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Estimating Private Equity Returns from Limited Partner Cash Flows

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1. Introduction

Private equity is a major institutional asset class and represents a significant fraction of investments by colleges, foundations, pension funds, and sovereign wealth funds, among others.[1] A major drawback of private equity is the lack of transactions-based performance measures. This greatly hampers the use of optimal portfolio allocation, which requires information about the risk, return, and covariance of asset classes. In liquid markets, these estimates are typically derived from statistical analysis of time-series returns. Most private equity time series are based on infrequently updated non-market estimates or on multi-year internal rates of return broken down by fund vintage years.

We develop a methodology to estimate a time series of private equity returns based on cash flows accruing to limited partners. We analyze the dynamics of private equity over 1993 to 2011, as well as investigate private equity returns for different subclasses: venture capital, buyout, real estate, and credit funds. We decompose returns into a component due to exposure to traded factors and a time-varying private equity premium. The latter can be interpreted as the unique value-added by private equity which cannot be replicated by passive, liquid instruments. Given assumptions on the traded factors, the private equity premium can be interpreted as the time-varying private equity alpha.

Our methodology identifies private equity realized returns by using a net present value (NPV) framework. Under the null that the realized returns are correct, the (realized) present value of the capital calls paid into the fund must equal the (realized) present value of the distributions from the fund. The NPV equation involving all limited partner (LP) cash flows should thus be zero, on average, both across time and across funds (cf. Driessen, Lin, and Phalippou (2012)). Using a Bayesian Markov Chain Monte Carlo (MCMC) procedure, we filter the time-varying private equity returns using the fund-level NPV equations as observation equations. As long as we have at least one fund in existence at a given time we can econometrically identify the private equity return prevailing at that time, given additional assumptions about the data-generating process of

[1] In 2011, institutional investors had over $2 trillion worth of investments in private equity funds worldwide, up from less than $0.4 trillion just ten years earlier. These funds are structured as private partnerships, invest in non-traded assets, and specialize in buyout, venture capital, real estate, etc. In these partnerships, investors commit capital ex ante and fund managers call this capital at their own discretion. The total amount of capital committed but uncalled in private equity funds stands at $1 trillion. This makes it, in a sense, a $3 trillion asset class.
Source: https://www.preqin.com/docs/quarterly/PE/Private_Equity_Quarterly_Q3_2012.pdf

the private equity returns. Intuitively, a change in the estimated realized return for a particular quarter affects the NPVs of those funds 'alive' during that quarter. The estimation procedure can be interpreted as finding the set of realized quarterly returns which produces the smallest errors (statistically defined with respect to a distribution of those errors) in the NPV equations. Through the appropriate use of priors and some parameterization of the return process dynamic, the procedure is robust to sparse data and can handle unbalanced panels of contributions and distributions.

We find that the estimated time series of private equity returns are more volatile than standard industry indexes. For example, the volatility of our cash flow-based return time series for buyout funds is 25% per annum compared to 11% for the Cambridge Associates buyout index. Similarly, the NCREIF real estate index has a volatility of only 5%, while our estimated volatility of private real estate funds is 19%, which is close to the volatility of publicly traded REITS. There is a smaller difference in volatilities for venture capital, at 35% for our sample and 27% for the Venture Capital index produced by Cambridge Associates; but the volatility of the latter is largely due to a sharp spike in 1999.

In addition, we find that our private equity return time series exhibit less serial dependence than industry indexes, even after allowing for a persistent component specific to private equity. This result is consistent with strong smoothing biases due to the appraisal process or delayed and partial adjustment to market prices, which often arise in illiquid asset markets (cf. Gelter (1991) and Ross and Zisler (1991)).

The second major contribution of this paper is to introduce and apply a methodology for decomposing the time series of private equity returns into systematic and idiosyncratic components. The systematic component involves factor loadings on standard equity benchmarks including large-cap, small-cap, value, and liquidity factors. In this specification, we find that the most important systematic variable is the market factor, on which the private equity returns have a beta loading significantly greater than one. We estimate the market factor exposure for different types of private equity and find that they vary considerably, with venture funds having a high exposure and real estate funds having a low exposure. Private equity returns exhibit strong factor loadings on small stocks. We also find significant covariation of private equity returns with the

Pástor and Stambaugh (2003) liquidity factor constructed from public equity markets, but there is little evidence that private equity returns covary strongly with a value factor.

We term the remaining idiosyncratic portion of private equity returns the "private equity premium." To the extent that the returns on traded factors can be elsewhere earned by investors, this private equity premium can be interpreted as a time-varying private equity alpha. We find that this premium is highly persistent and exhibits strong cyclicality. The cycles we uncover differ according to fund type and coincide with both anecdotal evidence and the time-series variation in private equity fundraising. For instance, we find that venture capital returns were high in the second half of the 1990s and low in the first half of 2000s, as was fundraising for this asset class. We also find that the buyout premium was low from 1998 to 2002 and then increased sharply from 2003 until 2007, which coincides with the well-known boom in buyout fundraising. In addition, we find a low correlation in the premiums across types of private equity funds, which provide some support for diversification across types of private equity funds.

Our broad finding about the private equity premium is that it contributed positively to total returns in the first half of the sample period and negatively in more recent years. In addition, this time-series variation allows us to identify macroeconomic variables which significantly co-move with private equity returns, including the spread in the free-cash flow yield (EBITDA/Enterprise value) over the junk bond yield which was proposed and studied by Kaplan and Strömberg (2009), and behavioral variables proposed by Baker and Wurgler (2007) indicative of aggregate corporate mispricing. We find evidence consistent with the Kaplan and Strömberg hypothesis that capital market segmentation is a potential driver of the private equity premium, and that the cyclicality of the private equity premium may be related to behavioral frictions.

The rest of this paper is organized as follows. Section 2 describes the methodology. Section 3 details the data. In Section 4 we present the empirical results, focusing on the estimated time-series of private equity returns and how they differ from industry benchmarks. We conclude in Section 5.

2. Methodology

The estimation procedure requires only the cash flows paid and received by investors (called Limited Partners; LPs) in different funds. The funds start and end at different periods in time, which allows us to identify the underlying unobservable returns. We convey the intuition of our approach with a simple example in Section 2.1 and show how it nests existing approaches to estimating illiquid asset returns, particularly in real estate. A complementary approach in Appendix A conveys additional intuition with a numerical example. We present the model in Section 2.2 and the estimation procedure in Section 2.3.

2.1 Simple Example

This section presents a series of realized net present value relations, with their corresponding series of realized returns. We build progressively from the simplest case, publicly traded equities, to the most complex private equity cash flow series. All cases work from restating the standard holding period return formula, which we define as g_t:

$$1+g_{t+1}=\frac{CF_{t+1}+V_{t+1}}{V_t}, \tag{1}$$

where g_t is the return in period *t*, CF_t is the net cash flow generated by the asset during period *t*, with a cash flow received (paid) by the investor signed positively (negatively), and V_t is the asset value at time *t*.

We can rewrite the period return as a realized present value:

$$V_t-\frac{CF_{t+1}+V_{t+1}}{1+g_{t+1}}=0, \tag{2}$$

which states that the holding period return, g_t, can be defined so that the realized net present value of the investment over the period equals zero. Note that g_t is a realized return; it does not represent a forward-looking discount rate.[2] Equation (2) can be iterated forward to state a net present value relation in terms of a time-series of realized returns:

$$V_t-\left(\frac{CF_{t+1}}{1+g_{t+1}}+\frac{CF_{t+2}}{(1+g_{t+1})(1+g_{t+2})}+...\right)=0. \tag{3}$$

[2] We can define a discount rate, δ_t, by the relation $V_t=E_t(CF_{t+1}+V_{t+1})/(1+1+\delta_t)$, where the cash flow expectation and the discount rate are estimated at the beginning of the period. In contrast, the realized return in equation (2) is known only at the end of the period.

Special Case: Equity Returns

The special case when there is only one asset and end-of-period values and cash flows are observed each period corresponds to publicly traded equity. In this case, equation (1) is used directly each period to estimate returns, $\{g_t\}$.

Special Case: Bond Returns

In over-the-counter markets like corporate and municipal bonds, investors can pay markedly different prices for the same security.[3] Suppose we observe the following cash flows on transactions *i* and *j* on the same underlying security:

$$V_{i,t} - \frac{CF_{i,t+1} + V_{i,t+1}}{1+g_{t+1}} = 0$$
$$V_{j,t} - \frac{CF_{j,t+1} + V_{j,t+1}}{1+g_{t+1}} = 0,$$

where we wish to estimate one return, g_{t+1}, for the asset over the period *t* to *t+1*. This is an over-identified system because there are two equations but only one unknown return. We could estimate the unknown return, g_{t+1}, by minimizing an objective function involving the errors of the orthogonality conditions. In the case of minimizing squared error terms, we obtain a least squares estimate using the residuals:

$$\begin{bmatrix} V_{i,t} \\ V_{j,t} \end{bmatrix} - \begin{bmatrix} CF_{i,t} + V_{i,t+1} \\ CF_{j,t} + V_{j,t+1} \end{bmatrix} \delta_{t+1} = 0, \tag{4}$$

where $\delta_{t+1} = 1/(1+g_{t+1})$ and we can recover an estimate of the return by $\hat{g}_{t+1} = 1/\hat{\delta}_{t+1}$, where $\hat{\delta}_{t+1}$ represents the OLS estimate. This is in effect the methodology used to estimate returns from repeat-sales transactions of real estate, to which we now turn.

[3] Green, Hollifield, and Schürhoff (2007), for example, report large dispersion in markups from the reoffering price in municipal bond markets. The reoffering price is often represented to issuers as the price at which bonds are being sold to the public. Retail investors typically pay markups of 3-5% above the reoffering price, but some large institutional investors obtain prices close to the reoffering price at virtually the same time. Because of (infrequent) violations of Regulation NMS in equity markets, stock market investors also may pay different prices for the same security.

Special Case: Real Estate Property Returns

In constructing a real estate return index, we need to build a single return representing many heterogeneous asset transactions. We now change the subscript *i* index to represent different properties (as opposed to the bond market case where the subscript *i* represented different transactions of the same security). Suppose there are three houses, *i, j,* and *k*, with the following cash flows:

$$\begin{array}{llll} V_{i,t} & -\dfrac{CF_{i,t+1}+V_{i,t+1}}{1+g_{t+1}} & & =0 \\ & V_{j,t+1} & -\dfrac{CF_{j,t+2}+V_{j,t+2}}{(1+g_{t+2})} & =0 \\ V_{k,t} & -\dfrac{CF_{k,t+1}}{1+g_{t+1}} & -\dfrac{CF_{k,t+2}+V_{k,t+2}}{(1+g_{t+1})(1+g_{t+2})} & =0, \end{array}$$

where sales price is indicated by an end-of-period V_t and intermediate cash flows are observed on each property. This is the standard set up of the repeat-sales index developed by Bailey, Muth and Nourse (1963) with hedonic and other adjustments introduced by Goetzmann (1992) and Geltener and Goetzmann (2000), among others.

We can rewrite the system in regression notation:

$$\begin{bmatrix} V_{i,t} \\ 0 \\ V_{k,t} \end{bmatrix} - \begin{bmatrix} CF_{i,t+1}+V_{i,t+1} & 0 \\ -V_{j,t+1} & CF_{j,t+2}+V_{j,t+2} \\ CF_{k,t+1} & CF_{k,t+2}+V_{k,t+2} \end{bmatrix} \begin{bmatrix} \delta_{t+1} \\ \delta_{t+2} \end{bmatrix} = 0, \qquad (5)$$

where $\delta_{t+1} = 1/(1+g_{t+1})$ and $\delta_{t+2} = \delta_{t+1}/(1+g_{t+2})$. Equation (5) is also an over-identified system because there are three equations with two unknowns. A common method of estimating $\delta = [\delta_{t+1} \quad \delta_{t+2}]'$ is by OLS, after which estimates of the total returns each period can be obtained using

$$\hat{g}_{t+1} = \frac{1}{\hat{\delta}_{t+1}} - 1 \quad \text{and} \quad \hat{g}_{t+2} = \frac{\hat{\delta}_{t+1}}{\hat{\delta}_{t+2}} - 1.$$

As in most OLS applications, large numbers of observations (transactions) are needed to ensure stable estimates of realized returns in real estate repeat-sales indexes. Note also that round-to-round venture capital returns (at the portfolio company level) fall into this category and modifications of the above setup have been used in this context by Cochrane (2005) and Korteweg and Sorensen (2010).

In the NPV setup in equation (5), the unknown returns $\{g_t\}$ represent common shocks across all properties. That is, the identification assumption to estimate one series of real estate returns is that all heterogeneous assets are subject to the same underlying shocks. We now generalize this approach to private equity.

Private Equity Fund Returns

There is one important difference between private equity fund returns and real estate property returns. Because private equity funds hold multiple underlying portfolio companies, cash flows received by an investor usually cannot be assigned to underlying investments. For example, a series of cash flows like [-100, 200, -100, 400] cannot be decomposed into two transactions [-100, 200, 0, 0] and [0, 0, -100, 400] because some of the final $400 cash flow might be due to investments made with the first $100 paid into the fund. The separation into underlying transactions (or deals) enables the real estate indexes to use standard OLS techniques. This is not possible with private equity cash flows.

Suppose we observe the following three private equity fund cash flows for funds *i, j,* and *k*:

$$\begin{aligned} V_{i,t} - \frac{CF_{i,t+1}}{1+g_{t+1}} + \frac{CF_{i,t+2}}{(1+g_{t+1})(1+g_{t+2})} &= 0 \\ V_{j,t} - \frac{CF_{j,t+1}}{1+g_{t+1}} + \frac{CF_{j,t+2}}{(1+g_{t+1})(1+g_{t+2})} &= 0 \\ V_{k,t} - \frac{CF_{k,t+1}}{1+g_{t+1}} + \frac{CF_{k,t+2}}{(1+g_{t+1})(1+g_{t+2})} &= 0. \end{aligned} \tag{6}$$

Like the special cases of bonds and real estate, this is an over-identified system with two unknowns, g_{t+1} and g_{t+2}, and three equations representing the zero NPV condition,

$$PV(I) = PV(D), \tag{7}$$

where $I = \{I_{i,t}\}$ denotes the series of investments paid into the fund by the LP and $D = \{D_{i,t}\}$ the distributions received by the LP. Written in this fashion, it is clear that our approach is a generalization of a repeat-sales real estate index. The NPV condition in equation (7) holds on average, across funds, just as in the repeat-sales real estate context the NPV condition holds across properties.

Our strategy is to filter the private equity returns, $\{g_t\}$, such that the NPV condition is satisfied across funds and across time. We treat the returns as latent parameters and obtain best estimates by over-identifying NPV relations. The NPV equation (7) represents an observation equation. (Below, we modify the NPV condition slightly in our empirical work to express it as a log ratio.) To complete the filtering problem, we need to specify a state equation, also called an equation of motion, for the latent returns. Common to the special cases of bond and real estate indexes covered earlier is that they rely on the assumption that each individual asset's (or fund's) return is a function of a common factor, or set of factors, plus uncorrelated observation error in the NPV equation. The NPV observation errors average to zero — which is the identification assumption required to estimate the latent returns. Like any problem with over-identifying restrictions, our technique will be accurate only for large numbers of funds (observations) relative to unknown common returns (parameters). This is the case in our sample, confirmed in simulations, and our estimation methodology discussed below incorporates sampling error into (posterior) distributions of our parameters.

2.2 Model

The key assumption of the model is that the cash flows associated with any investment market are generated by a time-varying portfolio of assets that have unobserved but continuous latent values. While the assets (funds) are heterogeneous, we assume their returns are a linear function of an underlying systematic factor structure. Thus, if the latent asset values were observable, some portion of their return variance could be explained by common factors using standard regression methods. In addition, we allow (and test) for asset class-specific latent factors.

