Debt Crises: For Whom the Bell Tolls*

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PRELIMINARY AND INCOMPLETE

Abstract

What a country has done in the past, and what other countries are doing in the present can feedback for good or for ill. We develop a simple model that can address hysteresis and contagion in sovereign debt markets. When a country's fundamentals change, those changes affect information acquisition about that country but also affect the allocation of investment funds worldwide, inducing changes in the dynamics of sovereign spreads in seemingly unrelated countries.

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1 Introduction

Several features of sovereign debt markets are difficult to explain.

First, contagion. Sovereign debt crises tend to be highly correlated across countries and sovereign spreads (the sovereign's cost of external funding), tend to co-move strongly. The most recent example is the 2010 debt crisis in Europe. Beirne and Fratzscher (2013), using information for 31 advanced and emerging economies during the crisis, find that there was a sharp and simultaneous increase in sovereign spreads across in both European and non-European countries. Similar forces were at play in the debt crises initiated by Poland in 1981, Mexico in 1994, Thailand in 1997, Russia in 1998, and Argentina 2001[1].

Previous work has attempted to explain contagion by appealing to different types of linkages between countries. One branch of the literature focuses on *real linkages*. For example, trade in goods or financial assets between countries may transmit negative shocks from one country to the next and lead to co-movements in sovereign spreads (e.g., Alter and Beyer (2014) and Gross and Kok Sorensen (2013)). A second branch focuses on *belief linkages* through learning and herding. In this view (e.g., (De Santis 2012)), contagion is driven by the correlation of beliefs about fundamentals in different countries, so that bad news about one country make investors pessimistic about other countries. Of course, a prerequisite for belief correlation to cause contagion is that observations about one country hold information about other countries. This requires correlation in fundamentals across countries, or the existence of a common unobservable variable linking all countries. Theories of contagion based on belief linkages therefore also require real linkages between countries. Finally, a third set of explanations relies on the rationalization of crises as self-fulfilling roll-over problems a la Cole and Kehoe (1996). To explain contagion, however, this literature requires a correlated structure of sunspots to induce simultaneous roll-over crisis episodes in many countries at the same time.

Because many extant theories of contagion rely on the existence of structural links across countries, finding evidence for such linkages is imperative in providing support for them. Problematically, however, it is often difficult to empirically identify linkages that are plausibly powerful enough to induce the degree of contagion observed in many debt crisis episodes. Again taking the recent European experience

[1] For a survey of these cases see Reinhart and Rogoff (2009).

as an example, Beirne and Fratzscher (2013) explore empirical models with economic fundamentals and find that "the market pricing of sovereign risk may not have been fully reflecting fundamentals prior to the crisis."

Second, sovereign risk premia seem only loosely connected to the country's fundamentals more generally: they frequently exhibit sudden changes without obvious changes in underlying fundamentals, and sometimes fluctuate without any observable changes in fundamentals at all. Indeed, sovereign risk premia seem to react differently to a given change in fundamentals at different points in time.

Third, there seems to be history dependence in the borrowing conditions faced by different countries: the same change in fundamentals may have different effects in different countries, and these differences are persistent over time. Indeed, a given country's *past* behavior seems to matter for how sovereign spreads react to changes in fundamentals. Consider, for example, the diverging experiences of Argentina and the United States. The U.S. seems to be in a "stable" environment that allows it to accumulate high debt levels without triggering increases in spreads, while Argentina, in constrast, seems to be in an "unstable" environment in which slight changes in fundamentals cause large and sudden changes in spreads.

To jointly accommodate all of these features within a single framework, we construct a model of sovereign bond markets with many countries and two key elements. First, there is a global pool of risk-averse investors who freely allocate funds across sovereign bond markets. Second, these investors can choose to produce information about a country's fundamentals at a cost. This information is valuable because informed investors are able to exploit their superior knowledge of a country's fundamentals to outbid uninformed investors in particularly attractive states of the world. In equilibrium, this benefit is exactly offset by the cost of becoming informed.

Our first result is that the free flow of capital across countries can generate contagion across countries, even in the absence of any real linkages, correlation of fundamentals, or belief updating about one country due to equilibrium outcomes in another country. Specifically, when investor preferences exhibit *prudence* (that is, $u'''(c) > 0$, as is the case for CRRA utility functions), an increase in the probability of default in one country increases sovereign spreads for all sovereign bonds held by the investor. This is because an increase in the default risk of a given country increases the *background risk* inherent in the entire portfolio of sovereign bonds, and thereby reduces the investor's appetite to invest in sovereign debt more generally. Hence, sovereign bond prices fall across all countries when one country becomes more likely to default. If this effect is sufficiently large and the increase in spreads is severe enough, it may no longer be feasible for countries to roll over their debt, causing a wave of debt crises.

Our contagion result relies only on investor prudence and the fact that there is a common pool of investors for all countries. Hence it does *not* rely on changes in investors' wealth (as in Kyle and Xiong (2001) or Goldstein and Pauzner (2004)), borrowing constraints (as in Yuan (2005)) or short-selling constraints (as in Calvo and Mendoza (1999)). Indeed, contagion stems only from the portfolio rebalancing of prudent investors in response to an increase in the riskiness of a subset of assets at their disposal. For empirical evidence about the importance of portfolio effects on contagion see Broner, Gelos, and Reinhart (2004). For empirical evidence about the importance of risk aversion to explain sovereign spreads see Lizarazo (2013).

Our second result is that the option to produce information about countries' fundamentals can generate multiple equilibria. In particular, an *uninformed equilibrium*, in which no investor acquires information about the country's fundamentals, may co-exist with an *informed equilibrium*, in which some investors do acquire information about the country's fundamentals. These information regimes have real effects: taking as given the stochastic process for fundamentals, the average level and the volatility of spreads differ across regimes. In the uninformed equilibrium, spreads are stable and low on average, because investors are relatively insensitive to variation in fundamentals. In the informed equilibrium, in contrast, spreads are volatile and high on average, because investors strongly react to variation in fundamentals and demand very high risk premia in bad states of the world. For this reason, sovereigns strictly prefer an uninformed equilibrium to an informed equilibrium. Because information acquisition is costly, and information rents come at the expense of other investors, the same is true for investors.

An important upshot from our analysis is that, because investors' optimal portfolio choice and the information regime *jointly* determine the mapping from country fundamentals to sovereign bond spreads and the likelihood of debt crises, there need not exist a unique mapping from economic fundamentals to spreads in sovereign bond markets even in the absence of roll-over crises driven by coordination failures. Indeed, since investors choose their portfolio by taking the fundamentals and information regimes in *all* countries into account, the mapping from fundamentals to prices in a single country depends on equilibrium outcomes in all other countries. To the extent that a given pool of investors prices sovereign bonds in multiple countries, understanding contagion and default risk therefore requires a "global" view of bond markets.

Finally, to the extent that informational regimes are persistent (in the sense that there is a change in regime only if the only if the old regime can no longer be sustained), only large changes in fundamentals can force a transition across regimes. This implies that a country starting out in an uninformed equilibrium begins to attract informed investors only if its fiscal situation worsens substantially, while a country starting out in an informed equilibrium requires a substantial improvement of their fiscal situation to discourage information acquisition. In the absence of such large shocks, two given countries may therefore be in different informational regimes, and thus have to pay different spreads, even when their *current* fundamentals are similar. A country's past sins or virtues may therefore be important determinants of current borrowing conditions, and may remain with the country for a long time. We call this phenomenon *hysteresis*.

In the next section we present a model with an arbitrary number of countries and endogenous default probabilities. In Section 3 we discuss multiplicity of equilibria in terms of information acquisition and the outcome in terms of sovereign spreads for the special case of a single country and exogenous default probabilities. In Section 4 we extend the results for a special case of two countries and exogenous default probabilities. In this section we discuss the main source of contagion in its purest form, without any fundamental linkage and no information. In Section 5 we discuss extensions and in Section 6 we conclude.

2 Model

This is a two period model with a mass 1 of investors and J countries (indexed by j). Investors start with initial wealth W and only care about second period consumption c. Their preferences over consumption are given by the concave utility function $u(c)$, such that $u'(c) > 0$ and $u''(c) < 0$. Since investors only care about consumption in period 2, their choice problem is deciding how to invest their wealth in period 1, choosing between a safe asset that has gross return 1, and a combination of risky government debt of the J countries. We describe the source of this risky debt next.

