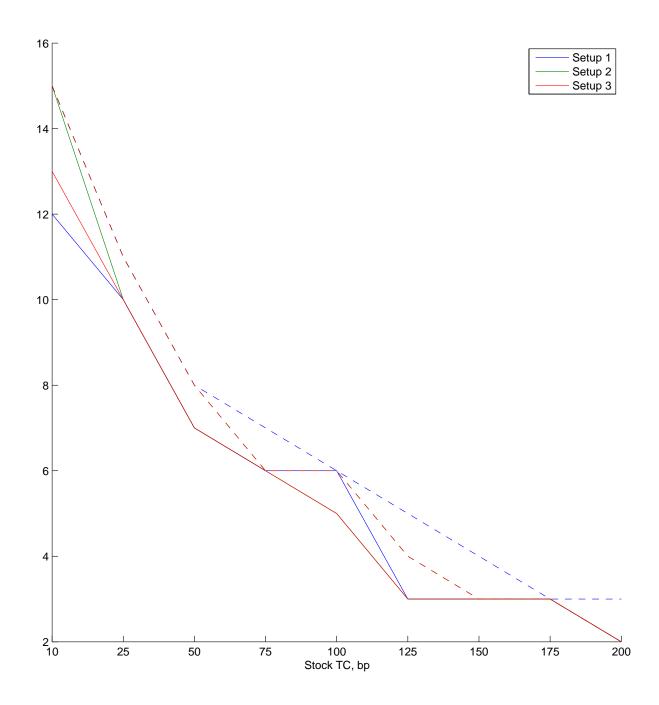


Figure 5: One Stock Case: Bond Price

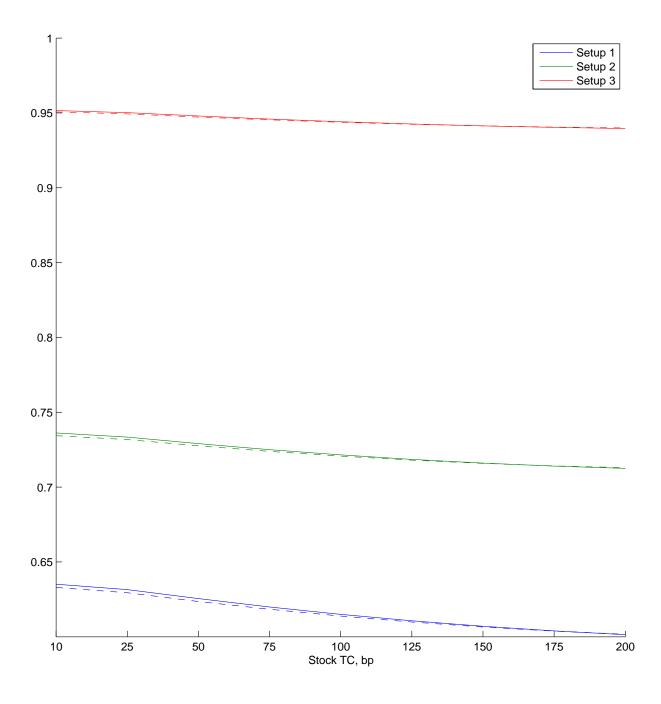


Figure 6: One Stock Case: Stock Price

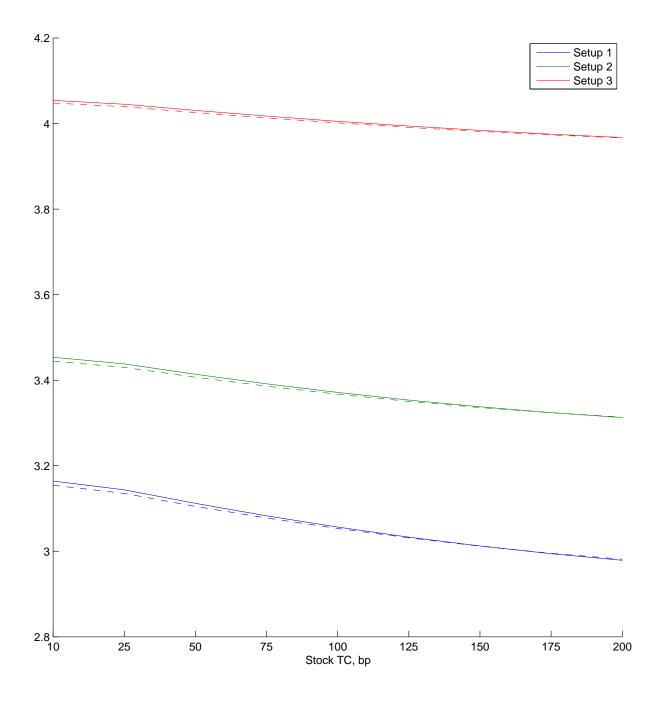


Figure 7: One Stock Case: Expected Interest Rate