Let g_t denote the return of private equity at time *t*, and g_t^e the excess return relative to the risk-free rate r_t^f.[4] We specify:

$$g_t = g_t^e + r_t^f. \tag{8}$$

The underlying return process, g_t, cannot be directly observed in the private equity data. We specify that private equity returns are driven by a set of *J* common tradable factors, $F_t = [F_{1,t},...,F_{J,t}]$, which are observable in public markets. We consider factors like the equity market, the Fama and French (1993) factors, and the liquidity factor of Pástor and Stambaugh (2003). In addition, we allow for an asset class-specific latent factor, f_t. This potentially makes private equity non-redundant in the space of tradable assets.

Combining the two sources of return, we consider the following model for the private equity risk premium, g_t^e :

$$g_t^e = \alpha + \beta' F_t + f_t, \tag{9}$$

where β are the loadings (betas) on the common factors, F_t.[5] As is standard in factor models, we assume that f_t is orthogonal to the traded factors, F_t. We specify that the private equity return component, f_t, follows an AR(1) process:

$$f_t = \phi f_{t-1} + \sigma_f \varepsilon_t. \tag{10}$$

We specify that f_t is mean zero so that the α in equation (3) reflects the average level of private equity returns in excess of its systematic (and liquid) component of the private equity return. The error, ε_t, is drawn from an i.i.d. standard normal distribution. Equations (9) and (10) constitute the state equation dynamics of the filtering problem. They can be restated as an AR(1) for the private-

[4] Note here that as in our examples in Section 2.1, g_t is not an estimate of the *ex-ante* expected rate of return to a given investment. We do not model an expectations process but estimate *ex-post* realized returns.

[5] It is equivalent to model the total private equity return, g_t, as opposed to the private equity return in excess of the risk-free rate, g_t^e. We choose the latter because we are interested in the properties of the risk premium.

equity specific return, f_t, with exogenous variables represented by the systematic components, $\beta' F_t$.

The latent factor process, f_t, can be viewed as the idiosyncratic component of private equity returns. Usually, traditional factor models for liquid asset returns specify that both systematic and idiosyncratic returns are i.i.d. This is driven by the assumption of market efficiency; predictable returns in a liquid market would be rapidly arbitraged away. In our specification, the f_t process is not exposed to the forces of arbitrage because, by design, it is not tradable and is orthogonal to factors in the public markets. Instead, it is intended to capture such features as persistent manager skill, the inter-temporal variation in good investment opportunities or the trends in performance due to non-constant returns to scale.[6] The specification allows us to test for trends in the private-equity-specific factor by testing whether $\phi = 0$ and also to more formally address the intuition that certain classes of private equity, like venture capital or buyouts, have different return premium properties after controlling for market effects.

The model nests the following special cases:

1. Constant expected returns, when $\beta = 0$, $\phi = 0$, and $\sigma_f = 0$;
2. CAPM, when $\alpha = 0$, $\phi = 0$, and $\sigma_f = 0$, and F_t contains only market excess returns as the systematic factor;
3. Constant excess returns above the CAPM model can be captured by $\alpha \neq 0$ when $\phi = 0$, and $\sigma_f = 0$, and F_t contains only market excess returns;
4. Private equity returns unrelated to public, systematic factors, when $\beta = 0$; and
5. The performance of private equity is explained entirely by liquid market returns, when $\sigma_f = 0$.

The full model allows for a rich set of dynamics for private equity returns. In the full model, private equity returns are related to systematic factors ($\beta \neq 0$) and they have characteristics

[6] Imperfect information environments combined with the inability to immediately deploy capital can lead to large persistence in returns (see, for example, Abreu and Brunnermeier (2003), Brunnermeier (2005), and Duffie (2010)).

unique to private equity, which may be persistent ($\phi \neq 0$, $\sigma_f \neq 0$). Private equity may offer risk-adjusted returns in excess of what is available in traded markets ($\alpha \neq 0$).

Our task is to estimate the latent factor, f_t, with dynamics given in equation (10). If the private equity returns were directly observable as they are for listed equity returns, then the system can be simplified to regular OLS with autocorrelated residual terms. If the private equity returns can be decomposed into individual deals, then we could use similar estimation techniques to those used to construct real estate indexes. The private equity returns are not directly observable; the non-observability can be thought of as a censoring process which renders the estimation a signal-extraction problem conditional on censoring. We discuss a Bayesian method of estimation to filter the returns.

2.3 Estimation

We observe cash flows to LPs across *N* private equity funds indexed by *i*. The cash flows include investments I_{it} paid into fund *i* at time *t* and distributions D_{it} received from fund *i* at time *t*. If the model is correctly specified, the cash flows satisfy a NPV condition of

$$E\left[\sum_t I_{it}\delta_{it}\right] = E\left[\sum_t D_{it}\delta_{it}\right], \tag{11}$$

where δ_{it} is defined recursively as:

$$\delta_{it} = \delta_{i,t-1}(1+g_t)^{-1},$$

with $\delta_{i,\tau} = 1$ at the inception of fund *i* when $t = \tau$ and g_t is the private equity return given in equation (8). We take each period to be one quarter in our estimation.

We write equation (11) with a present value operator, *PV(.)*, so it is equivalent to

$$PV(D) = PV(I).$$

We do not work directly with equation (11). Instead, we follow Kaplan and Schoar (2005) and use the public market equivalent (PME) ratio,

$$\frac{PV(D)}{PV(I)} = 1,$$

where our ratio of present value of distributions to the present value of investments is defined using a time-varying series of returns with systematic and private-equity specific components (equations (9) and (10)). Using a ratio has the advantage that it is robust to different periods used to compute the present values. That is, if the valuation date is taken to be the first date of the sample, then present values of cash flows for funds formed at the end of the sample are smaller than present values of cash flows for funds started at the beginning of the sample. Taking a ratio removes these timing effects. We make one more transformation to the PME by taking a log transformation, in which outliers have less effect:

$$\ln \frac{PV(\mathrm{D})}{PV(I)} = 0. \tag{12}$$

Equation (12) holds approximately across funds, which are distributed across time. Equation (12) represents the observation equation. In our Bayesian estimation, we specify that the observation equation is the ratio of the present value of investments to the present value of distributions. We assume the observation error is log normally distributed:

$$\ln \frac{PV(I)}{PV(D)} \sim N(-\tfrac{1}{2}\sigma^2, \sigma^2). \tag{13}$$

The mean of the log distribution is set at $-\tfrac{1}{2}\sigma^2$ so that the raw ratio PV of investments to the PV of distributions is centered at one. That is, this assumes that the log ratio has zero mean, and takes into account the Jensen's inequality induced by taking the log transformation.

We estimate the model using a Bayesian MCMC procedure described in Appendix B. We use equation (13) as the likelihood function and treat the unobserved returns as parameters to be estimated (which is called "data augmentation"), along with the other parameters of the data generating process, $\theta = (\alpha, \beta, \phi, \sigma_g, \sigma)$.

In Appendix C we report sensitivity analysis of the procedure to a range of assumptions, including robustness to different priors. We also show the small sample properties of the estimated parameters using Monte Carlo simulations. The procedure is acceptable for as few as 200 funds for our sample length and accurate for the number of funds we employ in our empirical work.

This estimation procedure is similar to that of Cochrane (2005), Korteweg and Sorensen (2010), Driessen, Lin, and Phalippou (2012), Franzoni, Nowak, and Phalippou (2012), and Korteweg and Nagel (2013). The key difference with respect to their work is that, in addition to estimating factor loadings, we estimate a quarterly time series of returns for private equity—both systematic and idiosyncratic—from investor cash flows, while the previous papers only estimate average private equity returns and risk exposures.

There are several caveats to our approach. First, a natural interpretation of the index is that it is the net return to investing in each of the private equity funds in the database.[7] This interpretation implicitly assumes that the returned capital D_t in any given period is immediately re-investable in all existing funds as opposed to only new funds. This is typically not the case. This assumption, however, only affects interpretation of the premium factor— the latent factor series f_t component of the total return index. The passive component due to $\beta' F_t$ comprises only marketable factors, in which investors can re-invest or rebalance.

A more subtle point that is generally true in all manager performance studies which rely on estimated linear factor exposures is that, by presuming that the passive component is accessible to an investor, we are also implicitly assuming that leverage may be used to achieve a factor exposure greater than one. As we show below, a significant amount of the variation in the g_t^e series is explained by large exposures to public equity factors. Private equity may offer a means to relax borrowing constraints and this convenience may be priced (cf. Frazzini and Pedersen (2010)). We also use long-short factors, and implicitly assume that short-selling is feasible and costless in replicating the performance of such factors.

Third, our procedure solves for the best fit of the private equity returns given fund cash flows. We are not solving for expected returns, but for estimates of realized private equity returns. We take the cash flows as given to solve for the realized returns (see equation (3)). To obtain estimates of forward-looking discount rates, we would need to embed an expectation process into a valuation

[7] Value-weighting does not affect results consistent with Harris, Jenkinson, and Kaplan (2013) results.

model and tie the discount rates with estimates of our realized returns. This is an interesting avenue for future research.[8]

Finally, as we infer private equity returns from LP cash flows, we require high quality data on cash flows. In theory, we would take funds that have terminated so that complete histories of cash flows are observable. In our empirical work, we relax this stringent constraint to take funds with a small portion of unrealized investments in a way we make more precise below. Part of our contribution is methodological, and our procedure can be used on any suitable dataset. An advantage of our estimation technique is that we can estimate private equity returns on data with very sparse cash flows, say a particular institutional investor track record, by using priors set from estimations on more extensive data sets which collate information across many investors.

3. Data

We use the cash flow dataset of Preqin purchased in March 2012; data are as of June 2011. Preqin collects the quarterly aggregated investments, distributions, and Net Asset Values (NAVs) made by private equity funds as recorded by U.S. pension funds. Preqin collects this data from public reports and routine Freedom of Information Act requests.

The Preqin sample has some desirable characteristics and some limitations. Cash flows are likely to be accurately reported; pension funds would face serious sanctions if they deliberately misreport or only selectively report returns. In addition, data on a given fund can be cross-checked between the different pension funds which invest in it. One of the potential limitations is that, by conditioning on pension fund investments, we may not be picking up investments made by other institutional groups such as college endowments.[9]

Preqin data have similar characteristics, including similar average and median returns, as those reported in other studies such as those of Robinson and Sensoy (2011) and Harris, Jenkinson, and

[8] Recent work by Sorensen and Jagannathan (2013) and Korteweg and Nagel (2013) explore the relations between PMEs and discount rates. The problem of correlated forward-looking discount rates and cash flows is also considered by Brennan (1997) and Ang and Liu (2004). We use only realized cash flows in our net present value relations to estimate realized cash flows.

[9] Lerner, Schoar, and Wongsunwai (2007) show that endowments have earned higher returns than other investors in private equity investments. Sensoy, Wang, and Weisbach (2013) show that the better performance of endowments is concentrated over the earlier part of the sample and in early stage venture capital.

Kaplan (2013) (cf. Phalippou (2013)).[10]

To assess the risk profile of funds, we need to observe the cash flows of a sufficient number of funds at any point in time. Because the number of funds in the dataset increases rapidly over the time period, we start in a year with at least five funds. This is 1992 for both venture capital and buyout funds. Ideally, we would include all funds from that point on. This approach would, however, assume that the reported NAVs are market values.

Funds serving fiduciaries such as pension funds report their audited calculations of portfolio value (NAV) every year. In the U.S., FASB 157 requires fund assets to be fair market-valued, however the private nature of these investments and varying methodologies for evaluation leaves significant uncertainty. Ultimately reported fund NAVs represent the opinion of the fund manager about the assets.[11] It may therefore be problematic to take these NAVs at face value when trying to assess the underlying 'true' returns.

One solution to this problem would be to include only funds that have passed their eight or tenth anniversary in order to both minimize the impact of NAVs and guarantee a representative sample for each of the included years. But doing so would result in the frequency of cash flows significantly decreasing in the later part of the sample. We thus include all (post-1992) funds as long as they have a relatively low NAV (50% of fund size or less). We exclude funds that do not have at least one distribution of at least 10% of fund size (which can be a cash distribution or the final NAV). These criteria mean that we keep a few funds each year in the sample all the way to 2008. This is essential in order to estimate the quarterly return of private equity. We label this sub-sample of funds the "quasi-liquidated" sub-sample.

When reporting abnormal performance, we will show the results for both the quasi-liquidated sample of funds and the full sample of funds. The estimation of risk loadings, however, necessitates the use of the quasi-liquidated sample. This constraint has some potential effect if, for example, younger funds have different characteristics, or the management style and factor exposures for funds launched in the mid-2000s was not representative, then our estimates will be weighted away from these and towards more mature funds (cf. Barrot (2012)).

[10] An additional and unique advantage of Preqin data is that they are publicly available.

[11] The process typically involves a valuation committee and for audited funds, the additional valuation assumptions made by the auditing firm. Jenkinson, Sousa, and Stucke (2013) and Brown, Gredil, and Kaplan (2013) find that fund valuations are conservative except when follow-on funds are raised.

Table 1 reports descriptive statistics for our data.[12] Panel A shows the number of funds entering the sample in each year. The Preqin sample appears to be similar to that of Harris, Jenkinson, and Kaplan (2013) in terms of size and years covered. Panel B compares the number of observations of the full sample and the quasi liquidated sample. It also breaks down the statistics of the quasi-liquidated sample per fund categories.

The venture capital category includes funds classified as general venture capital, balanced, seeds, start-up, early stage, expansion and late stage. The buyout category includes funds classified as buyout and turnaround. The credit category includes funds classified as Mezzanine and Distressed debt. The real estate category includes funds classified as such. Note that the number of real estate and debt funds is relatively small.

< Table 1 >

4. Empirical Results

4.1 Time-series estimates of private equity total returns and premiums

In this subsection, we discuss and plot the time series of our estimated private equity total returns and premiums. (Appendices B and C offer further details on the methodology, the choice of priors, and robustness checks.)

We apply the methodology to the sample of 630 quasi-liquidated funds described in Table 1. Figure 1 plots the cumulated total return index, g_t, obtained with a four-factor model for systematic risk. The four factors are the market portfolio, the Fama and French (1993) small-large and value-growth factors, and the Pástor and Stambaugh (2003) liquidity factor that goes long illiquid stocks and shorts more liquid stocks. Figure 1 plots our estimated private equity returns expressed as an index, which starts with the value 1.0 in March 1993.

< Figure 1 >

[12] Selected funds are "closed" or "liquidated," and based in the United States. We exclude GCP California Fund, a partnership between Leonard Green and CalPERS to invest in "California-related industries and underserved markets."

Figure 1 compares the return of our private equity index, g_t, to the returns of Vanguard S&P 500 index fund. Consistent with the findings of Harris, Jenkinson, and Kaplan (2013), and Robinson and Sensoy (2011), private equity beats the index portfolios over the time period 1993 to 2010. Part of the private equity performance, however, may be replicable using some passive factor exposures. Indeed, Figure 1 shows that there is significant co-movement between the private equity total returns and the Vanguard S&P 500 index funds.

Figure 2 plots the total return, g_t, the return of the passive factor exposures, $\beta' F_t$, and the spread between the two which is the private equity return premium, f_t. The latter can also be interpreted as private equity's time-varying alpha. Over the sample, the cumulated private equity premium, f_t, is zero, so private equity has had an excess return of zero (when using the four factor asset pricing model of Pástor and Stambaugh (2003); see below). Nevertheless, over some time periods, there is a significant spread between the total return and the systematic return, $\beta' F_t$, indicating that there is a non-negligible idiosyncratic component of private equity returns.