In period 1 each government has an amount of outstanding legacy debt D_j coming due, which is net of the country's income in period 1. This implies that, in order to repay D_j country j, has to roll over the debt. We assume that the government rolls over this debt using pure discount bonds via an auction-type market. In this market, investors specify combinations (possibly menus) of prices P and quantities B they wish to purchase in each country. The government sells debt to the highest bidder until it either exhausts the bids or sells enough to roll over its debt. If the government cannot roll over its debt then it must default, a situation we call a *debt crisis*.

In period 2 the debt issued in period 1 comes due. The government then chooses whether to repay its debt using its income Y generated in period 2, or to default. If the government defaults in either period the total output that remains is $(1-\theta_j)Y_j$ where $\theta_j \in [0,1]$ is the cost of default in terms of lost income. Both the government's default cost factor θ_j and its income Y_j are random. While the realization of Y_j is drawn in period 2, the realization of θ_j is drawn in period 1 from a discrete distribution with S elements $\Theta = \{\theta_{j,1}, .., \theta_{j,S}\}$, such that $\theta_{j,1} > .. > \theta_{j,s} > .. > \theta_{j,S}$. In period 1, the investors who plan to invest in a country's bond can choose to acquire information about θ_j at a utility cost $u(K)$.

By design, the model has a number of features that make it analytically very tractable. Since the cost of default is independent of whether the government defaults in both periods or only 1, it will turn out that it will always default in period 2 if it has defaulted in period 1. Since the government is just seeking to roll over its debt in period 1, it will always do so if it can; reserving the decision to default for the second period. These features allow us to focus on the impact of the investors decisions.

2.1 The problem of one country

For simplicity, and without loss of generality, we focus on the construction of equilibria in a single country that we will refer as the *domestic country* (for notational simplicity we get rid of the subindex j in what follows). Investors bidding in the country can be of two types: informed and uniformed, which is an investor's choice. Denote by n the fraction of informed investors bidding in the domestic country and by P the marginal price of government debt in period 1. If there are informed investors then this marginal price will depend upon the realized θ, and in this case we will denote it by $P(\theta)$. Because informed traders know θ, they know the price that the marginal investor must pay for government debt and hence bid the price $P(\theta)$ along with the (conditional) quantity that they wish to purchase at that price, $B^I(\theta)$. The uninformed traders may (and will) find it advantageous to bid heterogeneous price-quantity pairs. Because they know the set of possible marginal prices, $\{P(\theta_1), ..., P(\theta_J)\}$ they will choose the quantities to bid at each one of these prices. Let $B^U(\theta)$ denote the amount that an uninformed trader bids if he chooses to bid at price $P(\theta)$.

The auction arrangement leads to the following budget constraints for the government. In period 1, and for a given θ, if it can roll over its debt in period 1, then

$$nB^I(\theta)P(\theta) + (1-n) \sum_{\left\{\widehat{\theta}: P(\widehat{\theta}) \geq P(\theta)\right\}} B^U(\widehat{\theta})P(\widehat{\theta}) = D. \tag{1}$$

If the government cannot roll over the debt in period 1, then

$$nB^I(\theta)P(\theta) + (1-n) \sum_{\left\{\widehat{\theta}: P(\widehat{\theta}) \geq P(\theta)\right\}} B^U(\widehat{\theta})P(\widehat{\theta}) < D,$$

in which case it must default. We will refer to this second case as a *debt crisis*.

If the government hasn't defaulted in period 1, its debt coming due in period 2 is

$$R(\theta) = nB^I(\theta) + \sum_{\left\{\widehat{\theta}: P(\widehat{\theta}) \geq P(\theta)\right\}} (1-n)B^U(\widehat{\theta})$$

In this case the government's payoff if it doesn't default in period 2 is $Y - R(\theta)$, while it is $(1-\theta)Y$ if it does default in period 2. This leads to a simple cut-off rule in which the government defaults in period 2 if and only if $Y < \bar{Y}(\theta)$, where

$$\bar{Y}(\theta) \equiv \frac{R(\theta)}{\theta}. \tag{2}$$

Since we assume that the government's income is $(1-\theta)Y$ if it has already defaulted in period 1; irrespective of whether it defaults in period 2, the government will always default in period 2 if it has defaulted in period 1. In addition, it is always weakly better off waiting to default in period 2 if possible (there are no gains from defaulting in the first period rather than rolling over with the possibility of repaying in the second period).

Since $\bar{Y}(\theta)$ denotes the government's cut-off rule for defaulting as a function of θ, the realized return to an investor is 1 if $Y \geq \bar{Y}(\theta)$ and 0 otherwise. In other words, it is 1 with probability $\Pr\left\{Y \geq \bar{Y}(\theta)\right\}$ and 0 with probability $1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}$. Then, so long as the total amount coming due, $R(\theta),$ is weakly decreasing in θ (the higher the cost of default, the higher the price of debt and the less debt comes due in period 2), it follows that the default cut-off is strictly decreasing in θ and the default probability is also weakly decreasing in θ. In words, the higher the cost of default θ the less likely is that the country defaults, this decreases the repayment needs and reduces the probability of default, which is consistent with a lower repayment need.

An informed agent knows θ and takes as given the marginal price of debt $P(\theta)$. Therefore, their maximization problem is given by

$$\begin{aligned} U^I(\theta) = \max_{B^I(\theta)\geq 0} \quad & u\left(W + \left[1 - P(\theta)\right] B^I(\theta)\right) \Pr\left\{Y \geq \bar{Y}(\theta)\right\} \\ & + u\left(W - P(\theta) B^I(\theta)\right)\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] - u(K), \end{aligned} \tag{3}$$

which implies that their first-order condition is,

$$\begin{aligned} & u'\left(W + \left[1 - P(\theta)\right] B^I(\theta)\right)\left[1 - P(\theta)\right] \Pr\left\{Y \geq \bar{Y}(\theta)\right\} \\ & + u'\left(W - P(\theta) B^I(\theta)\right)\left[-P(\theta)\right]\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \leq 0, \end{aligned} \tag{4}$$

and with strict equality if $B^I(\theta) > 0.$

An uniformed agent must choose how much to bid at each one of the possible marginal prices $P(\theta)$. The maximization problem of an uninformed agent is then

$$U^U = \max_{\{B^U(\widehat{\theta}_1),\ldots,B^U(\widehat{\theta}_S)\}} \sum_{\theta\in\Theta} \Pr(\theta) \left\{ \begin{array}{c} u\left(W + \sum_{\{\widehat{\theta}:\widehat{\theta}\geq\theta\}} \left[1 - P(\widehat{\theta})\right] B^U(\widehat{\theta})\right) \Pr\left\{Y \geq \bar{Y}(\theta)\right\} \\ u\left(W - \sum_{\{\widehat{\theta}:\widehat{\theta}\geq\theta\}} P(\widehat{\theta}) B^U(\widehat{\theta})\right)\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \end{array} \right\}.$$

which implies that his first-order condition for $B^U(\widehat{\theta})$ is,

$$\sum_{\{\theta:\theta\leq\widehat{\theta}\}} \Pr\left\{\theta\right\} \left\{ \begin{array}{c} u'\left(W + \sum_{\{\theta':\theta\leq\theta'\leq\widehat{\theta}\}} \left[1 - P(\theta')\right] B^U(\theta')\right)\left[1 - P(\widehat{\theta})\right] \Pr\left\{Y \geq \bar{Y}(\theta)\right\} \\ u'\left(W - \sum_{\{\theta':\theta\leq\theta'\leq\widehat{\theta}\}} P(\theta') B^U(\theta')\right)\left[-P(\widehat{\theta})\right]\left[1 - \Pr\left\{Y \geq \bar{Y}(\theta)\right\}\right] \end{array} \right\} \leq 0, \tag{5}$$

where this condition holds as an equality if $B^U(\widehat{\theta}) > 0$. As the decision of the quantities to bid at different prices are linked through first order conditions, the bids in equilibrium are the solution to the system of equations (5) for all θ.

Finally, if any investor decides to become informed, investors must be indifferent between being informed or staying uninformed. Hence

$$\sum_{\theta} \Pr\{\theta\} U^I(\theta) - u(K) \leq U^U \text{ with strict equality if } n > 0. \tag{6}$$

The previous discussion summarizes the main elements of the problem of a single country, which is completely indexed by n from equation (6).