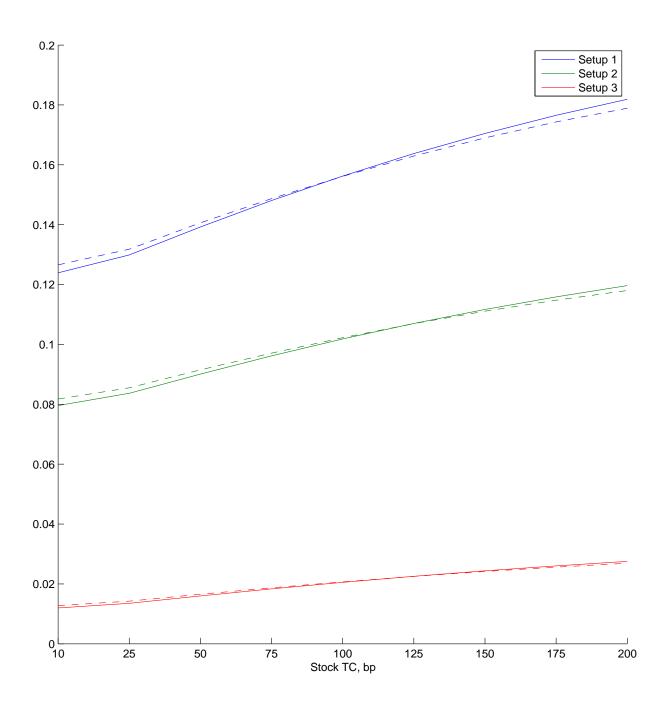


Figure 8: One Stock Case: Volatility of Bond Return

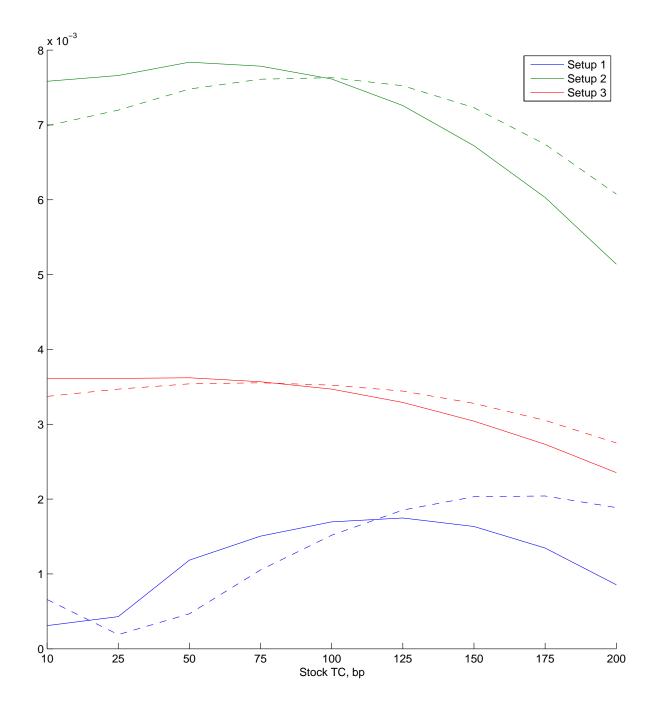


Figure 9: One Stock Case: Equity Premium Agent 1

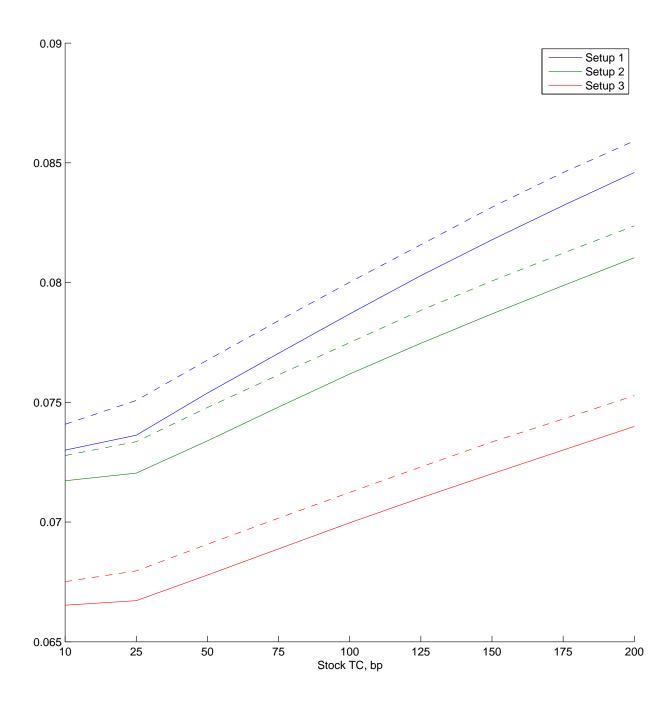


Figure 10: One Stock Case: Equity Premium Agent 2