< Figure 2 >

Figure 3 provides more detail about the timing of the premium, f_t, and shows the quarterly returns to the private equity premium as bars in each period (so they are not compounded like Figures 1 and 2). Although the overall average is zero, there is significant time variation. The premium is large and positive in the second half of the 1990s, approximately zero for the first half of the 2000s and then negative from 2006 including during the financial crisis. The pattern suggests that the private equity premium is cyclical, with as much as 10 years from peak to trough.

< Figure 3 >

The cyclical pattern of the private equity premium is most interesting when broken down into sub-asset classes. In Figure 4, we plot the premiums for our four subsets: buyouts, venture capital, real estate, and high yield (see Table 1, Panel B). The premiums for each asset types behave quite differently. Buyout funds experienced premiums of more than 1% in 1995-1996 and over 2005-2007, consistent with conventional beliefs as reflected in industry reports and press coverage. Venture capital funds had one very large peak of more than 5% in 1999-2000, coinciding with high valuations of internet companies during this time. Real estate peaked at more than 3% in

2006 (notably before available appraisal-based commercial property indexes captured a downturn). The premium to investing in credit funds had two peaks of 2%—one in 1995 and another in 2009, displaying little covariation with the premium to buyout funds.

< Figure 4 >

These imperfect co-movements suggest that the cycles to venture capital and real estate differ from those of buyout and credit funds, and that there are benefits to diversifying across private equity investment classes—for example, credit premiums were positive when venture capital premiums were negative. More generally the evidence suggests that, even conditional on differing exposures for systematic factors, private equity premiums in different asset classes are exposed to different underlying factors unrelated to publicly traded securities.

< Table 2 >

4.2 Factor exposures and private equity premium

The private equity premium displayed in Figures 1 to 4 is computed using a four-factor asset pricing model with market, size, value, and liquidity factors. We find that the estimated overall private equity premiums, g_t^e, are relatively insensitive to the assumed model for systematic risk.[13] The systematic factors, by construction, do affect the estimates of the private equity premium, f_t. In Table 2, we report parameter estimates of the factor loadings, β, the α coefficients, and the persistence of the private equity premiums, ϕ, with different asset pricing factor models. The table reports posterior means and standard deviations of the parameters.

We take models with one, three, and four systematic factors. The one factor model is the CAPM; the three-factor model is from Fama and French (1993) which adds SMB and HML factors, and the four-factor model is that of Pástor and Stambaugh (2003) which adds a liquidity factor. For robustness, we also report models for which we use the CRSP equally-weighted (EW) index instead of the CRSP value-weighted index as a measure of market returns. This is equivalent to the assumption that private equity funds acquire companies that are drawn from a pool resembling the

[13] This shows the robustness of our approach. We estimate (independently) the time-series of returns with six different models (see below and Table 2); the time-series pairwise correlation is lowest for the series derived using models 1 and 4 at 94.2% and the highest correlation is at 99.2% (between model 1 and 2).

CRSP sample; i.e. they are as likely to acquire a firm from the bottom decile as from the top decile of capitalization. This assumption is useful because the typical company purchased by a private equity fund is small compared to the firms in the S&P 500. The drawback is that the equal-weighted CRSP index is not investable—at least not by very large institutions.

Table 2 shows that the CAPM estimate of the beta of private equity is 1.41, which is almost unchanged using an EW market index. The estimates for the four-factor loadings on market, size, value, and liquidity factors are 1.49 for the market excess return, 0.41 for SMB, 0.03 for HML, and 0.36 for the liquidity factor. In the four-factor model, the posterior means of the market and SMB loadings are more than two posterior standard deviations away from zero, but this is not the case for the value and liquidity factor loadings. Nevertheless, the economic magnitude of 0.36 for the liquidity factor beta is relatively large.

We report two sets of alphas. The 'in-sample alpha' is the premium computed, given the estimated set of risk loadings, using the sample of funds on which the model was estimated (i.e. the 630 quasi-liquidated sample). The 'full-sample alpha' is the premium estimated using the full sample of 1,222 funds given the estimated set of risk loadings (from the quasi-liquidated sample). The two sets of alpha generally agree with each other, which indicates that our selection procedure does not lead to a bias towards better or worse performing funds. All alpha estimates in the table are annualized.

Consistent with Robinson and Sensoy (2011), Harris, Jenkinson, and Kaplan (2013)), and others, we find that the alphas with respect to the S&P 500 and a three-factor model accounting for size and value effects are positive at 0.05 and 0.04, respectively, using in-sample estimates. Adding the liquidity factor drives the alpha to zero. Also consistent with the literature, substituting an index which weights small companies heavily—the equal-weighted CRSP index—reduces the alpha estimates dramatically. The alpha in the EW CAPM specification is negative at -0.04, and this is largely unchanged when the SMB, HML, and liquidity factors are added.

In our model, the private equity premium is a non-arbitrageable factor which is auto-correlated. Table 2 reports the persistence in the private equity premium measured at the quarterly horizon. Depending on the specification, this value ranges from 0.40 to 0.47. In all cases, the estimates are significantly different from zero. The autocorrelation estimates are a potentially useful measure because a variable with a significant autocorrelation coefficient is potentially forecastable. Our auto-correlation estimates indicate there is fairly strong persistence in the aggregate private equity premium. Value-enhancing shocks have a half-life of about one-quarter. A year later, the effect is 1/16 of its original intensity but still contributes to the net return. For example, Figure 3 shows an upward trend at the end of the time period, indicative of a potential reversion to a positive value in the future.

4.3 Factor exposures and private equity premium broken down by fund type

Table 3 reports estimations on the different private equity sub-classes. Venture capital funds have the highest estimated CAPM beta, followed by buyout, real estate, and credit funds. The venture capital market beta is 1.67 in Panel A, which is a slight decrease from the previous estimates in the literature (see Appendix B and Appendix Table A.1). Venture capital has a significant negative loading on the Fama-French value factor, which is what we would expect from a strategy of buying high growth companies. The robustness specifications for venture capital use an equal-weighted portfolio of Nasdaq stocks instead of the equal-weighted CRSP. As expected, this change decreases the market beta and drives alpha to (close to) zero.

< Table 3 >

Two remarks are worth making on the venture capital alpha. First, there is a negative loading on the value premium. Venture capital strategies appear to be loading up on growth stocks, which have low average returns. Thus, the total returns of venture capital are relatively low, but the alpha is boosted up by the negative loading on the value factor. The second remark is that the value-weighted stock-market index used in the CAPM has low returns over our sample period, which sets a low bar in terms of performance. When the index is changed to the EW Nasdaq stocks, which have delivered better performance, then venture capital exhibits a negative alpha.

Panel B of Table 3 reports results for the largest fund type in terms of asset under management, buyout funds. The buyout fund market beta for the standard specification is around 1.3, similar to previous estimates in the literature (see Appendix Table 1). The coefficients on value and liquidity factors are positive. The single-factor CAPM alpha is 0.05 in the estimation sample and 0.04 in the full Preqin sample. The alpha drops to 0.03, but remains significant in the estimation sample in the standard three-factor Fama-French specification. The inclusion of the Pástor-Stambaugh liquidity factor, however, changes the sign of the alpha. This can be interpreted as buyout funds harvesting a liquidity risk premium in the Pástor-Stambaugh sense that buyout funds have exposure to a liquidity factor constructed from publicly traded equities (cf. Franzoni, Nowak, and Phalippou (2012)).

Although we have relatively few real estate funds, results in Panel C show that the estimation procedure generates intuitively reasonable results. Real estate market betas vary from 0.74 to 0.79, consistent with previous estimates of a beta less than 1.0 for real property. The beta on the REIT index is less than one as well, ranging from 0.49 to 0.75. Most specifications show a negative alpha for real estate funds.

For credit funds in Panel D, we estimate a CAPM beta of 0.66, and find that all three of the factor loadings on the Fama-French model are significant: credit has a beta greater than one for both size and value factors. As with real estate funds, most specifications show a negative alpha for these funds.

Note that we have estimated the exposure of private equity investment to factors commonly used in the analysis of equity returns; and we have modeled a private equity premium as an auto-correlated latent factor. In several specifications we reject the null that private equity assets are redundant with respect to the standard Fama-French and Pástor-Stambaugh equity factors. These systematic factors capture a large part of (and in some cases fully explain) the total returns to investing in private equity. This, however, does not necessarily imply that there is no value to private equity because none of these equity factors returns are available without incurring transaction costs. An open question is whether an investor can cheaply access the premiums of the tradable factors passively, or whether private equity investments are a more efficient way to access these factor premiums. This would involve an analysis of transactions costs (and investor size) that is beyond the scope of this paper.

Finally, it is interesting to note that the persistence in the premium is strongest in venture capital and real estate, as reported in Table 3. It is less strong in buyout and even lower in credit funds. These results are consistent with the idea that persistence is driven by non-scalability. Certainly venture capital and real estate are the most difficult investments to scale. The buyout and credit strategies have more capacity.

4.4 Comparison to industry indexes

One practical advantage of our cash flow-based index is that it seeks to attribute returns to the time period in which they occur. In practice, there are some industry indexes with the same objective but they use estimated asset values. These estimated values are potentially subject to inertia—for example anchoring on prior appraisal values. The econometrics of appraisal-based indexes have been well-studied for commercial real estate (cf. Geltner (1991)). Among other things, they have volatilities which under-estimate true volatilities and lag market values. In this section we examine the relationship of our cash-flow based index to industry indexes.

In Table 4, we label our estimated index the "CF PE index," which is produced using the four-factor model for systematic risk (see also Figures 1-3). The table shows the annualized mean, standard deviation, inter-quartile range and autocorrelation coefficient for some standard industry indexes and for our cash flow-based indexes. For buyout and venture capital we use the Cambridge Associates indexes; they are the most prominent ones in practice. For real estate we use the NCREIF index. This is the industry-standard appraisal-based index of unlevered property returns, which is computed using data reported by institutional investors to the National Council for Real Estate Investment Fiduciaries. All the mean and volatility estimates in Table 4 are annualized.

< Table 4 >

Table 4 shows that the cash flow-based indexes are more volatile than the industry indexes. The difference is particularly dramatic for real estate. We estimate a volatility of 19% per annum for real estate, compared to the NCREIF index volatility of 5%. The 19% is closer to the volatility of publicly traded real estate portfolios, REITS. This suggests that our estimated index may provide a more realistic estimate of real estate portfolio risk for investment managers.

For buyouts, the volatility of our cash flow-based return time series is more than twice as high as that of Cambridge Associates (25% compared to 11%). There is a smaller difference in volatilities for venture capital, at 35% for our sample and 27% for the venture capital index produced by Cambridge Associates; but the latter is solely driven by a sharp spike in 1999. These results indicate that existing private equity return time series exhibit smoothing biases likely due to the appraisal process and the fact that valuations of illiquid assets may only partially adjust to market prices. In addition, we find that our private equity return time series exhibit much less serial dependence, if any, than industry indexes. Of particular interest in this regard is the volatility relative to the LPX 50. Europe's key insurance companies' regulator, the Solvency II Committee, has been criticized for using this index as a basis for value-at-risk parameters rather than less-volatile appraisal-based indices in their calculations of private equity capital requirements. Our estimates derived from private equity funds cash flows lie between these two and is generally closer to those of the LPX 50.

4.5 Vintage year comparisons

Industry participants and academic researchers have traditionally used vintage year IRRs, multiples of returned cash to investment, and PMEs to identify the cyclical behavior of private equity. Table 5 examines the relationship between these measures, our cash flow private equity indexes, and flows into private equity (changes in the number of funds and amount of capital entering the industry).

< Table 5 >

Vintage year returns are computed by first aggregating all the cash flows of the funds from a given vintage year, and then computing the IRR, multiple and public market equivalent (PME) of that aggregated cash flow stream. We compute the PME with the returns derived from the factor loadings estimated with the four-factor model. In this respect it differs from a standard PME calculation that uses only S&P 500 returns to discount cash-flows. We do this to highlight the difference between our private equity returns with the PME on the basis of how the measures are computed, rather than having different returns in each measure. The sample is all the Preqin quasi-liquidated private equity funds.

The IRR, multiple, and PME measures display some common trends (Panel A of Table 5). They all start to decrease from 1994 and reach a low in 1999. Then, they start to increase. After the 2005 vintage year, the measures are also low. These patterns are counterintuitive because 1999 was anecdotally the best year ever for venture capital, as was the 2003-2007 time periods for buyout funds. In contrast, the returns in our cash flow-based index show 1999 and 2003 as high return years, while 2008 was the worst year. In other words, by "unbundling" vintage year returns we are able to more accurately identify good and bad years for private equity.

To further demonstrate that our contrasting result is mainly due to unbundling, we use our index to simulate vintage year returns by constructing a forward moving average of our index return. This forward moving average of the cash flow-based private equity index, g_t, is the geometric yearly average return calculated from year t+1 to t+5. The four-year horizon reflects the typical duration of a private equity investment.[14] Our forward measure has a correlation of 0.87 with the vintage year IRR, which shows that our index is mainly an unbundled version of what is done in practice and in the literature. This unbundling is important because it allows the identification of the performance cycles. These cycles cannot be identified with vintage years IRR (e.g. our yearly index exhibits a correlation of -0.05 with the vintage year IRR).

To assess whether our index captures actual performance cycles, we study the correlation between capital flows in the private equity fund industry and different past performance measures. Results in Panel B of Table 5 show that our index correlates highly with capital flows.[15] The correlation is as high as 90% with year-on-year growth in the number of funds. In contrast, the vintage-based performance measures have correlations close to zero with industry growth.

Panel C of Table 5 breaks down results for venture capital and buyout fund sub-samples and compares them with existing industry annual returns. The Cambridge Associate indexes track our indices fairly closely although significant differences occur for the venture capital series in 1999 and 2000. Our cash flow-based venture capital index is 114% in 1999 (while the Cambridge Associates index reaches 293%) and is -29% in 2000, while the downturn manifested itself in the Cambridge Associates indexes only in the following year.

Panel D documents the relationship between capital flows and different return series. We regress the measures of industry growth on the various industry return measures as well as our cash flow-based index. With only 16 years of data, the regression should be carefully interpreted and is only suggestive evidence of relative significance. With this caveat, we note that the coefficient on our index based private equity index and capital flows is positive and strongly statistically significant in all cases. In contrast the coefficient on both the vintage-based return measures and Cambridge Associates return series are only significant for the buyout sub-sample.

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[14] See Lopez-de-Silanes, Phalippou, and Gottschalg (2013). Note: The last year we use is 2009 and the forward moving average is not computed for 2008 as it is not meaningful.

[15] Growth in the number of funds and in capital raised have a 92% correlation with one another. The metric using number of funds is less sensitive to one large fund missing. Data source is the full Preqin sample (Table 1).

4.6 Private equity return cycles

Table 6 uses our cash flow-based private equity premium indexes—the alphas, f_t—to identify peaks and troughs in returns specific to private equity. The time-varying alpha is the 'pure' private equity return component. Table 6 shows the start and end of private equity cycles broken down by fund types. In Panels A and B, a boom period is one that has more than two quarters in a row with 'time-varying alpha' *above* one standard deviation above the mean. A bust period is one that has more than two quarters in a row with the 'time-varying alpha' *below* one standard deviation below the mean. We thus identify cycles in a similar way various economic institutions, like the NBER, define economic cycles.

< Table 6 >

In Panel A of Table 6, the alpha is derived from the four-factor model of Pástor and Stambaugh (2003). In Panel B, alpha is computed using the CAPM model. In Panel C, the definition of a boom or bust is the same as the other panels, except that we use the Cambridge Associates NAV-based quarterly return series.