2.2 General Equilibrium

An equilibrium will consist of a set of cut-offs $\bar{Y}_j(\theta)$, prices $P_j(\theta)$, quantities for the informed and uninformed ($B^I_j(\theta)$ and $B^U_j(\theta)$ respectively), a fraction of informed investors (n_j) for all countries $j \in \{1, ..., J\}$ such that the following conditions are satisfied.

1. The period 1 bond market from equation (1) clears in each country for each state θ, or $\bar{Y}_j(\theta) = 0$ and $P_j(\theta) = 0$ and there is a debt crisis in state θ in country j.
2. The set of cut-offs, $\bar{Y}(\theta)$, satisfy the threshold condition (2).
3. The choices of $B^I_j(\theta)$ and $B^U_j(\theta)$ are solutions to the informed and uniformed investors' problems (first order conditions (4) and (5) respectively).
4. The fraction of informed investors in each country n_j must satisfy the indifference condition (6). A country is an *informed equilibrium* when $n_j > 0$ and in an *uninformed equilibrium* when $n_j = 0$.

There is a variety of equilibria. For example, *no-lending* with $P_j(\theta) = 0$ and $\bar{Y}_j(\theta) = 0$ for all θ and all j is always an equilibrium. In the next section we present a simplified *special case* to characterize the other (potentially multiple) equilibria in a tractable and intuitive way.

3 A Special Case

Here we introduce several simplifications that are useful to understand the forces at play in the model. First we assume just two countries (that is, $J = 2$). Second we assume just two possible costs of default in each country (that is, $S = 2$), such that $0 < \theta_L^j < \theta_H^j < 1$, where θ_H^j is realized in period 1 with probability a_j (situation that we denote as *good state*) and θ_L^j is realized in period 1 with probability $1 - a_j$ (situation that we denote as *bad state*). Third, we assume just three possible income realizations in period 2, $Y_L^j < Y_M^j < Y_H^j$, where Y_L^j happens with probability x_j and Y_M^j with probability z_j. Finally, we assume θs and Ys are such that default cutoffs are exogenous in each country. Formally,

Assumption 1

$$Y_L^j < \bar{Y}(\theta_H^j) < Y_M^j < \bar{Y}(\theta_L^j) < Y_H^j \qquad \forall\ j$$

This assumption guarantees that when the cost of default is high (good state), the country only defaults when the income is low, which implies a default probability of $\kappa_H^j \equiv x_j$. When the cost of default is low (bad state), the country only repays when the income is high, which implies a default probability of $\kappa_L^j \equiv x_j + z_j$. Naturally this assumption depends on endogenous variables (the prices of debt $P_j(\theta_L^j)$ and $P_j(\theta_H^j)$, but in equilibrium these will be expressed in terms of primitives and we will have to guarantee these are fulfilled).

We first characterize the set of equilibria of this *special case* for a single country (and thus dispense with the labeling j). One interpretation focusing on a single country only is that the second country does not have any outstanding debt to roll over in period 1. Then we discuss equilibria when investors can invest in both countries and discuss the source in determining the strength of this contagion.

3.1 A Single Country

3.1.1 Uninformed Equilibrium

First we study the conditions for the existence of an equilibrium in which no investor is informed about the state of the country. In this case, we define the expected probability of default as

$$\widehat{\kappa} \equiv ax + (1-a)(x+z)$$

Since there is no information about the country's state there is a single marginal price P. Given this price, we can rewrite the first order condition (4) as

$$\frac{u'(W + [1-P]B)}{u'(W - PB)} = \frac{P\widehat{\kappa}}{(1-P)(1-\widehat{\kappa})} \tag{7}$$

The next proposition displays properties of this first-order condition in terms of how bid quantities depend on parameters.

Proposition 1 *The investors' demand of sovereign bonds is decreasing in the price the default probability.*

Proof Rewriting the first order condition (7) as

$$F(B|P, \widehat{\kappa}) \equiv \frac{u'(W + [1-P]B)}{u'(W - PB)} - \frac{P\widehat{\kappa}}{(1-P)(1-\kappa)} = 0$$

define $u'(+) \equiv u'(W + [1-P]B)$ and $u'(-) \equiv u'(W - PB)$. Differentiating with respect to $\widehat{\kappa}$, $\frac{dB}{d\widehat{\kappa}}$ is negative as

$$\frac{\partial F}{\partial B} = \frac{(1-P)u''(+)u'(-) + Pu''(-)u'(+)}{u'^2(-)} < 0$$

and

$$\frac{\partial F}{\partial \widehat{\kappa}} = -\frac{P}{(1-P)(1-\widehat{\kappa})^2} < 0$$

Similarly, differentiating with respect to P, $\frac{dB}{dP}$ is negative if

$$\frac{\partial F}{\partial P} = \frac{B}{u'^2(-)}[u''(-)u'(+) - u''(+)u'(-)] - \frac{\widehat{\kappa}}{(1-P)^2(1-\widehat{\kappa})}$$

is negative. A sufficient condition for this to be the case is that $\frac{u''(-)}{u'(-)} \leq \frac{u''(+)}{u'(+)}$, which is always the case for CRRA and CARA preferences. Q.E.D.

Notice that under risk-aversion, $P < 1 - \widehat{\kappa}$ as $u'(+) < u'(-)$ in equation (7).

The first-order condition together with the resource constraint pins down the price in equilibrium. Substituting the resource constraint $PB = D$ into the first-order condition,

$$\frac{u'(W-D+\frac{D}{P})}{u'(W-D)} = \frac{P\widehat{\kappa}}{(1-P)(1-\widehat{\kappa})} \tag{8}$$

Proposition 2 *With Inada conditions, there is always a trivial equilibrium with $P = 0$. If there are other equilibria, the highest price in equilibrium decreases with the probability of default $\widehat{\kappa}$ and the country's debt D.*

Proof Define

$$F(P|\widehat{\kappa}) = \frac{u'(W-D+\frac{D}{P})}{u'(W-D)} - \frac{P\widehat{\kappa}}{(1-P)(1-\widehat{\kappa})}$$

A P^* in equilibrium is given by $F(P^*|\widehat{\kappa}) = 0$.

At the one extreme, for $P = 0$, under Inada conditions (this is $\lim_{c\to\infty} u'(c) = 0$), $F(P = 0|\widehat{\kappa}) = 0$, then $P^* = 0$ is always an equilibrium.

At the other extreme, for $P = 1-\widehat{\kappa}$, $F(P = 1-\widehat{\kappa}|\widehat{\kappa}) < 0$ (the first term on $F(P|\widehat{\kappa})$ is less than one and the second term is equal to one), then as discussed above, $P = 1-\widehat{\kappa}$ is never an equilibrium with risk aversion.

If parameters are such that $F(P|\widehat{\kappa}) < 0$ for all $P \in (0, 1-\widehat{\kappa}]$, then the only equilibrium is given by $P^* = 0$. If $F(P|\widehat{\kappa}) > 0$ for some $P \in (0, 1-\widehat{\kappa}]$, then there are other equilibria besides $P^* = 0$. Among those, the maximum P^* sustainable in equilibrium is such that $\frac{\partial F}{\partial P} < 0$ (recall $F(P^*|\widehat{\kappa}) = 0$ and $F(P = 1-\widehat{\kappa}|\widehat{\kappa}) < 0$).

The maximum sustainable price in equilibrium is decreasing in $\widehat{\kappa}$ and D/W as, on the on hand, $\frac{dP}{d\widehat{\kappa}} = -\frac{\frac{\partial F}{\partial \widehat{\kappa}}}{\frac{\partial F}{\partial P}}$ and $\frac{\partial F}{\partial \widehat{\kappa}} = -\frac{P}{(1-P)(1-\widehat{\kappa})^2} < 0$ whereas on the other hand, $\frac{dP}{dD} = -\frac{\frac{\partial F}{\partial D}}{\frac{\partial F}{\partial P}}$ and $\frac{\partial F}{\partial D} = -\frac{\frac{1-P}{P}u''(+)+\frac{u'(+)}{u'(-)}u''(-)}{u'(-)} < 0$ Q.E.D.

To provide intuition, the next figure plots the left hand side of equation (8), in black and the right hand side in different colors for three different levels of $\widehat{\kappa}$. The equilibrium price is determined by the intersection of the two curves. The higher is the expected probability of default, the higher is the right hand side and the smaller is the price P in equilibrium. When $\widehat{\kappa}$ is large enough, the only feasible equilibrium is a $P^* = 0$.