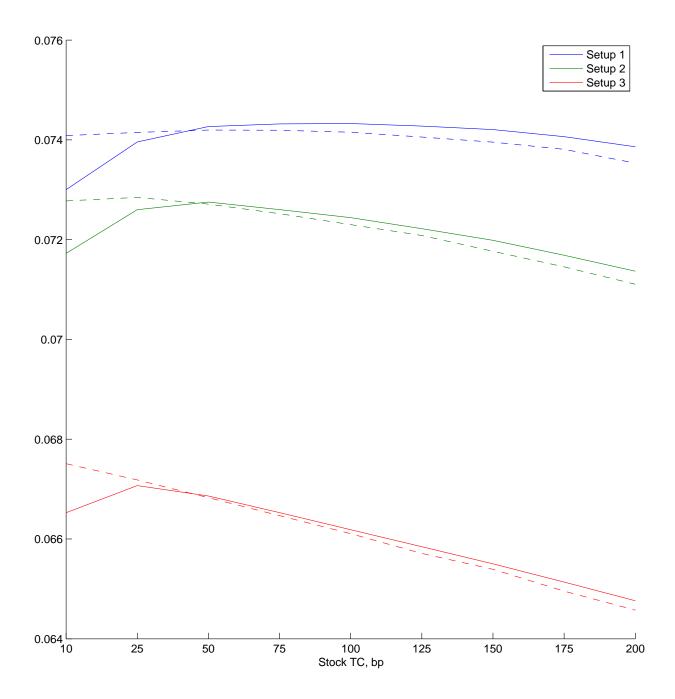


Figure 11: One Stock Case: Volatility of Stock Return Agent 1

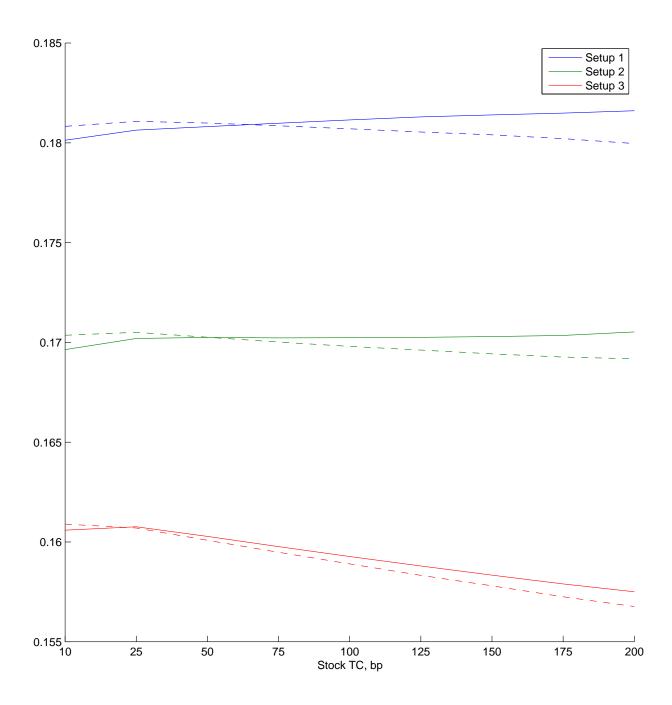


Figure 12: One Stock Case: Volatility of Stock Return Agent 2

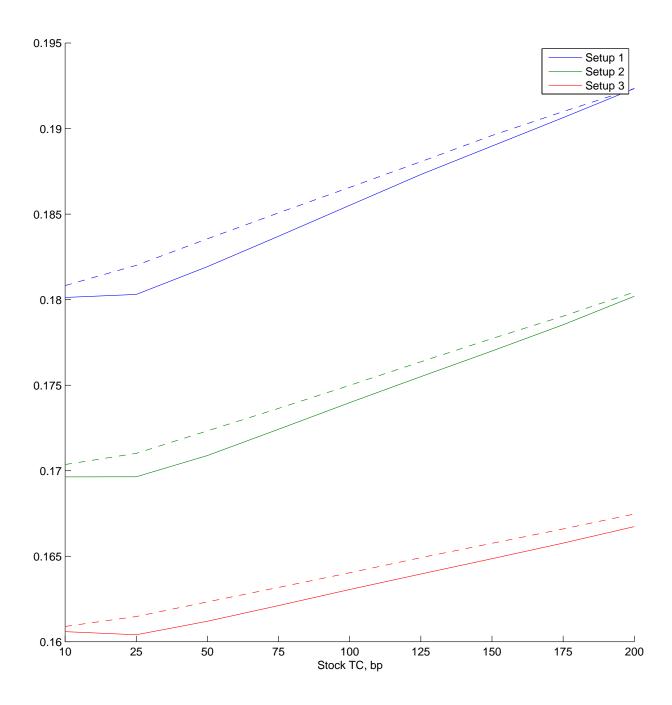