Table 6 clearly identifies the venture capital boom of late 1990s, along with the buyout boom of the mid-2000s. The real estate boom in the mid-2000s coincides with the buyout boom. The real estate bust around the crisis can also be seen in the data. These results are similar if we use a single-factor CAPM model to derive alpha or a four-factor model (Panel B). In contrast, cycles identified from the Cambridge Associates returns do not exhibit much boom-bust dynamics, if at all (Panel C).

It is also interesting to compare the time-series of our returns with the aggregated cash-flow liquidity/return measure devised by Robinson and Sensoy (2011). Following them, we compute the ratio of total cash distribution to committed capital for each quarter and each fund. We regress this variable onto three sets of fixed effects (based on fund age, uncalled capital and time). The R^2 s are similar to Robinson and Sensoy (2011). The residuals from this regression—which proxy for abnormal liquidity/return in a given quarter—are then aggregated across funds to obtain a unique time-series. Our premium time-series, f, has a coefficient of correlation of approximately 50% with the Robinson-Sensoy series, which is statistically significant at the 1% level.

4.7 Test of the market segmentation hypothesis

The cyclicality of private equity represents a challenge to private equity investors who are faced with the decision to time their investments, or to maintain a continuous commitment to the asset class and manage expectations about short-term performance. This pattern is also difficult to explain in a standard economic framework. Kaplan and Strömberg (2009) introduce a novel theory of boom and bust cycles in private equity. They propose that buyout funds exploit segmentation between the debt and equity markets.[16] Kaplan and Strömberg (2009) extend the insights of the behavioral corporate finance literature to explain this correlation. In particular, Baker, Greenwood, and Wurgler (2003), and Baker and Wurgler (2000) present evidence that corporations choose financing channels based on the relative capital market demand for equity vs. debt. Kaplan and Strömberg (2009) argue that the ultimate source of the variation in relative demand for debt vs. equity is market sentiment, and they report suggestive evidence of this by charting a variable defined as the EBITDA/enterprise value minus the high yield spread. When this variable is high, private equity buyouts should be relatively profitable because the cost of debt financing in low compared to the return on asset.

Our cash flow-based private equity indexes allow us to empirically test the behavioral market segmentation hypothesis. In particular, we test whether private equity is profitable when the Kaplan-Stromberg asset-debt yield spread is higher. Table 7 reports the results of regressions in which our private equity cash flow returns are dependent variables and the independent variables include the asset-debt yield spread, the Baker-Wurgler sentiment index and a set of macro-economic variables that capture credit conditions (the default spread, which is the difference in yields on AAA and BAA AAA rated debt, and a survey of loan officers) and the health of the economy (growth in industrial production, inflation, and the change in the VIX index).[17]

< Table 7 >

Our specification jointly tests the theory that market sentiment provides the opportunity for private equity managers to create value, and that the source of that value is the asset-debt yield spread. If market sentiment is a significant determinant of the private equity return premium, we expect a

[16] Prior researchers have noted the connection between low interest rates and buyout fundraising, such as Ljungqvist, Richardson, and Wolfenzon (2008), Demiroglu and James (2010), Ivashina and Kovner (2011), Axelson et al. (2013).
[17] We use Newey and West (1987) standard errors and have 72 quarters of observations.

positive sign on the sentiment index and a negative sign on the change in the VIX. In our specification the sign on the default spread may go either way since, by construction, it is negatively correlated to the asset-debt spread.

Chen, Roll, and Ross (1986) argue that the default spread captures investor confidence about the economy. Another measure of confidence is the survey of loan officers. This and industrial production growth should be positively associated with the aggregate cash flow private equity indexes since buyout funds are, in effect, a levered exposure to the corporate sector of the economy. Innovations in these macroeconomic variables are rapidly priced in public capital markets, but not necessarily incorporated in private capital markets. Likewise, inflation is likely to have negative effects on nominal cash flow measures. The key prediction is that the asset-debt spread should be a positive determinant of the private equity return premium.

Panel A of Table 7 reports results for three specifications using both the aggregate cash flow-based private equity indices (g_t) and the private equity premium series (f_t). The specifications include either the asset-debt spread, industrial production or both. The first three columns of coefficients show that the index is significantly positively related to the asset-debt spread and the sentiment index, consistent with the Kaplan-Strömberg hypothesis. It is negatively related to the VIX inflation and the default spread and positively correlated to production and the survey of loan officers. The coefficients on production and inflation are insignificantly different from zero. These results are consistent with the hypothesis that private equity does well when the economy does well and when sentiment about the economy is positive. The second set of regressions repeats the estimate using the private equity premium. In this specification, only two variables are significant: growth in industrial production and the VIX.

One problem with interpreting the results based on the aggregate indices is that the Kaplan-Strömberg theory is actually about buyout funds. Our aggregate cash flow-based private equity indices are comprised of the returns for all four types of funds. To the extent that all asset types are similarly exposed to macro-economic conditions, this improves the power for estimate the relationship of private equity to the general economy, but it adds noise to the estimate of the co-variates of the premium series, f_t. We have seen that the premium cycles for buyout, venture capital, private equity and high yield funds differ significantly. Panel B uses only the cash flow-based buyout return indices. It also reports results for the Cambridge Associates buyout index.

The first two specifications show the results for our private equity total return index. As with the broad index results, the asset-debt yield variable is positive and significant, the survey of loan officers is positive and significant, the default spread and VIX coefficients are negative and significant. Sentiment and production lose significance for the buyout sub-index. Turning to the buyout premium index results, we see that the asset-debt coefficient is positive and significant as predicted, while the signs on the default spread and the VIX change. The significant, positive coefficient on the buyout premium represents a rejection of the null hypothesis that cheap relative financing terms are not a source of value-creation by private equity managers.[18]

One qualification of these findings is that we are measuring the contemporaneous effects of the asset-debt yield spread. The proposed channel by which this adds value is via the purchase of a higher yielding asset financed by issuing cheap debt. The fund cash flows we observe are deployment or realization of capital and are thus conditional on such a transaction occurring. Nevertheless, our premium index assumes that all firms in operation at a given date experience the same shocks. If we could separate transacting firms from firms that were not exploiting the spread, we may find a larger effect.

5. Conclusion

Researchers and practitioners interested in understanding private equity investment have been limited by the structure and nature of the data. This has made it particularly difficult to evaluate its time-series characteristics. We present a methodology for extracting a latent performance measure from non-periodic cash flow information, and demonstrate how it may be further decomposed into passive and active components. We find that private equity returns are only partially spanned by investable passive indices. Our estimate suggests that private equity is, to a first approximation, a levered investment in small and mid-cap equities.

We model the residual component of private equity returns which cannot be replicated in traded, public markets as an orthogonal variable with cyclical characteristics. We find that in the first part of our sample period the private equity premium contributed positively to returns and in the

[18] In the last two columns of Panel B we report regressions for the Cambridge buyout index. The results are consistent with theory and with the estimates from our aggregate cash flow private equity index.

second period it detracted from returns. Our estimated autocorrelation coefficient is consistent with long-horizon cyclical behavior. We estimate the private equity premium for separate classes and show that their cycles are not highly correlated. This suggests that a diversified strategy across sub-asset classes of private equity may be beneficial.

Our cash flow-based private equity indexes allow us to test current theories about the cyclical nature of private equity returns. In particular, we test the Kaplan and Strömberg (2009) hypothesis that relative yields on corporate asset compared to high-yield debt explain returns of buyout investment. We find evidence that the buyout premium is higher in quarters for which the asset-debt yield spread is higher. Consistent with the conjecture that this investment opportunity is related to behavioral frictions, we also find that the Baker-Wurgler (2007) sentiment variable is correlated to total private equity returns.

Our methodology and results also have potential regulatory implications. Volatility measures for private equity with our cash flow-based return series are at least as volatile as standard aggregate equity market indexes. In contrast, estimates of private equity volatility constructed from appraisal-based indexes are much lower. The Solvency II Committee, the European Union's flagship project to harmonize European insurance supervision and set capital requirements (similar to Basel II), has chosen to use a publicly traded proxy for private equity returns, and our results suggest the volatility estimates from such an index is close to the volatility of true private equity returns. Investors and regulators all benefit from more accurate estimates of returns and risk of illiquid private equity.

Appendix A: Identification of Private Equity Returns

Consider the following four funds, which live between times $t=0$ and $t=4$:

Times	PE return (g)	Fund 1	Fund 2	Fund 3	Fund 4
0		-100	-100		
1	5.90%	**53.0**	0	-100	-100
2	17.50%	**31.1**	**124.4**	**117.5**	0
3	-4.80%	**29.6**		-100	**111.9**
4	31.70%			**131.7**	

All the cash flows represent money paid or received by Limited Partners (LPs). The contributions into the funds are denoted by negative signs and have all been normalized to 100. Distributions from the funds are marked in bold and are represented by positive numbers. Each of the four funds begins with an initial investment of 100. Funds 1 and 2 start at time 0, and funds 3 and 4 start at time 1. Fund 1 pays intermediary dividends and pays half of the fund value out each year, except in the last year where it pays out the remainder. Fund 3 invests in two projects sequentially and the other funds dissolve after only one project.

We do not observe the private equity return, g_t. If the funds are correctly priced, then the fund investments must satisfy a NPV condition, which is

$$PV(I) = PV(D), \qquad \text{(A.1)}$$

where PV denotes present value, *I* represents the investments made, and *D* the distributions received.

The NPV conditions for the four funds are:

$$\begin{aligned}
&\text{Fund 1: } 100 = \frac{53}{(1+g_1)} + \frac{31.1}{(1+g_1)(1+g_2)} + \frac{29.6}{(1+g_1)(1+g_2)(1+g_3)}, \\
&\text{Fund 2: } 100 = \frac{124.4}{(1+g_1)(1+g_2)}, \\
&\text{Fund 3: } 100 + \frac{100}{(1+g_2)(1+g_3)} = \frac{117.5}{(1+g_2)} + \frac{131.7}{(1+g_2)(1+g_3)(1+g_4)}, \\
&\text{Fund 4: } 100 = \frac{111.9}{(1+g_2)(1+g_3)}.
\end{aligned} \qquad \text{(A.2)}$$

In this highly unrealistic example, there are four equations with four unknowns: g_1, g_2, g_3, and g_4. Treating each of these returns as separate parameters, we can solve this system (with a non-linear root solver). This yields a unique solution and it coincides exactly with the private equity returns listed in the table. Hence, using the NPV conditions allows us to estimate the private equity returns each period using LP cash flows.

Although highly stylized, this example conveys some intuition of the identification strategy in our more general problem. The NPVs of all the funds do not involve the complete set of returns. Since only funds 1 and 2 are alive at time 1, only those funds identify g_1. All funds are alive at time 2, so all their NPVs involve the time 2 return, g_2. As only fund 4 is alive at time 4, g_4 enters only the NPV equation of fund 4. Intuitively, identification is obtained because when we change a particular return, like g_4, only certain NPVs are affected by that change. At a given time, all funds that are alive at that time are subject to the same return. If the return at that time changes, the NPVs of the funds alive at that time are affected.

Suppose that the same return applies at all periods, so $g_t = \overline{g}$ for $t = 1 \ldots 4$. Then, there are four NPV equations but only one return. Thus, the system is over-identified. We can estimate the constant return by assuming an orthogonality condition or distribution for the NPV equations. In our empirical work, we work with the log ratio

$$\ln \frac{PV(I)}{PV(D)} = 0, \tag{A.3}$$

which is equivalent to equation (A.1).[19] If we use the NPV itself in equation (A.1), the error in fitting the NPV condition may be large simply because the fund size is large. In equation (A.3), the ratio of the present value of investments to the present value of distributions does not have this problem as the size of the cash flows roughly cancels out in both the numerator and denominator.

[19] Equation (A.3) is similar to the ratios introduced by Ljungqvist and Richardson (2003) and Kaplan and Schoar (2005). In the public market equivalent (PME) ratio of Kaplan and Schoar (2005), the present values of the investments and distributions are computed with the market return, or equivalently it is assumed that the private equity's return is the same as the aggregate equity market. They interpret private equity as out-performing the market if the PME is greater than one. In equation (A.3), we compute the present values using discount rates which are endogenously determined. Nevertheless, the same intuition as Kaplan and Schoar holds in the sense that when private equity investments are fairly priced with appropriate discount rates, the ratio of the PV of investments to the PV of distributions should equal one.

There is an intermediate case between assuming that each return for each period is a free parameter and the case of a constant return for all periods. We parameterize the private equity return to be a persistent process, so that we could accommodate fewer funds than returns for identification and increase the over-identification, which is what makes our procedure robust.

Across our funds, and across time, the log ratio of present values of investments to present values of distributions in equation (A.3) will approximately equal zero. In our empirical work, we have many more funds than returns, which ensures good econometric identification for our return estimates (see Table 1). We estimate the returns so that the errors from the present value relations in equation (A.3) are "small," defined in terms of a likelihood distribution.

It is useful to contrast the returns with the IRR. Our returns apply to all funds. In contrast, the internal rate of return (IRR) commonly used as a return heuristic by private equity industry participants is usually computed at the fund level. Funds are often grouped into separate vintages, and the IRRs associated with funds in different vintages are taken as performance measures. Our approach differs in two ways from the IRR. First, we estimate the same set of returns across funds, rather than inferring one rate of return, the IRR, from each fund. Second, by using many simultaneous funds with different cash flows in different periods, we can identify a time series of returns which are common to all private equity projects. The only time variation that can be achieved by fund-level IRRs is to examine ten-years overlapping IRRs of funds in different vintage years.

The literature has used various estimation procedures when the system is over-identified. In the real estate literature, estimation has typically involved (generalized) least-squares procedures. These techniques have been applied to residential real estate (Bailey, Muth, and Nourse (1963), Case and Shiller (1987)) and commercial property (Geltner and Goetzmann (2000)). Similar procedures have been used by Peng (2001) and Hwang, Quigley, and Woodward (2005) to estimate returns to venture capital. In the private equity literature, Driessen, Lin, and Phalippou (2012) employ a generalized method of moments estimator to a set of constant returns, similar to the assumption that $g_t = \overline{g}$ for all *t*.

Our innovation is to introduce a way to extract multiple latent factors—and factor loading—from infrequent transactions data when the latent factor can be persistent, and there are also observable factors. The estimation is detailed in Appendix B.

Appendix B: Estimation of the Model

We re-state the model here for convenience. We can merge equations (9) and (10) into one equation containing only the latent state variable, g_t^e, which is the state equation:

$$g_t^e = (1-\phi)\alpha + \phi g_{t-1}^e + \beta'(F_t - \phi F_{t-1}) + \sigma_g \varepsilon_t, \tag{B.1}$$

where the systematic factors, F_t, are observable.

We assume that the zero NPV condition in equation (13) holds, and we specify that the log ratio of the PV of the distributions to the PV of investments is normally distributed:

$$\ln \frac{PV_i(D)}{PV_i(I)} \sim N(\mu, \sigma^2), \tag{B.2}$$

Equation (B.2), which repeats equation (13), represents the likelihood function of the cash flows. To ensure that the ratio of the present value of distributions and the present value of investments are centered at one, we set $\mu = -\frac{1}{2}\sigma^2$. This is equivalent to assuming that the errors of the log ratio of the PV of distributions to the PV of investments have zero mean.

Equations (B.1) and (B.2) constitute a state equation and a non-linear observation equation. The following algorithm filters the latent state variable g_t^e given the observation equations. Once g_t^e is estimated, we can infer the private equity-specific return, f_t, using

$$f_t = g_t^e - (\alpha + \beta' F_t). \tag{B.3}$$

We denote the parameters $\theta = (\alpha, \beta, \phi, \sigma_g, \sigma)$ and let θ_- denote the full set of parameters less the parameter that is being estimated in each conditional draw. We collect the exogenous private equity cash flow data and the common tradable factors F_t as $Y_t = \{\{I_{it}\}, \{D_{it}\}, \{F_t\}\}$.