The next figure shows the right hand side of equation (8) in black and the right hand side in different colors for three different levels of D/W. As before, the equilibrium

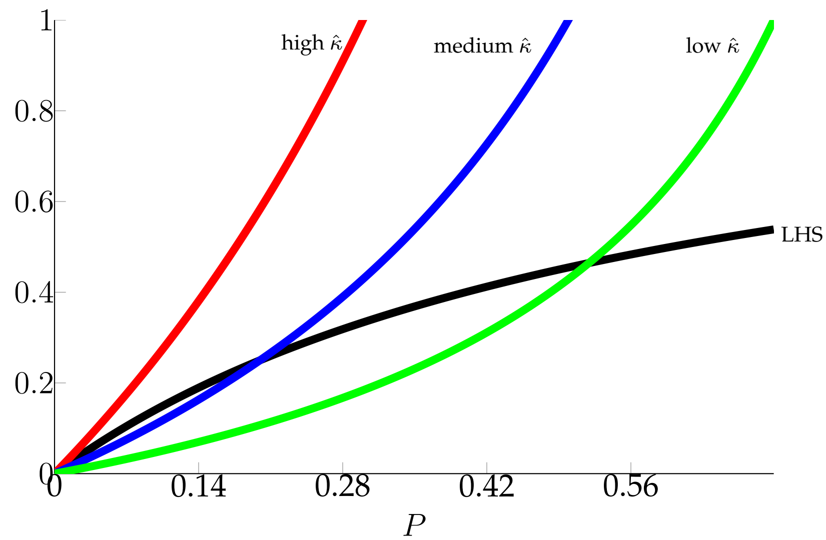

Figure 1: Price Determination for Different Levels of $\widehat{\kappa}$

price is determined by the intersection of the two curves. The higher is the relative indebtedness of the country, the higher is the left hand side and the smallest the price P in equilibrium. When D/W is large enough, the only feasible equilibrium is a $P^* = 0$.

When is an uninformed equilibrium sustainable? To answer this question, we have to determine the incentives for a single uninformed investor to deviate and acquire information, paying a utility cost $u(K)$. Because a single investor's bidding behavior does not impact equilibrium prices, the benefits of acquiring information come from the possibility of re-optimizing the quantities the investor bids at the marginal price P in equilibrium. If the investor learns the state is good, he would like to bid more than uninformed individuals. This is immediate from the first order condition 4 evaluated at P and κ_H, as the bid is decreasing in the probability of default and $\kappa_H < \widehat{\kappa}$. Similarly, If the investor learns the state is bad, he would like to bid less than if he were uninformed.

Defining the expected benefits of acquiring information as

$$\chi^U \equiv a\left[U(B(\kappa_H, P)) - U(B(\widehat{\kappa}, P))\right] + (1-a)\left[U(B(\kappa_L, P)) - U(B(\widehat{\kappa}, P))\right]$$

As $U(B(\kappa, P))$ is obtained by re-optimizing the quantities bid, it is clear that χ^U cannot be negative (as the investor can always replicate his uninformed bid). Then, the

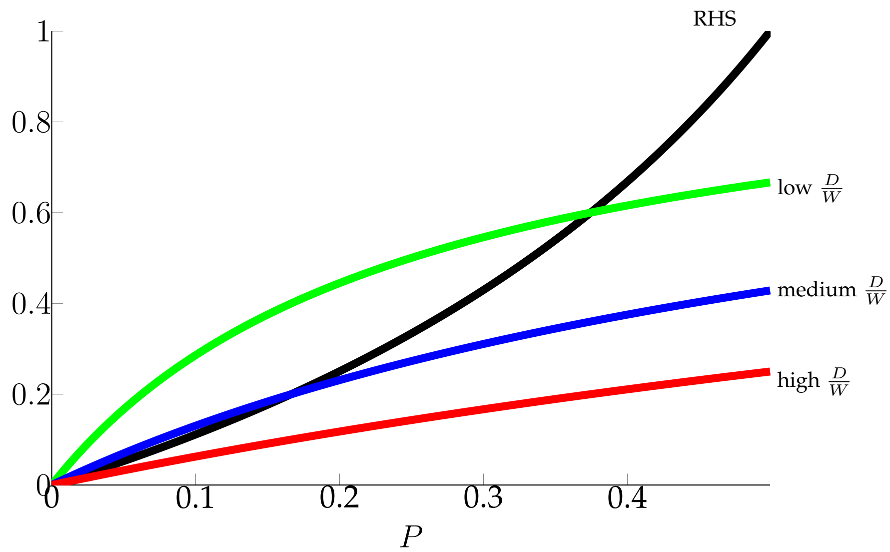

Figure 2: Price Determination for Different Levels of D/W

uninformed equilibrium is feasible as long as

$$u(K) > \chi^U \geq 0$$

Notice that the difference between the optimal bid in each state and the bid without information (this is, $B(\kappa_s, P) - B(\widehat{\kappa}, P)$) is increasing in the absolute difference $\kappa_s - \widehat{\kappa}$. Since $\widehat{\kappa} - \kappa_H = (1-a)z$ and $\kappa_L - \widehat{\kappa} = az$, the gap increases with z and it is maximized at intermediate levels of a. In the extremes, when $a = 0$, $U(B(\kappa_L, P)) = U(B(\widehat{\kappa}, P))$ and $\chi^U = 0$. This is also the case for $a = 1$.

The incentives to acquire information is also increasing in D/W as more exposure to the risky asset increases the differences in utility from knowing the probability of default in each state.

3.1.2 Informed Equilibrium

Denoting the two prices $P_L \equiv P(\theta_L)$ and $P_H \equiv P(\theta_H)$ we can rewrite the first order condition (4) as

$$\frac{u'(W + [1 - P_s]B_s^I)}{u'(W - P_s B_s^I)} = \frac{P_s \kappa_s}{(1 - P_s)(1 - \kappa_s)} \tag{9}$$

where $\kappa_s \in \{\kappa_L, \kappa_H\}$ are the expected probabilities of default in each state s and $P_s \in \{P_L, P_H\}$ are the prices in each state s.

As in the uninformed equilibrium, the next proposition describes the features of these first order conditions, which are identical to those in Proposition 1, as is the proof.

Proposition 3 *Informed investors' demand of sovereign bonds is decreasing in the price the default probability.*

For uninformed investors bidding in the informed equilibrium, we can rewrite the first-order condition (5) for the bid in the low state b_L^U as

$$P_L\kappa_L u'(W - P_H B_H^U - P_L B_L^U) = (1-P_L)(1-\kappa_L)u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U) \tag{10}$$

and for the bid in the high state b_H^U as

$$\begin{aligned} &a\left[P_H\kappa_H u'(W - P_H B_H^U)\right] + (1-a)\left[P_H\kappa_L u'(W - P_H B_H^U - P_L B_L^U)\right] = \\ &a\left[(1-P_H)(1-\kappa_H)u'(W(1-P_H)B_H^U)\right] + (1-a)\left[(1-P_H)(1-\kappa_L)u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U)\right] \end{aligned} \tag{11}$$

Note that, because $P_H > P_L$, the sovereign will sell b_H^U to the uninformed even in the low state.

By comparing these first order conditions, the next proposition describes general properties of the total expenditures on sovereign debt by uninformed investors.

Proposition 4 *Uninformed investors spend more than informed investors in the bad state and less than informed investors in the good state.*

Proof First, we prove that uninformed investors spend less than informed investors in the bad state, that is $P_L B_L^I < P_H B_H^U + P_L B_L^U$.

Suppose not, so that $P_L B_L^I \geq P_H B_H^U + P_L B_L^U$. Then

$$P_L\kappa_L u'(W - P_L B_L^I) \geq P_L\kappa_L u'(W - P_H B_H^U - P_L B_L^U)$$

From the first-order conditions for informed investors in the bad state (9) and the first-order condition for uninformed investors in the bad state (10), this implies

$$u'(W + (1-P_L)B_L^I) \geq u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U)$$

or

$$B_L^I - (B_H^U + B_L^U) \leq P_L B_L^I - (P_H B_H^U + P_L B_L^U) < B_L^I - (\frac{P_H}{P_L} B_H^U + B_L^U)$$

where the second inequality is the result of $P_L < 1$. This is a contradiction for all $P_H > P_L$.