Figure 13: One Stock Case: Sharpe Ratio Agent 1

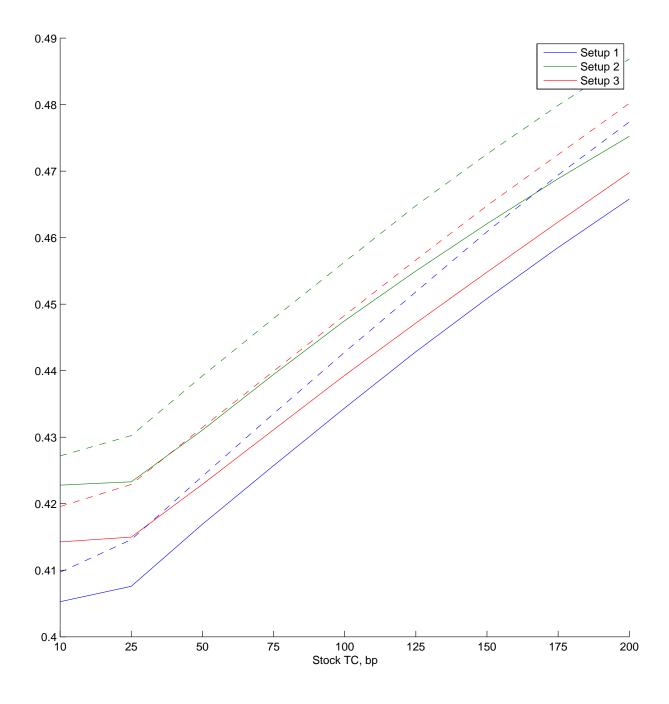
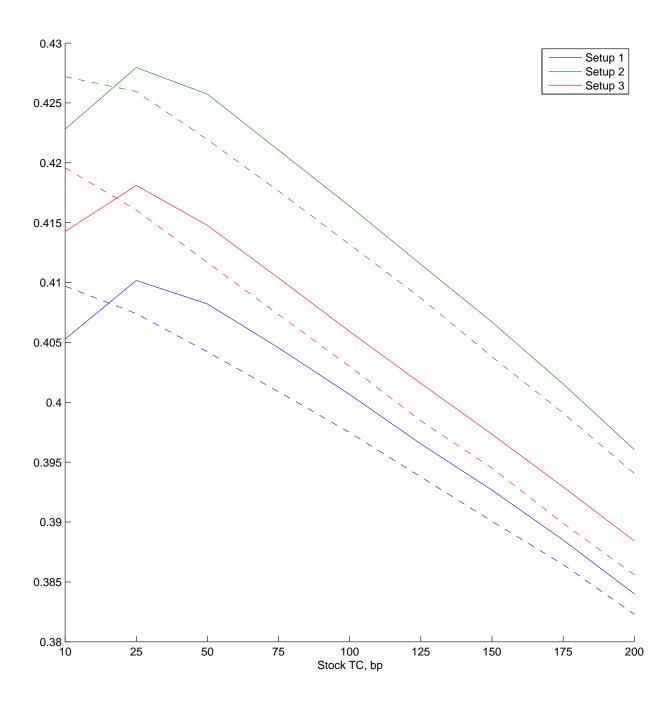


Figure 14: One Stock Case: Sharpe Ratio Agent 2



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