We estimate the model described by MCMC and Gibbs sampling. Other similar models are estimated by Jacquier, Polson, and Rossi (2004), Jacquier, Polson, and Rossi (1994), Ang and Chen (2007), which involve latent state variables. These papers are able to directly use observable returns. In contrast, we use non-linear NPV equations to infer returns. This makes our estimation more similar to Chen (2013), who infers latent returns and cash flow factors from price-dividend ratios. A textbook exposition of Gibbs sampling is provided by Robert and Casella (1999).

In each of our estimations, we use a burn-in period of 20,000 draws and sample for 80,000 draws to produce the posterior distributions of latent state variables and parameters. With this large number of sampling, our estimation converges in a sense of passing the Geweke (1992) convergence test.

The Gibbs sampler iterates over the following sets of states and parameters conditioned on other parameters and states variables, to converge to the posterior distribution of $p(\{g_t^e\},\theta \mid Y)$:

1. Private equity returns: $p(\{g_t^e\} \mid \theta, Y)$,
2. Parameters of the private equity-specific return: $p(\beta,\phi,\alpha \mid \theta_{-},\{g_t^e\},Y)$,
3. Standard deviation of the private equity return shocks: $p(\sigma_g \mid \theta_{-},\{g_t^e\},Y)$, and
4. Standard deviation of likelihood errors: $p(\sigma \mid \theta_{-},\{g_t^e\},Y)$

We discuss each one in turn.

B.1 Private equity returns, $p(\{g_t^e\} \mid \theta, Y)$

We draw g_t^e using single-state updating Metropolis-Hasting algorithm (see Jacquier, Polson, and Rossi (1994), Jacquier, Polson, and Rossi (2004)). For a single state update, the joint posterior is:

$$\begin{aligned} & p(g_t^e \mid \{g_t^e\}_{i\neq t},\theta,Y) \\ & \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T,\theta)\, p(\{g_t^e\}_{t=1}^T,\theta,Y) \\ & \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T,\theta)\, p(g_t^e \mid g_{t-1}^e, g_{t+1}^e,\theta,Y) \\ & \quad \propto p(Y \mid \{g_t^e\}_{t=1}^T,\theta)\, p(g_t^e \mid g_{t-1}^e,\theta,Y)\, p(g_{t+1}^e \mid g_t^e,\theta,Y)\, p(g_t^e) \end{aligned} \qquad \text{(B.4)}$$

We can go from the second to third line in equation (B.4) because g_t^e is Markov. In equation (B.4), the distribution $p(Y \mid \{g_t^e\}_{t=1}^T,\theta)$ is the likelihood function in equation (B.2). The distribution of $p(g_t^e \mid g_{t-1}^e,\theta,Y)$ and $p(g_{t+1}^e \mid g_t^e,\theta,Y)$ are implied by the dynamics of g_t^e in equation (B.1).

They can be expressed as:

$$p(g_t^e \mid g_{t-1}^e, \theta) \propto \exp\left(-\frac{1}{2\sigma_g^2} (g_t^e - (1-\phi)\alpha - \phi g_{t-1}^e - \beta'(F_t - \phi F_{t-1}))^2 \right)$$
$$p(g_{t+1}^e \mid g_t^e, \theta) \propto \exp\left(-\frac{1}{2\sigma_g^2} (g_{t+1}^e - (1-\phi)\alpha - \phi g_t^e - \beta'(F_{t+1} - \phi F_t))^2 \right) \quad \text{(B.5)}$$

Collecting terms and completing the squares, we obtain

$$p(g_t^e \mid \{g_t^e\}_{i\neq t}, \theta, Y) \propto p(Y \mid \{g_t^e\}_{t=1}^T, \theta) \exp\left(-\frac{(g_t^e - \mu)^2}{2\sigma_g^2} (1+\phi^2) \right) p(g_t^e), \quad \text{(B.6)}$$

where

$$\mu_t = \frac{\phi(g_{t-1}^e + g_{t+1}^e + (1-\phi)\alpha + \beta'((1+\phi^2)F_t - \phi(F_{t+1} + F_{t-1}))}{1+\phi^2}. \quad \text{(B.7)}$$

For the prior of g_t^e, we impose an uninformative prior, $p(g_t^e) \propto 1$.

We use a Metropolis-Hasting draw with the proposal density

$$q(g_t^e) \propto \exp\left(-\frac{(g_t^e - \mu_t)^2}{2\sigma_g^2} (1+\phi^2) \right). \quad \text{(B.8)}$$

The acceptance probability for the (*k+1*)-th draw, $g_t^{e,(k+1)}$, is

$$\min\left(\frac{p(Y \mid g_t^{e,(k+1)}, \{g_i^e\}_{i\neq t}, \theta)}{p(Y \mid g_t^{e,(k)}, \{g_i^e\}_{i\neq t}, \theta)}, 1 \right). \quad \text{(B.9)}$$

When drawing g_t^e at the beginning or the end of the sample, we integrate out the initial and end values drawing from the process in equation (B.1).

B.2 Parameters of the private equity-specific return $p(\beta, \phi, \alpha \mid \theta_{-}, \{g_t^e\}, Y)$

Consider the factor loadings, β. We can write the posterior

$$\begin{aligned} & p(\beta \mid \theta_{-}, \{g_t^e\}, Y) \\ & \quad \propto p(Y \mid \beta, \theta_{-}, \{g_t^e\}) p(\{g_t^e\} \mid \beta, \theta_{-}) p(\beta) \\ & \quad \propto p(\{g_t^e\} \mid \beta, \theta_{-}) p(\beta), \end{aligned} \tag{B.10}$$

because β does not enter the dynamics of the private equity returns, g_t^e.

We can rewrite equation (B.1) as

$$g_t^e - (1-\phi)\alpha - \phi g_{t-1}^e = \beta'(F_t - \phi F_{t-1}) + \sigma_g \varepsilon_t, \tag{B.11}$$

which implies a standard regression draw for β. We use a normal conjugate prior.

The draws of ϕ and α are similar. Although they could be drawn directly in a multivariate conjugate regression draw, we separate them. This allows us to place separate priors on each parameter.

B.3 Standard deviation of the private equity return shocks, $p(\sigma_g \mid \theta_{-}, \{g_t^e\}, Y)$

We draw σ_g^2 using a conjugate Inverse Gamma draw. We select a truncated conjugate prior by confining the range of σ_g between 0.1% and 100% per quarter. We assume the prior

$$p(\sigma_g^2) \sim IG\left(\frac{a_0}{2}, \frac{b_0}{2}\right) 1_{[10^{-6},1]}, \tag{B.12}$$

where $a_0 = 2$ and $b_0 = 10^{-6}$. The peak of this prior is far left to the lower bound of our range; therefore, the truncated prior is approximately a uniform distribution on the range.

We draw the posterior distribution of σ_g^2 from its truncated conjugate posterior:

$$p(\sigma_g^2 \mid \theta_{-}, Y) \sim IG\left(\frac{a_1}{2}, \frac{b_1}{2}\right) 1_{[10^{-6},1]}, \tag{B.13}$$

where $a_1 = a_0 + T - 1$ and $b_1 = b_0 + u$, and u is given by

$$u = \sum (g_t^e - (1-\phi)\alpha - \phi g_t^e - \beta'(F_t - F_{t-1}))^2. \tag{B.14}$$

B.4 Standard deviation of likelihood errors: $p(\sigma \mid \theta_{\_}, \{g_t^e\}, Y)$

We draw σ^2 using a conjugate truncated Inverse Gamma distribution. This follows a similar method to the draw for σ_g. We assume the prior

$$p(\sigma^2) \sim IG\left(\frac{A_0}{2}, \frac{B_0}{2}\right) 1_{[10^{-6},1]}, \tag{B.15}$$

with $A_0 = 10^{-6}$ and $B_0 = 10^{-6}$. Denote

$$s = \sum_i \left(\ln \frac{PV(D_i)}{PV(I_i)}\right)^2.$$

Then the posterior distribution is

$$p(\sigma^2 \mid Y, \theta_{\_}) \sim IG\left(\frac{A_0 + N}{2}, \frac{B_0 + s}{2}\right) 1_{[10^{-6},1]}. \tag{B.15}$$

B.5 Priors

Like any Bayesian procedure, the estimation requires assumptions on the prior distributions of parameters. The prior on betas are taken from the current literature on private equity as listed in Appendix Table A.1 (Brav and Gompers (1997), Driessen, Lin, and Phalippou (2012), Derwall et al. (2009), Ewens, Jones, and Rhodes-Kropf (2013), Korteweg and Sorensen (2010), Cao and Lerner (2009), Franzoni, Nowak, and Phalippou (2012), Jegadeesh, Kräussl, and Pollet (2009), Chiang, Lee, and Wisen (2005), Lin and Yung (2004), Elton et al. (2001)). These studies estimate a three factor Fama-French model for venture capital, buyout, real estate or high yield bonds.[20] Real estate estimates are derived from REITs, and credit estimates are derived from Industrial

[20] Note that Jegadeesh, Kräussl, and Pollet (2009) use a dataset that contains predominantly but not exclusively buyout related vehicles (the rest of their sample is venture capital related).

BBB-rated bonds of 10-year maturities. The weighted average across sub-classes takes the four sub-classes averages and weights them by the number of funds in each sub-classes. The loadings are rounded at 0.05. The average loading in each category is used as priors.

< Table A.1 >

For the loading on the liquidity factor, there is only the Franzoni, Nowak, and Phalippou (2012) estimate available in the literature and it is available only for buyout. They report a beta of 0.7, which we then use as a prior for buyout funds. For the other sub-samples and the pooled private equity fund samples we use a prior of 0.5.

From the estimates in Table A.1, we set prior means for market, size, value, and liquidity factors at 1.30, 0.55, 0.05, and 0.50, respectively, for the private equity sample. When we estimate factor loadings per fund category (venture capital, buyout etc.) we use the average estimate in the literature in the corresponding category.

The prior for alpha is set at zero for simplicity. Since are computed at the end of each iteration by setting the NPV to zero from the factor loadings in that iteration, the estimation is insensitive to the alpha priors.

To compute the prior for the premium persistence, we use the individual buyout investment return database of Lopez-de-Silanes, Phalippou, and Gottschalg (2013). We compute the correlation between successive investment IRRs of the same private equity firm (IRR of investment i and IRR of investment $i+1$) and find it to be 0.25. The average spread in starting dates is around six months and investments last for four years. If the process is assumed to be AR(1), this means an autocorrelation coefficient of 0.5 at yearly frequency, which is what we use as a prior.

We set bounds of ±1 around the prior mean for betas. We impose no bounds on the alpha. The autocorrelation parameter, ϕ, is restricted to lie between 0 and 0.9 and the latent return is restricted to lie in between -0.50 and 1.00 per quarter.

The standard deviation of the prior captures how diffuse the priors are. We choose a large standard deviation for the priors equal to 10. This would represent an extremely diffuse prior in most contexts. We find, however, that the posterior distributions depend on the volatility of the latent factors more than the priors of the parameters. The volatility of the latent factor is equivalent to determining the R^2, that is how much of the private equity return is attributable to systematic factors. Since f_t is latent, we could exactly match any private equity return process if there were no restrictions. This can be clearly seen in a traditional linear context, but also occurs in the likelihood for our private equity cash flows. Thus, as expected, the volatility of the latent process influences the informativeness of the priors. We cap the volatility of the latent returns at a large 100% per quarter. For the prior for the volatility of the latent return, we use a mean of 20% per quarter. We examine robustness to these choice and the sample selection choices in Appendix C.

Appendix C: Robustness Checks and Small Sample Properties

In this appendix, we show the robustness of our estimation method. First, we report the sensitivity of the estimation procedure to different choices of the priors of the parameters, their informativeness, and sample selection; we find no substantial influence on the estimation results. Second, we run Monte Carlo simulations with known (i.e., pre-set) parameters and state factors. We find that our Bayesian Gibbs sampling method and identification strategy generates little small-sample bias.

C.1 Sensitivity to priors and assumptions

Since we use a Bayesian framework, it is useful to understand how the priors and their informativeness affect the results. In addition, an important decision in our framework is the threshold to use as a NAV cut-off for definition of a quasi-liquidated fund.

In Table A.2, we examine robustness of our estimations to different priors and NAV thresholds. The first line shows the results with the default specification; the estimates coincide with those reported in the main tables. Each line then shows the results of implementing one change. Panel A shows results for the CAPM for the sample of private equity funds. We first examine the effects of changing the priors about the likelihood standard error, σ, in equation (13). Raising the maximum prior for σ slightly increases the mean of the estimated betas to 1.43. The alpha estimates are unchanged. Note also that persistence estimates drop when the maximum on σ increases.

< Table A.2 >

Priors on beta obviously make a difference. The prior mean used in our estimation is 1.30. Increasing the prior mean to 1.80 results in a posterior mean estimate of 1.65, roughly one standard error above the default estimated mean value. Decreasing the prior by 0.5, i.e. from 1.3 to 0.8 yields a beta of 1.15. Thus, priors on betas matter to estimate the systematic risk exposure, although we cannot statistically reject the null that these are equal to the baseline estimation. Variation of the beta priors has little effect on the in-sample alpha. The fact that the posterior always move in the direction of our original prior can be seen as additional support for our original set of priors.

Turning to the NAV cutoff assumption we find that beta estimates are relatively unchanged if we lower the threshold for the percentage of the fund liquidated from 50% to 33%. The CAPM alpha increases to 0.07. Raising the threshold to 66% and 75% has little effect. In other words, restricting the sample to funds that have more liquidated investments makes private equity look more attractive (in that sample). This is consistent with private equity funds holding on to losers, causing better performing funds to liquidate faster (cf. Lopez-de-Silanes, Phalippou, and Gottschalg (2013)). This may reflect an upward bias in estimated NAVs. More comforting is the evidence from increasing the threshold. Including more funds leaves the estimate unchanged. This suggests that higher alpha based on looser censoring is upward-biased but that results using the 50% threshold are representative. Persistence estimates decrease with a greater threshold. This may be due to the decreasing sample size.[21]

Panels B, C, and D show results for the four-factor Pástor and Stambaugh (2003) model for the sample of private equity funds, venture capital funds, and buyout funds, respectively. In general, changes in priors about σ have similar effects to those noted for the CAPM specification. Raising the maximum σ increases the posterior mean of the market beta, while decreasing the prior lowers the posterior mean of beta. Interestingly, widening the σ prior increases the in-sample alpha (even though beta goes up), while tightening the sigma prior decreases the in-sample alpha. This latter result appears to be due to an increase in the size and illiquidity betas. As with the single factor model, varying the priors on beta changes the beta estimates, although it does not push them beyond approximately one posterior standard deviation from the estimates under the default assumptions. The beta priors do, however, significantly affect the estimated in-sample

[21] Goetzmann (1992) shows that, in a similar estimation procedure, negative autocorrelation is induced by thin data.

alphas. Naturally, decreasing betas by 0.5 raises the posterior alpha mean estimate to 0.07 and raising betas by 0.5 lowers the posterior in-sample mean alpha to -0.04. The posterior standard deviations of the alphas are about 0.01, so this difference is large. The effect of varying the NAV threshold is similar to that observed for the CAPM model, raising the threshold lowers the alpha. It is likely that the higher NAV threshold reduces the likelihood of upward bias due to stale marks.