Second, we prove that uninformed spend more that informed in the good state. This is, $P_H B_H^I > P_H B_H^U$. Notice the first-order condition for uninformed investors in the good state (12) can be rewritten as

$$(1-a)\left[P_H \kappa_L u'(W - P_H B_H^U - P_L B_L^U) - (1-P_H)(1-\kappa_L)u'(W + (1-P_H)B_H^U + (1-P_L)B_L^U)\right] =$$
$$a\left[(1-P_H)(1-\kappa_H)u'(W(1-P_H)B_H^U) - P_H \kappa_H u'(W - P_H B_H^U)\right]$$

From equation (10) and $P_H > P_L$ the left hand side is positive. This implies

$$\frac{u'(W + [1-P_H]B_H^U)}{u'(W - P_H B_H^U)} > \frac{P_H \kappa_H}{(1-P_H)(1-\kappa_H)}$$

Comparing with the first order conditions for informed investors in the good state (9), then $B_H^U < B_H^I$. Q.E.D.

The intuition for this result is as follows. On the one hand, if uninformed investors spend the same amount as informed investors in the bad state, they pay a higher price than informed investors on a fraction $\frac{B_H^U}{B_L^U + B_H^U}$ of the debt that they purchase. This implies that the uninformed incur the same losses as the informed in case of default, but receive smaller gains in case of repayment: $B_L^U + B_H^U < B_L^I$. The marginal benefits of spending more in the bad state are thus larger than the marginal costs, which induces the uninformed to spend more than informed in that state.

On the other hand, whatever uninformed spend in the good state they also spend in the bad state. As they are overexposed to sovereign debt in the bad state they rather reduce their exposure in the good state when compared to the informed investors.

In principle, of course, B_H^U could be negative. The next proposition shows this is not possible. More specifically, there can be no short-selling of bonds in equilibrium.

Proposition 5 *There are no short-sales in equilibrium:* $B_H^U \geq 0$.

Proof Suppose not, so that $B_H^U < 0$. This implies that uninformed investors want to short-sell sovereign bonds at P_H, which they consider highly over-priced given that they receive any bond purchased in P_H in both states of the world. Given that the government is a borrower, the only potential counterparty to the uninformed's short-selling are informed investors. But the informed investors are willing to buy at bonds P_H only if the state is good. Hence, whenever the informed are willing to fill the short-sales by the uninformed, the uninformed can infer that the state must be good. But in the good state, the uninformed do not want to short-sell the bond at P_H. Hence, bidding $B_H^U < 0$ reveals information and is not sustainable in equilibrium. Q.E.D.

This result is reminiscent to the impossibility of informationally efficient market result in Grossman and Stiglitz (1980). When informed investors (in this case the only possible source of demand for short-sold bonds) are willing to participate at a price that uninformed investors thought excessive, they reveal their information about the correct price. In other words, informed investors reveal the state is good when uniformed want to sell bonds and they are willing to buy, but revealing that information uninformed investors would not like to sell, but rather buy. Notice this is not the case when uninformed investors are always bidding positive amounts as the government is always the other side of the market and then does not reveal information about the state by its actions.

We refer to the set of parameters under which $B_H^U = 0$ (that is, parameters such that uninformed investors bid nothing at P_H, and, thus, do not purchase any bonds in the high state), as the *partial participation region* (partial because only informed investors participate). For completeness, we refer to the set of parameters under which $B_H^U > 0$, so that the uninformed investors also get to buy sovereign bonds on the good state, as the *full participation region*.

Notice that, in the *partial participation region*, the uninformed investors know the default probability *conditional on being able to purchase the bond in equilibrium*, because they know they are only able to buy sovereign debt in the bad state. Hence, the informed and the uninformed behave symmetrically in the bad state. This is straightforward from replacing $B_H^U = 0$ in the first order condition (10) and comparing it with the first order condition (9). This implies that all information rents in the partial participation region stem from informed investor's ability to purchase bonds in both states of the world.

Now that we have characterized how informed and uninformed investors bid at different prices, we can characterize properties of the prices as a function of the fraction of investors that are informed, which we denote by n. Then we will endogenize the fraction of investors in equilibrium, n^*, by exploiting the free-entry condition under which investors are indifferent between being informed or uninformed.

Proposition 6 *Consider the equilibrium with the highest sustainable prices. The good state price, P_H, increases with the fraction of informed investors, n.*

Proof The resource constraint for the good state in the *partial participation region* is just

$$nP_H B_H^I = D$$

Increasing n is isomorphic to decreasing D, and as we showed in Proposition 2 this implies $\frac{dP_H}{dn} > 0$.

The resource constraint for the good state in the *full participation region* is just

$$nP_H B_H^I + (1-n)P_H B_H^U = D$$

We can rewrite it in terms of excess demand as

$$ED(P_H) = B_H^U + n(B_H^I - B_H^U) - \frac{D}{P_H} = 0$$

Then

$$\frac{dP_H}{dn} = -\frac{B_H^I - B_H^U}{n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H} - \left(-\frac{D}{P_H^2}\right)} > 0$$

This fraction is positive for the highest price in equilibrium because the numerator is positive (as we have shown $B_H^I > B_H^U$) while the denominator is negative for the highest price in equilibrium.

The slope of the demand (given by $n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H}$) and of the supply (given by $-\frac{D}{P_H^2}$) in the denominator are both negative, so in principle the denominator could be positive or negative. For the highest price in equilibrium (the lowest quantity bid or the lowest debt burden in equilibrium), however, the denominator is negative. To see this, notice that, when evaluated at $P_H = 1-\kappa$ there is an excess of supply, as $B_H^I = 0$ and $B_H^U = 0$ (then there is no demand), while the supply is given by $\frac{D}{1-\kappa}$. The highest price in equilibrium is computed at the highest price at which demand and supply equalize, which implies that $n\frac{\partial B_H^I}{\partial P_H} + (1-n)\frac{\partial B_H^U}{\partial P_H} < \left(-\frac{D}{P_H^2}\right) < 0$.

Q.E.D.

In Figure 3 we show how prices P_H and P_L depend on the fraction of informed investors n in the economy. We also show the price in the uninformed equilibrium (which we denote by P_U) for reference. The are two distinct regions in the graph. When n is low, the economy is in a full participation region and when n is high, the economy is in a partial participation region.

In the partial participation region, P_L does not change with n as $B_L^I = B_L^U$ and then the resource constraint in the bad state is just $P_L B_L^I = D$, which is independent of n. Even though in this particular graph it looks as if P_L always declines with n in the full participation region, this is not necessarily the case.

In contrast, P_H increases with the fraction of investors that are informed in the market for all n. In the full participation region the sensitivity of P_H to n is moderated by the participation of the uninformed investors, but in the partial participation region the sensitivity is larger (the rate of increase of P_H with n is larger) as there is a pure cannibalization effect among informed investors, in which the market in the good state is split among a larger fraction of informed investors, driving up demand and, thus, prices.

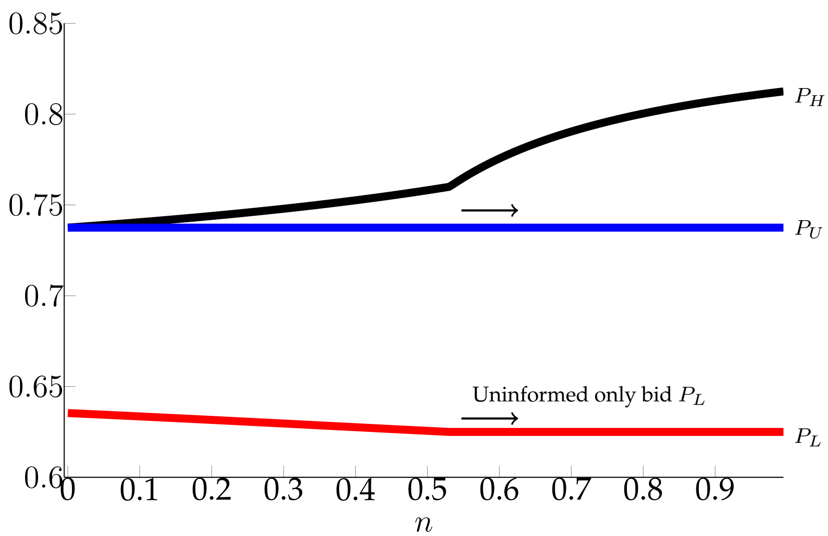

Figure 3: Prices and Information

In Figure 4 we show how the utility of informed investors and of uninformed investors depend on the fraction of informed investors in the market. These utilities depend on the evolution of prices, which we have shown depend on the fraction of informed investors. We also show the utility of investors in the uninformed equilibrium for reference. While the utility of uninformed investors decline with n in the full participation region, it is independent of n in the partial participation region as P_L is independent on n in this region. For informed investors, however, utility always declines in the partial participation region (because of the cannibalization effect), while the utility in the full participation region may increase and then decline. Even though in the figure the utility of informed investors always decline with n, the reason is that in this specific numerical example P_L always declines with n as well.

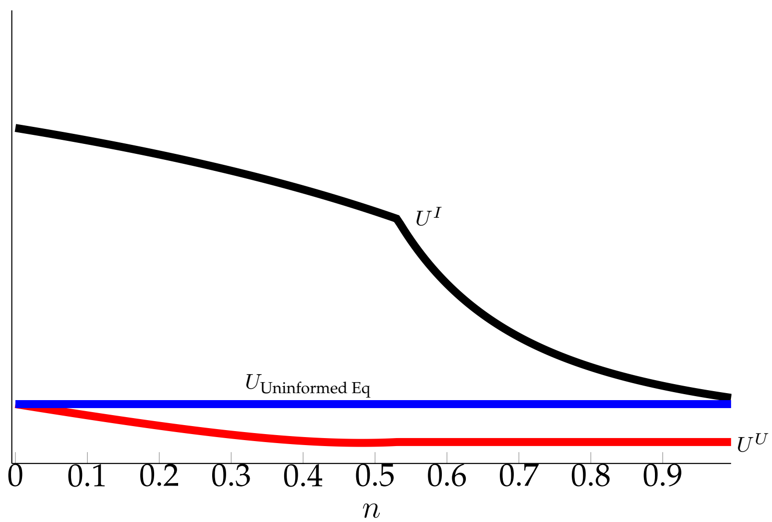

Figure 4: Utilities and Information

The utility of informed investors in the informed equilibrium is always above the utility of investors in the uninformed equilibrium and the utility of uninformed investors in the informed equilibrium is always below their utility in the uninformed equilibrium. This does not imply, however, that informed investors are better-off in the informed equilibrium, as they have to spend utility costs to become informed in the first place. In Figure 5 we show that the informed equilibrium is characterized by the fraction of investors n^* that make investors indifferent between being informed or uninformed, this is $U^I(n^*) - u(K) = U^U(n^*)$, which implies that all investors are always worse-off in the informed equilibrium.

Figure 5 also shows the possibility of multiple equilibria in our setting. The informed equilibrium, as discussed above, is the point at which the utility gap between informed and uninformed investors is equal to the utility cost of producing information $u(K)$. In this specific case, a situation where all investors are uninformed is also an equilibrium since $\chi^U < u(K)$.

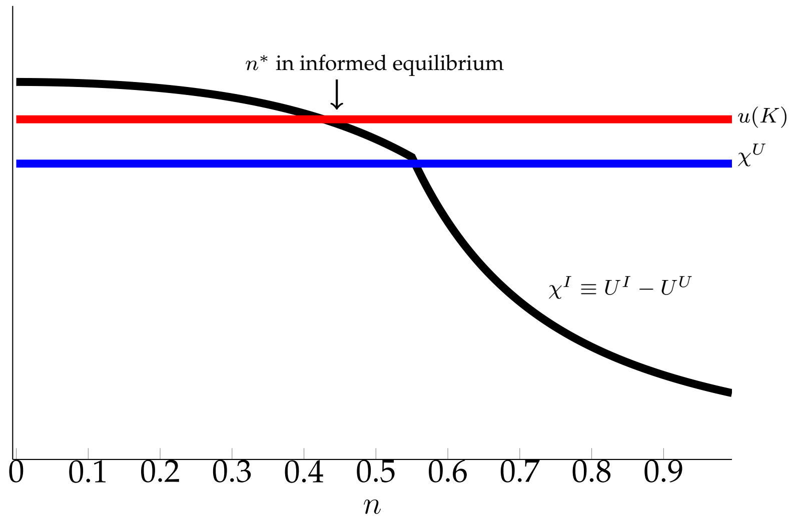

Figure 5: Equilibrium Multiplicity

Figure 6 shows how the equilibrium changes when there is an increase in z. An increase in the probability of default, which is generated by an increase in the gap between the two states, induces more information acquisition in the economy. The solid lines represent a low z and the dotted lines a higher z. One the one hand, an increase in z increases the individual incentives to deviate and become informed in the uninformed equilibrium (increasing χ^U). In the case of the numerical simulation this effect is large enough for the uninformed equilibrium to become unsustainable. On the other hand, it increases the gap between informed and uninformed investors in the informed equilibrium, thus increasing n^* in the informed equilibrium (the point at which the red solid line and the dotted black line cross).

Figure 7 shows the equilibrium fraction of informed investors, n^*, in the informed equilibrium, as we change the gap between the states in terms of default probabilities, z, and also as we increase D/W, the indebtedness of the country.

Now that we have characterized both the conditions for the uninformed equilibrium and the equilibrium fraction of informed investors in the informed equilibrium, we can compute the price P_U in the uninformed equilibrium for different levels of z, the difference in default probability between states, and the prices P_H and P_L in the

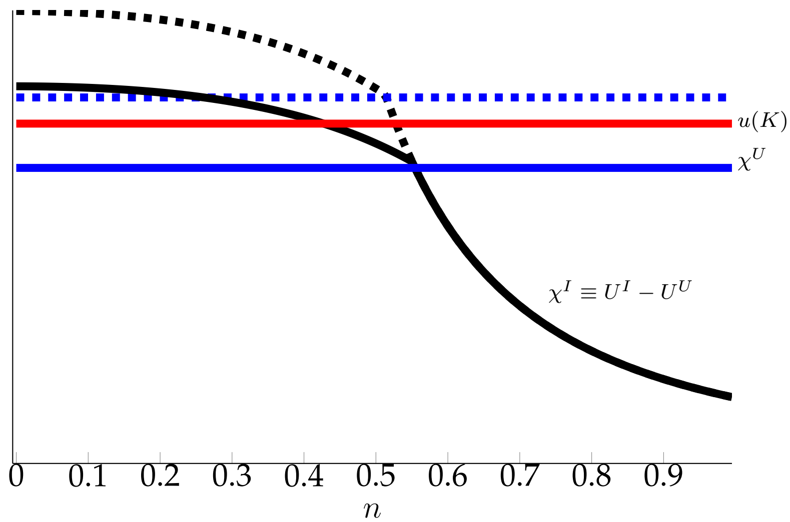

Figure 6: Effect of z on Equilibrium Multiplicity

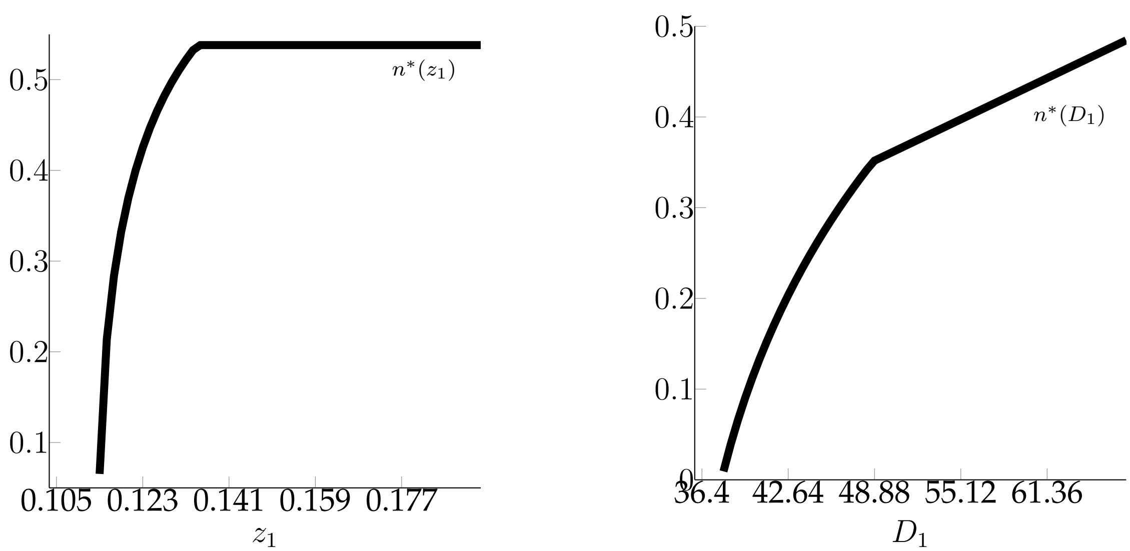

Figure 7: Effect of z and D on Information in Equilibrium

informed equilibrium. We displayed these prices in Figure 8.