C.2 Simulations with known alphas and betas

In order to test the precision and potential bias in the estimation procedure we conduct a simulation that constructs hypothetical funds for which we assume what the true parameters are. In addition, we allow for funds to have different rates of return over a given time period t, i.e. we allow for $g_{i,t,}$ to differ from $g_{j,t}$ where i and j denote two different funds.

As the structure of our panel of data may hamper our estimates, we preserve that structure in the simulation exercise. The timing and amount of the cash outflows (from the LP perspective) for each simulated fund is set to be the same as that of a randomly drawn fund from our sample. The timing of cash inflows is matched to that of the same drawn fund. We assume that each cash outflow corresponds to one unique investment. The first investment is assumed to liquidate at the time we observe the first cash inflow, the second investment is assumed to liquidate at the time we observe the second cash inflow etc. All the investments in a fund j earn the same rate of return in quarter t: $R_{j,t}$, which is set to be equal to a private equity index return (g_t) plus a fund-specific idiosyncratic component ($v_{j,t}$). The latter is equivalent to assuming the cash flows of each fund embed the private equity return, g_t, plus idiosyncratic error. We assume g_t is equal to an alpha of 1.25% per quarter (5% per year) plus a beta of 1.5 times the actual S&P 500 returns during quarter t plus the persistent private equity industry specific component f_t. For f_t, we draw an f_0 randomly and generate a time-series that follows an AR(1), with an autocorrelation coefficient (ϕ) of 0.5 and an idiosyncratic shock drawn from a normal distribution with mean zero and 5% per quarter volatility (σ_f). The first 100 observations of this time-series are discarded. The fund-specific idiosyncratic component, $v_{j,t}$, is independently drawn from a normal distribution with mean zero and 5% per quarter volatility.

By construction the net present value of the cash flows of each fund j equals zero if the time series of $R_{j,t}$ is used as discount rates. The econometrician, however, does not observe $R_{j,t}$ and computes ex-post present values with g_t and searches for the alpha, phi, beta, and σ_f that best fit the cross section of ln(PME) (see equation (13)). In the above setup the standard deviation of the cross-section of ln(PME) should be about 10% and is denoted σ.

This exercise is repeated 100 times for three different sample sizes (200, 400 and 1000 funds), all matching our 20 year sample. Table A.3 shows the results. The mean, standard deviation, and the quartile thresholds of the 100 estimations are reported in Panel A. The mean of the alpha distribution appears to be downward biased in small samples but the bias is modest at about 25 basis points per quarter. The autocorrelation coefficient is also slightly downward biased in small sample, which is consistent with the well-known bias of autocorrelation parameters from Kendall (1952). Beta and sigma estimates are less affected by small sample bias.

< Table A.3 >

Panel B shows the summary statistics for the simulated cash flow private equity total return index, g_t, and the time-varying private equity component, f_t , measured at the quarterly horizon. The mean of the simulated g_t is slightly higher than the true value for all sample sizes, as is the median of the time-varying private equity component, f_t.

The most important requirement of our index is that it captures the true dynamics of the private equity index. Since we know the actual returns of the true index by construction, we can measure its average correlation to the estimated indices. Panel C reports the average correlation between the true return g_t and the Gibbs sampler estimates. Even in small sample the correlation is greater than 50%. This may explain why our real estate index, measured with relatively few funds, appears to capture the dynamics of a broad-based commercial property return index. As sample size increases, the correlation increases rapidly towards one. The correlation of the true return g_t to other performance measures (IRR, multiple, and PME) is around zero, which is what we find in our empirical results.

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Table 1: Descriptive Statistics

This table shows the number of observations in different samples. Panel A compares the Preqin dataset to that of proprietary dataset recently used in the literature. Panel B shows statistics on Preqin full sample and sub-samples. Venture Capital (VC) funds include funds classified (by Preqin) as either: expansion, late stage, general venture, balanced or growth. Buyout (BO) funds include funds classified as either turnaround or buyout. Debt funds include funds classified as either distressed debt or mezzanine. Private Equity (PE) funds refer to funds that are either VC, BO, Credit (C) or Real Estate (RE). Funds are from vintage years 1992 to 2008. The quasi liquidated sample is the sub-sample of funds with the latest NAV reported that is less or equal to 50% of fund size, with at least one cash distribution, and with the latest NAV reported (or the largest distribution) larger or equal to 10% of fund size.

Panel A: Number of observations – Comparing Preqin and proprietary datasets

	Venture capital funds			Buyout funds		
	Harris, Jenkinson, & Kaplan	Robinson & Sensoy	Preqin, Full sample	Harris, Jenkinson, & Kaplan	Robinson & Sensoy	Preqin, Full sample
1992	17	4	10	5	4	7
1993	13	5	9	11	9	12
1994	20	7	12	13	24	15
1995	18	13	15	17	24	11
1996	20	13	16	9	41	18
1997	33	19	21	30	40	22
1998	46	36	31	38	59	39
1999	65	40	44	28	59	31
2000	80	55	79	39	68	38
2001	48	18	50	26	26	17
2002	18	7	28	21	5	19
2003	25		15	13	8	17
2004	32		28	46	3	29
2005	48	1	35	57	2	52
2006	62		49	67	8	52
2007	65	2	48	74	6	57
2008	45		26	68	12	42
Total	655	220	516	562	398	478

Panel B: Number of observations – Preqin samples

	Full sample	Preqin quasi-liquidated sub-sample of funds				
	All PE funds (VC,BO,RE,C)	All PE funds (VC,BO,RE,C)	VC funds	BO funds	RE funds	Credit funds
1992	21	21	11	6	1	3
1993	22	22	9	12	0	1
1994	31	30	11	15	1	3
1995	29	28	15	10	1	2
1996	41	39	15	17	3	4
1997	52	50	20	22	1	7
1998	78	74	27	39	4	4
1999	81	67	36	26	1	4
2000	126	97	60	28	6	3
2001	77	52	28	14	2	8
2002	55	33	15	10	3	5
2003	42	12	2	2	4	4
2004	74	17	3	6	7	1
2005	114	11	3	4	4	0
2006	140	19	7	5	4	3
2007	146	27	3	12	8	4
2008	93	31	7	15	4	5
Total	1222	630	272	243	54	61

Table 2: Private Equity Factor Exposures

This table shows the estimated risk loadings, abnormal returns and the persistence in abnormal returns using six different asset pricing factor models. All quasi-liquidated private equity funds are used in the analysis, irrespective of their type (venture capital, buyout, real estate, high yield). The quasi liquidated sample is the sub-sample of funds with the latest NAV reported that is less or equal to 50% of fund size, with at least one cash distribution, and with the latest NAV reported (or the largest distribution) larger or equal to 10% of fund size. The risk loadings are estimated using the quasi liquidated sample. The reported alpha is annualized (by compounding) and defined as the constant that makes the (equally weighted) average NPV equal to zero in either the full sample or the quasi-liquidated fund sample, given the estimated risk loadings. Underneath each coefficient, in italics, we report the posterior standard deviation of the estimated parameters. The factor models that we use are: the CAPM, the three factor model of Fama and French (1993), and the four factor model is that of Pástor and Stambaugh (2003). The equally weighted (EW) factor models are the same as the original model but with the CRSP equally-weighted index instead of the CRSP value-weighted index as a measure of market returns. The priors for the factor loadings are detailed in Appendix A.

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	Full sample Alpha	R-square
CAPM	1.41[a]				0.05[a]	0.40	0.04[a]	0.93
	0.24				*0.01*	*0.19*	*0.01*	
3 factors (FF)	1.49[a]	0.41	0.09		0.04[a]	0.43	0.03[a]	0.95
	0.23	*0.31*	*0.27*		*0.01*	*0.19*	*0.01*	
4 factors (PS)	1.41[a]	0.41	0.03	0.36	0.00	0.48	0.00	0.97
	0.21	*0.26*	*0.23*	*0.27*	*0.02*	*0.19*	*0.01*	
EW CAPM	1.42[a]				-0.04[a]	0.45	-0.04[a]	0.98
	0.18				*0.01*	*0.19*	*0.01*	
EW FF	1.47[a]	0.40	-0.11		-0.04[a]	0.47	-0.04[a]	0.98
	0.20	*0.25*	*0.21*		*0.01*	*0.19*	*0.01*	
EW PS	1.40[a]	0.33	-0.19	0.26	-0.05[a]	0.47	-0.05[a]	0.97
	0.22	*0.30*	*0.25*	*0.27*	*0.02*	*0.19*	*0.01*	

Table 3: Risk Exposures Broken Down by Fund Type

This is the same table as Table 2. Instead of using all the funds we use (independently) sub-samples of funds based on their type: venture capital, buyout, real estate and high yield. Venture capital funds include funds classified (by Preqin) as either: expansion, late stage, general venture, balanced or growth. Buyout funds include funds classified as either turnaround or buyout. Credit funds include funds classified as either distressed debt or mezzanine. We report posterior means. The reported alpha is annualized (by compounding) and defined as the constant that makes the (equally weighted) average NPV equal to zero in either the full sample or the quasi-liquidated fund sample, given the estimated risk loadings. Underneath each coefficient, in italics, we report the posterior standard deviation of the estimated parameters. The bottom three models in each Panel are the same as the original models but with a different proxy used for market returns. The proxies used in each of the panels are, respectively: Equally-weighted Nasdaq index, Equally-weighted AMEX/NYSE index, FTSE REITS index, 10 years T-bonds returns.

Panel A: Venture Capital funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	Full sample Alpha	R-square
CAPM	1.67[a]				0.05[a]	0.54[a]	0.04[a]	94.1%
	0.27				*0.01*	*0.19*	*0.01*	
3 factors (FF)	1.51[a]	0.45	-0.62[c]		0.08[a]	0.60[a]	0.06[a]	93.5%
	0.33	*0.42*	*0.38*		*0.02*	*0.17*	*0.02*	
4 factors (PS)	1.60[a]	0.53	-0.68[c]	0.16	0.06[a]	0.63[a]	0.05[b]	95.9%
	0.29	*0.42*	*0.37*	*0.36*	*0.02*	*0.17*	*0.02*	
NASDAQ CAPM	1.32[a]				-0.02	0.71[a]	-0.04[a]	97.3%
	0.19				*0.01*	*0.14*	*0.01*	
NASDAQ FF	1.11[a]	0.28	-0.59[c]		0.02	0.71[a]	0.00	96.0%
	0.25	*0.40*	*0.31*		*0.02*	*0.14*	*0.02*	
NASDAQ PS	1.15[a]	0.36	-0.57[b]	0.00	0.02	0.71[a]	0.00	97.0%
	0.23	*0.43*	*0.28*	*0.36*	*0.02*	*0.15*	*0.02*	

Panel B: Buyout funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	Full sample Alpha	R-square
CAPM	1.31[a]				0.05[a]	0.42[b]	0.04[a]	88.7%
	0.25				*0.01*	*0.20*	*0.01*	
3 factors (FF)	1.39[a]	-0.07	0.74[b]		0.03[a]	0.42[b]	0.01	92.3%
	0.21	*0.30*	*0.29*		*0.01*	*0.19*	*0.01*	
4 factors (PS)	1.33[a]	-0.04	0.57[b]	0.59[a]	-0.02[b]	0.50[a]	-0.03[a]	96.9%
	0.14	*0.25*	*0.22*	*0.21*	*0.01*	*0.17*	*0.01*	
AMEX/NYSE CAPM	1.30[a]				0.00	0.64[a]	-0.01	90.3%
	0.27				*0.01*	*0.16*	*0.01*	
AMEX/NYSE FF	1.29[a]	-0.38	0.46		0.00	0.62[a]	-0.01	90.5%
	0.25	*0.37*	*0.33*		*0.01*	*0.18*	*0.01*	
AMEX/NYSE PS	1.15[a]	-0.28	0.39	0.50[c]	-0.03[b]	0.70[a]	-0.04[b]	93.9%
	0.22	*0.36*	*0.32*	*0.30*	*0.02*	*0.15*	*0.02*	

Panel C: Real Estate funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	Full sample Alpha	R-square
CAPM	0.77[a]				0.00	0.72[a]	0.00	79.6%
	0.23				*0.01*	*0.11*	*0.01*	
3 factors (FF)	0.79[a]	0.21	0.76[b]		-0.03[a]	0.61[a]	-0.03[b]	87.1%
	0.22	*0.30*	*0.30*		*0.01*	*0.17*	*0.01*	
4 factors (PS)	0.74[a]	0.09	0.54	0.66[c]	-0.08[a]	0.54[a]	-0.07[a]	87.8%
	0.23	*0.37*	*0.37*	*0.38*	*0.02*	*0.19*	*0.02*	
REIT CAPM	0.75[a]				-0.04[a]	0.61[a]	-0.04[b]	83.8%
	0.18				*0.01*	*0.16*	*0.02*	
REIT FF	0.66[a]	0.10	0.49		-0.05[a]	0.58[a]	-0.04[b]	88.7%
	0.20	*0.29*	*0.30*		*0.01*	*0.18*	*0.02*	
REIT PS	0.49[b]	0.00	0.26	0.59[c]	-0.07[a]	0.58[a]	-0.07[a]	83.4%
	0.24	*0.39*	*0.37*	*0.36*	*0.02*	*0.18*	*0.02*	

Panel D: Credit funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	Full sample Alpha	R-square
CAPM	0.62[b]				0.03	0.36[c]	0.03[c]	66.3%
	0.27				*0.02*	*0.19*	*0.02*	
3 factors (FF)	0.88[a]	1.18[a]	1.05[a]		-0.02[b]	0.49[a]	-0.03[b]	96.7%
	0.20	*0.26*	*0.27*		*0.01*	*0.18*	*0.01*	
4 factors (PS)	0.87[a]	1.14[a]	0.97[a]	0.29	-0.06[a]	0.49[b]	-0.06[a]	96.5%
	0.22	*0.26*	*0.28*	*0.24*	*0.02*	*0.19*	*0.02*	
T-Bond CAPM	0.58[a]				-0.01	0.41[b]	-0.01	79.4%
	0.20				*0.02*	*0.18*	*0.02*	
T-Bond FF	0.44	1.02[a]	0.73[b]		-0.03	0.58[a]	-0.02	88.9%
	0.28	*0.38*	*0.36*		*0.02*	*0.19*	*0.02*	
T-Bond PS	0.36	0.99[a]	0.72[c]	0.16	-0.05[c]	0.57[a]	-0.04	89.6%
	0.24	*0.33*	*0.38*	*0.32*	*0.03*	*0.20*	*0.03*	

Table 4: Comparison of Private Equity Index with Industry Indices

Columns 2 to 6 show the following descriptive statistics for each index return: the annualized mean, volatility, 25th and 75th percentiles; and the autocorrelation coefficient (computed at quarterly frequency). Eight return indices are shown: Cambridge Associates index for buyout and venture capital, NCREIF for real estate, the LPX 50 listed equity index, and the four total return indices we derived (aggregated private equity, buyout, venture capital, and real estate). Time period is 1994-2010.

	Mean	Volatility	Percentiles		Autocorrelation
			25th	75th	
CF buyout index	0.15	0.26	-0.12	0.53	0.06
Cambridge Associates buyout index	0.16	0.12	0.04	0.33	0.41
LPX listed buyout index	0.16	0.30	-0.05	0.45	0.22
CF venture capital index	0.18	0.34	-0.18	0.67	0.03
Cambridge Associates venture index	0.19	0.28	-0.03	0.35	0.61
LPX listed venture capital index	0.13	0.39	-0.33	0.64	0.14
CF real estate index	0.05	0.17	-0.12	0.31	0.24
NCREIF (Real Estate) index	0.09	0.05	0.07	0.15	0.82
CF private equity index	0.15	0.29	-0.15	0.58	0.00
LPX 50 index	0.13	0.35	-0.21	0.52	0.18

Table 5: Alternative Performance Measures and Capital Flows

This table compares different performance measures. The forward moving average of our private equity index, g_t, is the geometric yearly average return calculated from year t+1 to t+5. It is computed using the four-factor Pástor - Stambaugh (2003) model for systematic risk. The last year is 2009 and the forward moving average is not computed for 2008 as it is not meaningful (n.m.). Vintage year returns are computed by first aggregating all the cash flows of the funds from a given vintage year, and then computing the IRR, multiple and public market equivalent (PME) of that aggregated cash flow stream. The PME is calculated using the four-factor model cost of capital to discount cash flows. The sample is all Preqin quasi-liquidated private equity funds. The number of funds / capital allocated in a given year is taken from the full Preqin sample (see Table 1). Growth in year t refers to the growth rate in number of funds / capital raised from year t to year $t+1$. Panel D shows results from an OLS time series regression. The t-statistics are reported in italics and are based on Newest-West (1987) standard errors with four lags. Superscripts denote statistical significance at 1%, 5%, and 10% levels are denoted by *a*, *b*, and *c*, respectively. Cambridge Associates (CA) publishes one buyout quarterly return index and one venture capital quarterly return index. In regression analysis with either the full sample (PE) or the buyout (BO) sample we use the CA buyout index; when we use the sub-sample of venture capital funds we use the CA venture capital index.

Panel A: Yearly time-series of returns, flow, and yield spread

	Vintage year			CF PE annual	Forward moving	Growth	Growth
Year	IRR	Multiple	PME	PE index (g_t)	average of g_t	N-funds	Capital
1993	0.27	2.71	1.13	0.19	0.27	0.41	0.58
1994	0.36	2.77	1.38	-0.01	0.38	-0.06	0.19
1995	0.30	2.33	1.27	0.44	0.23	0.41	0.36
1996	0.17	1.77	1.06	0.28	0.16	0.27	0.84
1997	0.13	1.69	1.14	0.45	0.02	0.50	1.18
1998	0.09	1.50	1.05	0.27	0.08	0.04	-0.05
1999	0.07	1.36	0.89	0.50	0.04	0.56	0.95
2000	0.12	1.61	0.98	-0.19	0.11	-0.39	-0.41
2001	0.20	1.68	1.05	-0.05	0.17	-0.29	-0.33
2002	0.26	1.77	1.14	-0.25	0.25	-0.24	-0.09
2003	0.12	1.25	0.97	0.68	-0.05	0.76	0.47
2004	0.22	1.53	1.25	0.25	-0.01	0.54	1.56
2005	0.07	1.16	1.02	0.12	-0.04	0.23	0.79
2006	-0.21	0.67	0.62	0.23	-0.11	0.04	0.03
2007	-0.18	0.94	0.69	0.04	-0.18	-0.36	-0.32
2008	0.09	1.12	0.80	-0.57	n.m.	-0.51	-0.70

Panel B: Correlation matrix

	Vintage year			CF PE annual	Forward moving	Growth	
	IRR	Multiple	PME	PE index (g_t)	average of g_t	N_{funds}	Capital
IRR	1.00	0.90	0.90	-0.05	0.87	0.16	0.21
Multiple	0.90	1.00	0.82	0.03	0.92	0.20	0.24
PME	0.90	0.82	1.00	0.17	0.75	0.37	0.46
Index (g_t)	-0.05	0.03	0.17	1.00	-0.16	0.90	0.78
Forward g_t	0.87	0.92	0.75	-0.16	1.00	0.02	0.06
Growth N-funds	0.16	0.20	0.37	0.90	0.02	1.00	0.92
Growth capital	0.21	0.24	0.46	0.78	0.06	0.92	1.00

Panel C: Venture capital funds and buyout funds

	Venture Capital funds				Buyout funds			
	CF index	Cambridge	Growth		CF index	Cambridge	Growth	
Year	($g_{vc,,t}$)	Associates index	N_{funds}	Capital	($g_{bo,,t}$)	Associates index	N_{funds}	Capital
1993	0.09	0.19	0.33	0.30	0.23	0.24	0.25	0.61
1994	0.00	0.17	0.25	0.93	0.00	0.13	-0.27	0.10
1995	0.52	0.47	0.07	0.18	0.40	0.24	0.64	0.21
1996	0.36	0.41	0.31	0.28	0.27	0.28	0.22	0.76
1997	0.46	0.34	0.48	0.71	0.52	0.31	0.77	1.61
1998	0.43	0.31	0.42	1.58	0.27	0.15	-0.21	-0.22
1999	1.14	2.93	0.80	1.43	0.06	0.44	0.23	0.79
2000	-0.29	0.20	-0.37	-0.39	-0.11	0.06	-0.55	-0.67
2001	-0.16	-0.40	-0.44	-0.68	0.05	-0.12	0.12	0.62
2002	-0.35	-0.34	-0.46	-0.42	-0.17	-0.08	-0.11	-0.14
2003	0.71	-0.04	0.87	0.77	0.50	0.22	0.71	0.36
2004	0.18	0.15	0.25	0.33	0.29	0.25	0.79	1.90
2005	0.06	0.07	0.40	1.48	0.26	0.28	0.00	0.71
2006	0.14	0.18	-0.02	-0.40	0.45	0.29	0.10	-0.13
2007	0.10	0.15	-0.46	-0.30	0.13	0.20	-0.26	-0.13
2008	-0.61	-0.16	-0.46	-0.30	-0.54	-0.22	-0.45	-0.71

Panel D: Regression analysis - capital flows and past performance

	Private equity		Venture capital		Buyout	
growth in	N_{funds}	Capital	N_{funds}	Capital	N_{funds}	Capital
Constant	-0.19[b]	-0.09	-0.03	0.11	-0.27[a]	-0.43[a]
	-1.99	*-0.38*	*-0.59*	*1.07*	*-2.60*	*-4.91*
Our CF index, year *t*	1.11[a]	1.51[a]	1.01[a]	1.06[a]	1.52[a]	1.53[a]
	4.67	*5.76*	*5.09*	*5.17*	*7.30*	*2.74*
IRR, vintage year *t*	0.11	0.51	0.25	0.72	1.87[a]	3.80[a]
	0.77	*1.48*	*0.85*	*1.02*	*6.11*	*5.54*
Cambridge Associates index, year *t*	0.01	0.08[c]	-0.10	0.08	0.21	1.82[a]
	0.39	*1.76*	*-1.56*	*0.89*	*0.53*	*2.93*
IRR, vintage year t-1	-0.02	-0.39	-0.20	-0.51	-1.06[a]	-2.18[a]
	-0.12	*-0.96*	*-0.52*	*-0.63*	*-5.16*	*-2.66*
Adjusted R-square	75%	34%	59%	22%	64%	47%
Number of observations	16	16	16	16	16	16

Table 6: Private Equity Premium Cycles

This table shows the start and end of private equity cycles broken down by fund types. In Panels A and B, a boom period is one that has more than two quarters in a row with the private equity return premium, f_t, more than one standard deviation above the mean. A bust period is one that has more than two quarters in a row with f_t more than one standard deviation below the mean. In panel A, alpha is derived from the four-factor model of Pástor and Stambaugh (2003). In Panel B, alpha is derived from the CAPM model. In Panel C, the definition of a boom/bust is the same except that we use the Cambridge Associates NAV-based quarterly return series for venture capital and buyout and the NCREIF index for real estate. The time series starts at 1993:Q1 and ends at 2010:Q4.

Panel A: Alpha cycle in private equity using the Pástor -Stambaugh four-factor model

	Boom		Bust		Boom		Bust	
	Starts	Ends	Starts	Ends	Starts	Ends	Starts	Ends
All private equity funds	Q1-1997	Q3-2000					Q4-2007	-
Venture capital funds	Q1-1998	Q1-2001					Q2-2008	-
Buyout funds	Q4-1995	Q1-1997	Q4-1998	Q2-2001	Q2-2005	Q3-2007		
Real estate funds					Q4-2005	Q4-2006	Q4-2007	Q3-2010
Credit funds	Q1-1995	Q2-1996	Q4-1999	Q2-2002	Q3-2007	Q1-2008		

Panel B: Alpha cycle in private equity using the CAPM

	Boom		Bust		Boom		Bust	
	Starts	Ends	Starts	Ends	Starts	Ends	Starts	Ends
All private equity funds	Q3-1998	Q4-2000					Q3-2007	-
Venture capital funds	Q1-1998	Q4-2000					Q3-2008	-
Buyout funds	Q2-1996	Q3-1996	Q3-1998	Q3-2000	Q1-2005	Q1-2007		
Real estate funds					Q4-2002	Q4-2006	Q4-2007	Q2-2010
Credit funds			Q1-1997	Q2-2001	Q2-2002	Q1-2004		

Panel C: Return cycles in private equity according to industry indices (Cambridge Associates & NCREIF)

	Boom		Bust		Boom		Bust	
	Starts	Ends	Starts	Ends	Starts	Ends	Starts	Ends
Venture capital funds	Q1-1999	Q1-2000	Q2-2002	Q4-2002				
Buyout funds							Q3-2008	Q1-2009
Real estate funds							Q4-2008	Q4-2009

Table 7: Private Equity Returns over the Business Cycle

This Table shows how our cash flow-based total private equity return indexes and those of industry relate to macroeconomic variables. There are three different dependent variables: i) Time varying alpha (f_t) for private equity funds or only buyout funds; ii) the total private equity return index (g_t), either for all private equity funds or only buyout funds; iii) Cambridge Associates quarterly NAV-based buyout returns. The factor model used to derive f_t and g_t is the four-factor model of Pástor and Stambaugh (2003). We compute *t*-statistics using Newey-West (1987) standard errors with four lags, which are shown underneath each coefficient in italics. Time period is from the first quarter of 1993 to the last quarter of 2010.

Panel A: Cash flow-based private equity indices

	CF PE index (g_t)			Premium (f_t)		
Constant	0.00	0.05[a]	0.00	0.00	0.00	0.00
	-0.05	*4.36*	*0.09*	*-0.13*	*-0.88*	*0.71*
Ebitda/EV - High Yield spread	2.82[a]		2.55[b]	0.02		-0.24
	3.49		*2.46*	*0.09*		*-1.55*
Industrial Production growth		2.01[a]	0.47		0.30[a]	0.44[a]
		2.67	*0.48*		*2.72*	*3.38*
Default Spread (BAA-AAA)	-1.64[a]	-2.18[a]	-1.61[a]	0.02	0.10	0.04
	-2.73	*-4.53*	*-2.71*	*0.24*	*1.60*	*0.66*
Inflation	-1.72	-1.79	-1.69	0.10	0.14	0.13
	-1.19	*-1.32*	*-1.22*	*0.55*	*0.80*	*0.76*
Sentiment index	0.73[a]	0.67[a]	0.70[a]	0.04	0.01	0.01
	3.50	*3.02*	*3.15*	*1.30*	*0.34*	*0.27*
Survey of Loan Officer	0.24[b]	0.26[b]	0.24[b]	0.02	0.01	0.02
	2.09	*2.40*	*2.13*	*1.19*	*1.05*	*1.16*
Return VIX	-0.22[a]	-0.26[a]	-0.22[a]	0.00	0.00	0.00
	-6.04	*-5.92*	*-4.85*	*0.30*	*-0.51*	*-1.32*
Adjusted R-square	67%	64%	67%	-2%	12%	17%
Number of observations	72	72	72	72	72	72

Panel B: Cash flow-based buyout indices

Dependent Variable:	CF BO index ($g_{bo,t}$)		Premium ($f_{bo,t}$)		Cambridge Associates index	
Constant	0.05[a]	-0.01	0.00	-0.01[b]	0.04[a]	0.01
	5.41	*-0.49*	*-0.21*	*-2.48*	*5.07*	*0.46*
Ebitda/EV - High yield spread		3.01[a]		0.51[a]		1.45[a]
		3.06		*2.78*		*3.41*
Industrial production growth	2.02[a]	0.21	0.04	-0.27[c]	1.51[a]	0.63[c]
	2.74	*0.20*	*0.35*	*-1.75*	*5.41*	*1.86*
Default spread (BAA-AAA)	-2.55[a]	-1.88[a]	0.06	0.17[b]	-0.37	-0.04
	-5.72	*-3.80*	*1.11*	*2.54*	*-1.45*	*-0.17*
Inflation	-0.85	-0.73	0.04	0.06	-0.22	-0.16
	-0.61	*-0.54*	*0.38*	*0.70*	*-0.46*	*-0.32*
Sentiment index	0.03	0.06	0.01	0.01	0.36[a]	0.38[a]
	0.20	*0.43*	*0.20*	*0.43*	*3.09*	*3.17*
Survey of loan officer	0.23[a]	0.21[a]	0.01	0.00	0.05	0.04
	3.12	*2.60*	*0.38*	*0.11*	*1.63*	*1.44*
Return VIX	-0.22[a]	-0.18[a]	0.00	0.01[b]	-0.06[a]	-0.04[b]
	-5.25	*-4.07*	*0.75*	*2.34*	*-3.55*	*-2.34*
Adjusted R-square	58%	64%	-7%	14%	57%	63%
Number of observations	72	72	72	72	72	72

Table A.1: Literature Estimates of the Risk Exposures of Private Equity Funds

This table shows the factor loading estimates shown in the literature. Selected papers are those that estimated a three factor model for venture capital, buyout, real estate or high yield bonds. Jegadeesh, Kraussl and Pollet (2010) use a dataset that contains predominantly but not exclusively buyout related vehicles (the rest of their sample is venture capital related). Real estate estimates are derived from real estate investment trusts, and credit estimates are derived from Industrial BBB-rated bonds of 10-year maturities. The weighted average across sub-classes takes the four sub-classes averages and weights them by the number of funds in each sub-class. The loadings are rounded to increments of 0.05. The average loading in each category is used as priors in our Bayesian estimations.

		Venture capital funds		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Brav, and Gompers	1997	1.10	1.30	-0.70
Driessen, Lin and Phalippou	2012	2.40	0.90	-0.25
Ewens, Jones and Rhodes-Kropf	2013	1.05	-0.10	-0.90
Korteweg, and Sorensen	2009	2.30	1.00	-1.55
Average venture capital funds		**1.70**	**0.80**	**-0.85**
		Buyout funds		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Cao, and Lerner	2007	1.30	0.75	0.20
Driessen, Lin and Phalippou	2012	1.70	-0.90	1.40
Ewens, Jones and Rhodes-Kropf	2013	0.80	0.10	0.25
Franzoni, Nowak and Phalippou	2012	1.40	-0.10	0.70
Jegadeesh, Kräussl and Pollet	2010	1.05	0.60	0.35
Average buyout funds		**1.25**	**0.10**	**0.60**
		Real estate		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Chiang, Lee and Wisen	2005	0.55	0.40	0.50
Derwall, Huij, Brounen, and Marquering	2009	0.65	0.40	0.60
Lin and Yung	2004	0.55	0.40	0.70
Average real estate		**0.60**	**0.40**	**0.60**
		Credit		
Authors	Year	β_{mkt}	β_{smb}	β_{hml}
Elton, Gruber, Agrawal, and Mann	2001	0.70	1.30	1.45
Average high yield debt		**0.70**	**1.30**	**1.45**
Weighted average across sub-classes		**1.30**	**0.55**	**0.05**

Table A.2: Risk Exposures of Private Equity Funds – Robustness Tests

The risk loading estimates are re-estimated separately with one change in our estimation methodology at a time. The first line shows the results with the default specification; the estimates coincide with those reported in the main tables. Each line then specifies the change made and the corresponding results. Panel A shows results for the CAPM for the sample of private equity funds. Panels B, C, and D show results for the four-factor model of Pástor and Stambaugh (2003) for the sample of private equity funds, venture capital funds, and buyout funds, respectively.