First, there are clearly three regions of equilibria as a function of z. For low levels of z there are low incentives to acquire information and only the uninformed equilibrium is sustainable. In contrast, for high levels of z there are high incentives to acquire information and only the informed equilibrium, with a large fraction of informed investors, is sustainable. For intermediate region of z both equilibria coexist. Interestingly, once we compute the weighted average of prices $E(P) =$

$aP_H + (1-a)[\omega P_H + (1-\omega)P_L]$, where $\omega = \frac{(1-n)B_H^U}{(1-n)(B_H^U+B_L^U)+nB_L^I}$ the informed equilibrium is not only characterized by volatility of prices (which can fluctuate between P_L and P_H), but also by a lower average price.

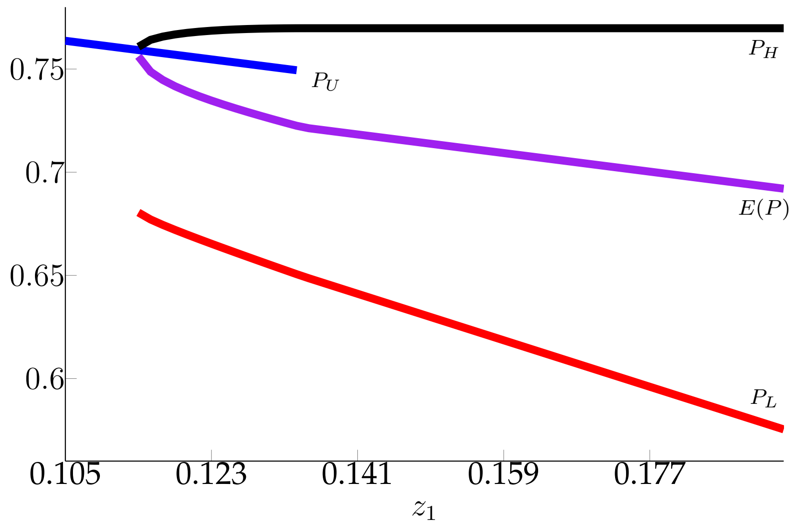

Figure 8: Equilibrium Prices

This is important because the informed equilibrium is not only worse for investors, but also for countries, who may dislike volatility per se, but definitely suffer from lower average debt prices. Indeed, as the expected prices at which a country raises funds in the informed equilibrium are lower than in the uninformed equilibrium, the expected debt burden is also higher for a country in the informed equilibrium, as shown in Figure 9. In other words, the informed equilibrium is inferior from both the country's and the investors' point of view.

This characterization of equilibria and potential multiplicity has implications for the interpretation of the effects of shocks to fundamentals on debt burden and debt prices of countries, as well as on the volatility that countries experience in their sovereign spreads. Assume for example a simple and plausible equilibrium selection under which a country remains in a given equilibrium as long as it is sustainable. This "conservative" equilibrium selection introduces history dependence, or *hytheresis*, such that small shocks to fundamental may generate large changes in the behavior of sovereign prices. In different words, the past matters and two countries with identical fundamentals can have different average price of their debt, different debt burdens and different price volatility just because they differ in their past.

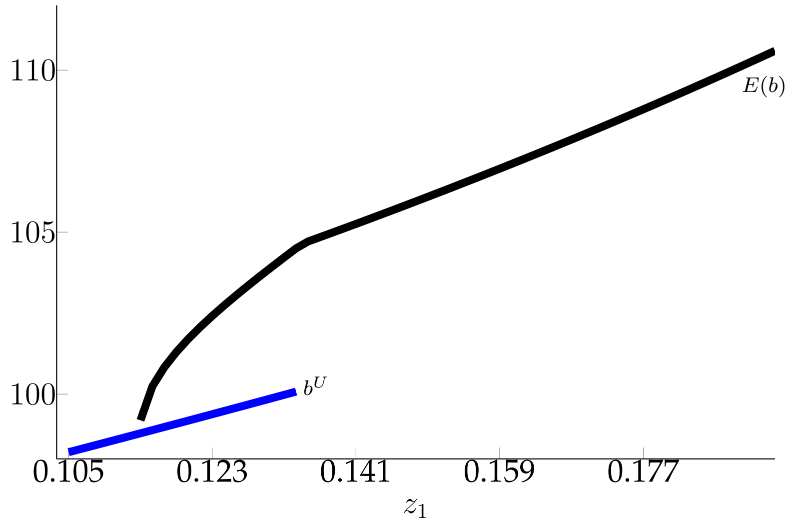

Figure 9: Equilibrium Debt Burden

These results are relevant in interpreting the mapping of fundamentals to sovereign debt prices. Periods of calm sovereign experiences do not necessarily imply that fundamentals are calm, as it may be that the country raises funds in an uninformed equilibrium, in which prices are simply not sensitive to movements in fundamentals. In contrast, periods of turbulent sovereign experiences do not necessarily imply that fundamentals have become more turbulent than normal, as it may be that the country transitioned to an informed equilibrium in which prices are more sensitive to movements in fundamentals.

3.2 Two Countries

So far we have studied the different informational equilibria under which a single country may raise funds, as well as the characterization of the behavior of prices and utilities in these equilibria. Now we study how investors bidding in different countries can induce co-movement of sovereign prices and contagion of debt crises in its purest form, without any fundamental linkage across countries other than a common pool of investors. Then we will discuss the role of different informational equilibria on contagion.

We start by analyzing the simpler case in which both countries are in the uninformed equilibrium, and hence no investor is informed about the state in either country. The maximization problem can be written simply as

$$\max_{B_1,B_2} U = \widehat{\kappa}_1 \left[\widehat{\kappa}_2 u(W - P_1B_1 - P_2B_2) + (1-\widehat{\kappa}_2)u(W - P_1B_1 + (1-P_2)B_2)\right]$$
$$+(1-\widehat{\kappa}_1)\left[\widehat{\kappa}_2 u(W + (1-P_1)B_1 - P_2B_2) + (1-\widehat{\kappa}_2)u(W + (1-P_1)B_1 + (1-P_2)B_2)\right]$$

The first-order condition for the quantities bid in country j is

$$\frac{E_j(u'(+))}{E_j(u'(-))} = \frac{P_j\widehat{\kappa}_j}{(1-P_j)(1-\widehat{\kappa}_j)}$$

where

$$E_j(u'(-)) = \widehat{\kappa}_{-j}u'(W - P_jB_j - P_{-j}B_{-j}) + (1-\widehat{\kappa}_{-j})u'(W - P_jB_j + (1-P_{-j})B_{-j})$$

and

$$E_j(u'(+)) = \widehat{\kappa}_{-j}u'(W+(1-P_j)B_j - P_{-j}B_{-j}) + (1-\widehat{\kappa}_{-j})u'(W+(1-P_j)B_j + (1-P_{-j})B_{-j})$$

The next proposition shows that, when utilities display prudence, such as CRRA utility functions, then an increase in the expected default probability in one country reduces the sovereign price in the other country. Notice we have constructed a simple portfolio problem where the returns on the two risky assets are i.i.d. and there is no feedback other than the one imposed by investors rebalancing their portfolio.