Panel A: CAPM

Change	β_{market}	In-sample Alpha	Persistence of Alpha	R-square	Nobs
Default	1.41^{a}	0.05^{a}	0.40	93.0%	630
	0.24	*0.01*	*0.19*		
Sigma max from 50% to 75%	1.43^{a}	0.05^{a}	0.27	81.1%	630
	0.40	*0.01*	*0.18*		
Sigma priors from 10 to 5	1.35^{a}	0.05^{a}	0.42^{a}	90.1%	630
	0.18	*0.01*	*0.16*		
Sigma priors from 10 to 2	1.31^{a}	0.05^{a}	0.46^{a}	86.6%	630
	0.10	*0.01*	*0.10*		
Beta priors increase by 0.5	1.65^{a}	0.04^{a}	0.40^{b}	95.5%	630
	0.22	*0.00*	*0.19*		
Beta priors decrease by 0.5	1.15^{a}	0.05^{a}	0.40^{b}	87.8%	630
	0.26	*0.01*	*0.19*		
NAV threshold from 50% to 33%	1.39^{a}	0.07^{a}	0.41^{b}	92.6%	484
	0.26	*0.01*	*0.20*		
NAV threshold from 50% to 66%	1.37^{a}	0.04^{a}	0.34^{c}	93.2%	790
	0.23	*0.00*	*0.18*		
NAV threshold from 50% to 75%	1.34^{a}	0.04^{a}	0.32^{c}	93.1%	868
	0.23	*0.00*	*0.17*		

Panel B: Four factor model

Change	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha	R-square	Nobs
Default	1.41^{a}	0.41	0.03	0.36	0.00	0.48	97.0%	630
	0.21	*0.26*	*0.23*	*0.27*	*0.02*	*0.19*		
Sigma max from 50% to 75%	1.46^{a}	0.40	-0.13	0.22	0.02	0.28	88.2%	630
	0.38	*0.52*	*0.47*	*0.53*	*0.02*	*0.18*		
Sigma priors from 10 to 5	1.36^{a}	0.50^{a}	0.03	0.46^{a}	*-0.01*^{b}	*0.49*^{a}	96.3%	630
	0.14	*0.15*	*0.15*	*0.15*	*0.01*	*0.13*		
Sigma priors from 10 to 2	1.31^{a}	0.54^{a}	0.04	0.49^{a}	*-0.02*^{a}	*0.50*^{a}	95.1%	630
	0.07	*0.07*	*0.07*	*0.07*	*0.01*	*0.07*		
Beta priors increase by 0.5	1.53^{a}	0.78^{a}	0.25	0.50^{c}	*-0.04*^{a}	*0.51*^{a}	98.5%	630
	0.18	*0.28*	*0.24*	*0.26*	*0.01*	*0.19*		
Beta priors decrease by 0.5	1.13^{a}	-0.01	-0.29	-0.01	*0.07*^{a}	*0.41*^{b}	88.5%	630
	0.32	*0.39*	*0.34*	*0.42*	*0.01*	*0.19*		
NAV threshold from 50% to 33%	1.41^{a}	0.42	0.03	0.36	*0.02*^{c}	*0.48*^{a}	96.7%	484
	0.22	*0.28*	*0.25*	*0.29*	*0.01*	*0.19*		
NAV threshold from 50% to 66%	1.40^{a}	0.41	-0.02	0.35	*-0.01*	*0.44*^{b}	97.0%	790
	0.20	*0.26*	*0.23*	*0.28*	*0.01*	*0.17*		
NAV threshold from 50% to 75%	1.37^{a}	0.39	-0.05	0.31	*-0.01*	*0.41*^{b}	96.5%	868
	0.22	*0.27*	*0.24*	*0.29*	*0.01*	*0.18*		

Panel C: Four-factor model for venture capital

Change	Alpha Full sample	Alpha Est. sample	Persistence	β_{market}	R^2	Nobs
Default	1.6%	3.9%	0.56[a]	1.62[a]	0.94	272
		0.6%	*0.18*	*0.26*		
Sigma max from 50% to 75%	1.6%	3.8%	0.38[c]	1.66[a]	0.82	272
		0.8%	*0.20*	*0.41*		
Sigma priors from 10 to 5	*1.6%*	*3.8%*	*0.52*[a]	*1.66*[a]	0.93	272
		0.6%	*0.16*	*0.19*		
Sigma priors from 10 to 2	1.6%	3.8%	0.49[a]	1.69[a]	0.90	272
		0.6%	*0.10*	*0.11*		
Beta priors increase by 0.5	1.7%	3.7%	0.54[a]	1.84[a]	0.96	272
		0.6%	*0.19*	*0.22*		
Beta priors decrease by 0.5	1.9%	4.4%	0.58[a]	1.35[a]	0.90	272
		0.6%	*0.18*	*0.30*		
NAV threshold from 50% to 33%	1.6%	7.3%	0.58[a]	1.63[a]	0.94	203
		0.6%	*0.19*	*0.26*		
NAV threshold from 50% to 66%	1.6%	2.6%	0.51[a]	1.62[a]	0.94	344
		0.6%	*0.19*	*0.27*		
NAV threshold from 50% to 75%	1.6%	2.4%	0.50[b]	1.60[a]	0.93	376
		0.6%	*0.19*	*0.27*		

Panel D: Four-factor model for buyouts

Change	Alpha Full sample	Alpha Est. sample	Persistence	β_{market}	R^2	Nobs
Default	4.7%	5.6%	0.41[b]	1.22[a]	0.87	243
		0.5%	*0.19*	*0.27*		
Sigma max from 50% to 75%	4.7%	5.6%	0.38[c]	1.20[a]	0.82	243
		0.5%	*0.20*	*0.29*		
Sigma priors from 10 to 5	4.7%	5.6%	0.42[b]	1.23[a]	0.85	243
		0.5%	*0.17*	*0.20*		
Sigma priors from 10 to 2	4.7%	5.6%	0.46[a]	1.24[a]	0.81	243
		0.5%	*0.11*	*0.12*		
Beta priors increase by 0.5	4.7%	5.5%	0.42[b]	1.42[a]	0.91	243
		0.5%	*0.20*	*0.26*		
Beta priors decrease by 0.5	4.9%	5.8%	0.42[b]	1.01[a]	0.79	243
		0.5%	*0.20*	*0.30*		
NAV threshold from 50% to 33%	4.7%	6.0%	0.36[c]	1.15[a]	0.86	193
		0.6%	*0.20*	*0.29*		
NAV threshold from 50% to 66%	4.7%	5.1%	0.40[b]	1.16[a]	0.88	306
		0.4%	*0.19*	*0.23*		
NAV threshold from 50% to 75%	4.8%	5.1%	0.42[b]	1.12[a]	0.86431	336
		0.4%	*0.19*	*0.23*		

Table A.3: Monte Carlo Simulations

The frequency is quarterly and the overall time span is 80 quarters. The simulation setup is described in details in Appendix C.2. The simulations are repeated 100 times. Panel A reports the mean, standard deviation and quartile thresholds across these 100 simulations for the estimated parameters. Panel B shows the summary statistics of the estimated latent total returns, g_t, and the private equity premium component, f_t. Panel C reports the correlations between the true return of g_t and our Gibbs sampling estimates, and other performance measures (IRR, multiple, and PME).

Panel A: Factor parameters

	True Value	N=200	N=400	N=1000
Mean α	0.0125	0.007	0.010	0.011
Std α		0.008	0.006	0.005
Mean ϕ	0.500	0.423	0.444	0.478
Std ϕ		0.083	0.076	0.074
Mean β	1.500	1.486	1.533	1.498
Std β		0.287	0.257	0.188
Mean σ_f	0.050	0.046	0.052	0.048
Std σ_f		0.008	0.007	0.005
Mean σ	0.100	0.110	0.129	0.120
Std σ		0.035	0.016	0.014
Mean $RMSE(g_t)$		0.051	0.050	0.045
Std $RMSE(g_t)$		0.008	0.007	0.004
Lower quartile α		0.002	0.004	0.004
Upper quartile α		0.011	0.014	0.015
Lower quartile β		1.363	1.344	1.402
Upper quartile β		1.636	1.702	1.675

Panel B: Index estimates

	N=200		N=400		N=1000	
	True value	Estimation	True value	Estimation	True value	Estimation
Mean g_t	3.76%	3.90%	3.72%	4.08%	3.91%	4.09%
Std g_t	14.32%	13.29%	14.44%	13.81%	14.09%	13.95%
Lower quartile f_t	-3.58%	-0.73%	-3.82%	-0.83%	-3.83%	-2.13%
Median f_t	0.20%	0.57%	0.03%	0.72%	0.06%	0.86%
Upper quartile f_t	3.87%	1.93%	3.82%	2.33%	3.98%	2.87%

Panel C: Correlation between true g_t and other performance measures

	N=200	N=400	N=1000
Estimated g_t	54.97%	94.55%	95.24%
IRR	1.91%	3.28%	3.77%
Multiple	-2.45%	-2.36%	-4.58%
PME	-1.15%	-0.89%	-3.00%

Figure 1: Private Equity Return Index vs. VanguardS&P500 Index Fund

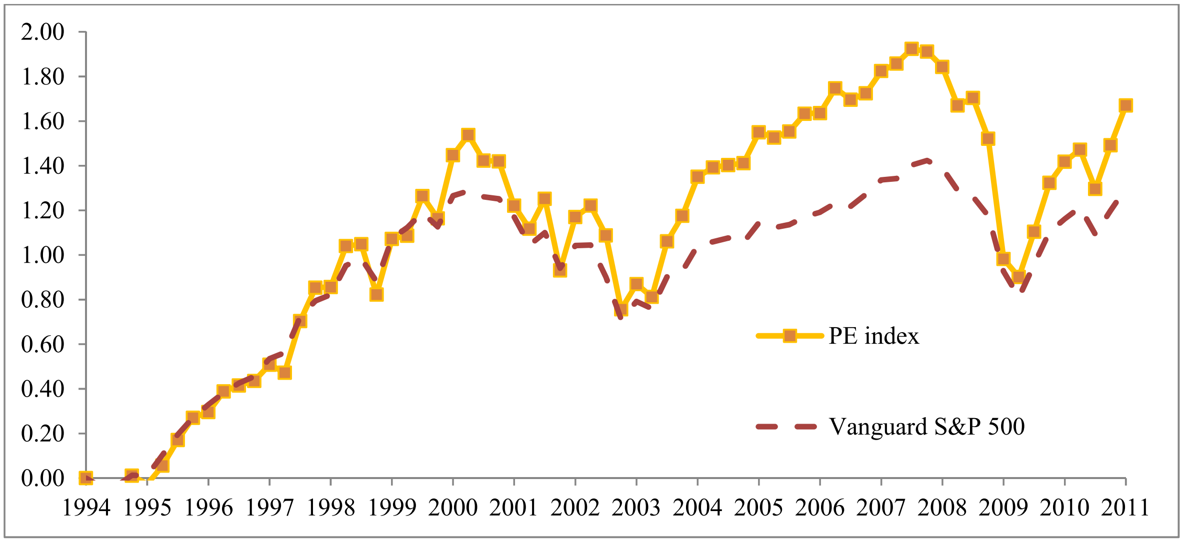

Figure 2: Decomposition of Private Equity Return Index into Passive and Premium Components

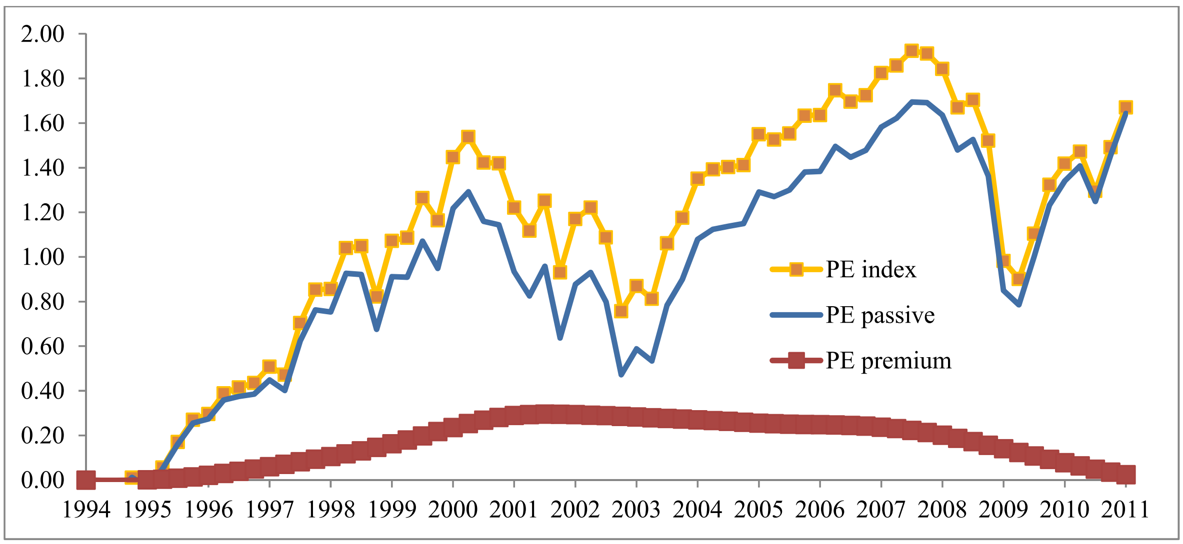

Figure 3: Quarterly Private Equity premium

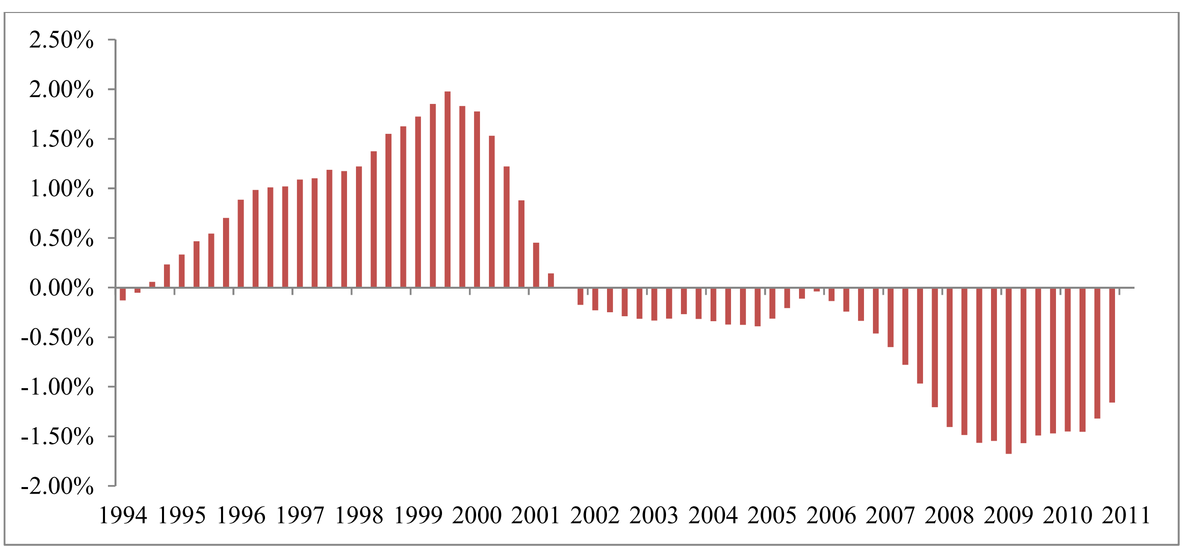

Figure 4
Quarterly Private Equity Premiums per Sub-Classes