Proposition 7 *There is contagion (this is $\frac{\partial P_j}{\partial \widehat{\kappa}_{-j}} < 0$) when preferences are CRRA.*

Proof Impose resource constraints $P_1B_1 = D_1$ and $P_2B_2 = D_2$ for each country in the first order conditions. Denoting $R = P_1B_1 + P_2B_2 = D_1 + D_2$, write first-order conditions as

$$\frac{\widehat{\kappa}_{-j}u'(W - R + \frac{D_j}{P_j}) + (1-\widehat{\kappa}_{-j})u'(W - R + \frac{D_j}{P_j} + \frac{D_{-j}}{P_{-j}})}{\widehat{\kappa}_{-j}u'(W-R) + (1-\widehat{\kappa}_{-j})u'(W - R + \frac{D_{-j}}{P_{-j}})} - \frac{P_j\widehat{\kappa}_j}{(1-P_j)(1-\widehat{\kappa}_j)} = 0$$

For simplicity

$$\frac{\widehat{\kappa}_{-j}u'(+-) + (1-\widehat{\kappa}_{-j})u'(++)}{\widehat{\kappa}_{-j}u'(--) + (1-\widehat{\kappa}_{-j})u'(-+)} - \frac{p_j\widehat{\kappa}_j}{(1-p_j)(1-\widehat{\kappa}_j)} = 0$$

where the first argument of u' corresponds to the repayment or not of country j and the second argument to the repayment or not of country $-j$.

$$\frac{dP_j}{d\widehat{\kappa}_{-j}} = -\frac{\frac{\left[u'(+-)-u'(++)-(1-\widehat{\kappa}_{-j})\frac{D_j}{P_{-j}^2}\frac{\partial P_{-j}^2}{\partial \widehat{\kappa}_{-j}}u''(++)\right]}{E_j(u'(-))} - \frac{E_j(u'(+))}{E_j(u'(-))}\frac{\left[u'(--)-u'(-+)-(1-\widehat{\kappa}_{-j})\frac{D_{-j}}{P_{-j}^2}\frac{\partial P_{-j}^2}{\partial \widehat{\kappa}_{-j}}u''(-+)\right]}{E_j(u'(-))}}{-\frac{D_j E_j(u''(+))}{P_j^2 E_j(u'(-))} - \frac{\widehat{\kappa}_j}{(1-P_j)^2(1-\widehat{\kappa}_j)}}$$

There is contagion, that is, $\frac{dP_j}{d\widehat{\kappa}_{-j}} < 0$, when the denominator is negative (which is the case, as discussed for the highest P_j^* in equilibrium) and the numerator is also negative, this is when,

$$\frac{\frac{u'(+-)-u'(++)}{1-\widehat{\kappa}_{-j}} - \frac{D_{-j}}{P_{-j}^2}\frac{\partial P_{-j}}{\partial \widehat{\kappa}_{-j}}u''(++)}{E_j(u'(+))} < \frac{\frac{u'(--)-u'(-+)}{1-\widehat{\kappa}_{-j}} - \frac{D_{-j}}{P_{-j}^2}\frac{\partial P_{-j}}{\partial \widehat{\kappa}_{-j}}u''(-+)}{E_j(u'(-))}$$

In words, the relative change in the gains from bidding in country j are smaller than the relative change in the losses. This implies a reduction in bidding in country j, a decline in the demand and then a decline in sovereign prices. Q.E.D.

Figure 10 is similar to Figure 2, but for different levels of risk aversion (which, for CRRA utility functions, also implies different levels of prudence) and with the left hand side computed by the ratio of marginal utilities in expectation (which depends on the probabilities of default in the country that suffers a shock). We can draw several conclusions from the figure. First, as we already discussed, the larger the level of risk aversion the smaller the sovereign price in equilibrium.

Second, we show in blue a situation in which the other country has a low expected probability of default and in red when the expected probability of default in the other country is higher. As is clear from the figure, given a shock in the probability of default in the other country, contagion is stronger the larger the risk aversion. This result arises for two reasons. On the one hand, the higher the prudence the larger is the reaction of investors to move investment away from risky sovereign bonds. On the other hand, the higher the level of risk aversion the lower the price in equilibrium and more sensitive it is to movements in the left hand side (this is, the left and right hand sides coincide in flatter regions).

Now that we have discussed the source of contagion, purely as the result of the re-optimization of investors' portfolio, regardless of the information underlying the

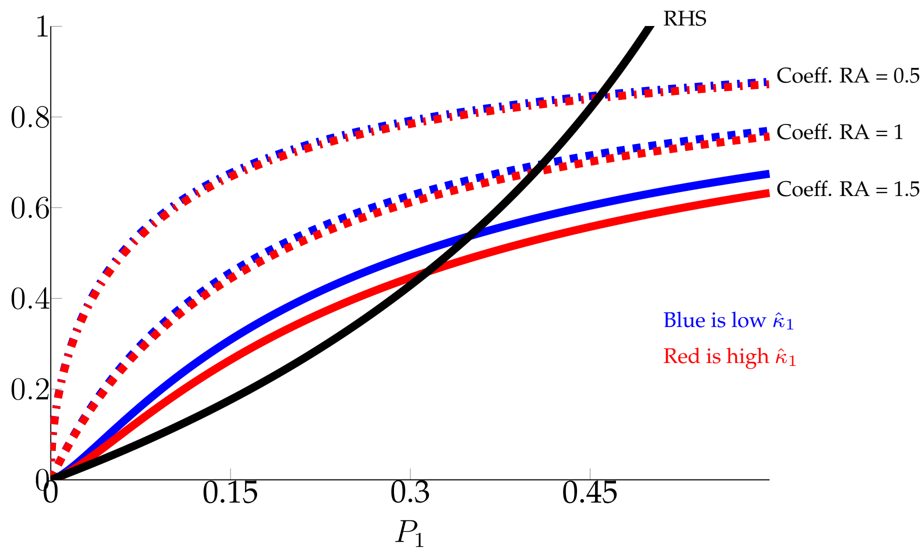

Figure 10: Contagion Depends on Risk Aversion

equilibrium, we can study how different information equilibria affect the strength and properties of contagion.

In Figure 11 we assume that the source country (the country that suffers a change in the expected probability of default, driven by a positive shock on z) can be either in the informed or uninformed equilibrium, and we study the contagion effect on the other country, that for simplicity we assume is only in an uninformed equilibrium. As can be seen, contagion is stronger when the source country is in an informed equilibrium, as expected prices are more sensitive to changes in its own fundamentals in that situation, translating the magnitude of the shock to the other country.

In Figure 12 we revert the assumptions on equilibrium multiplicity. We assume that the source country (the country that suffers a change in the expected probability of default) is in an uninformed equilibrium and we study how the change in the probability of default affect the equilibrium type in the other country. This figure is the same as Figure 6, but with the dotted lines showing an increase in z in the foreign country, not on the domestic country.

An increase in the probability of default in the source country increases the incentives to become informed in the informed equilibrium, thereby increasing the fraction of informed investors, n^*, and making the informed equilibrium more likely for a given level of information cost.

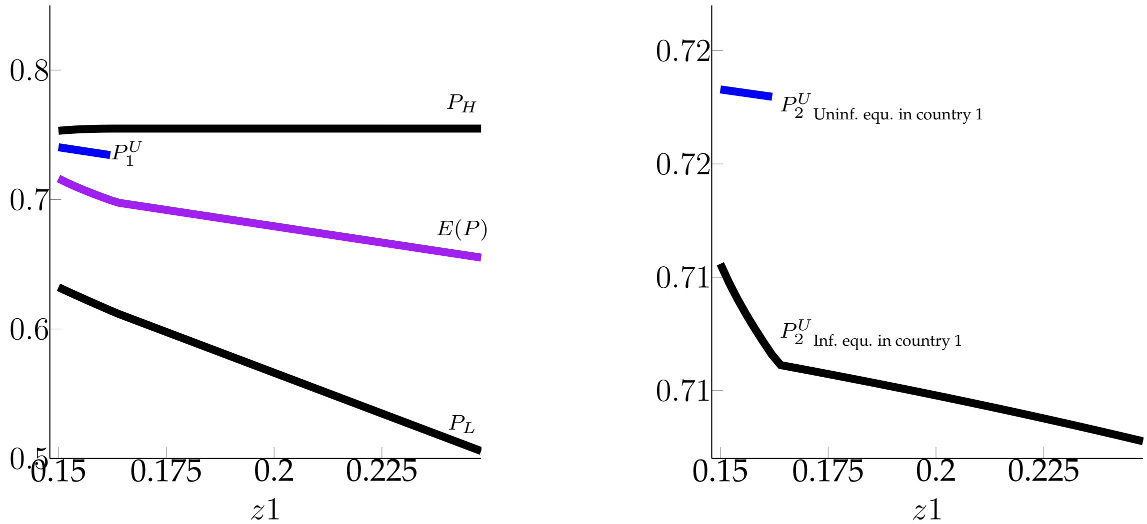

Figure 11: Contagion Depends on the Equilibrium Type in the Source

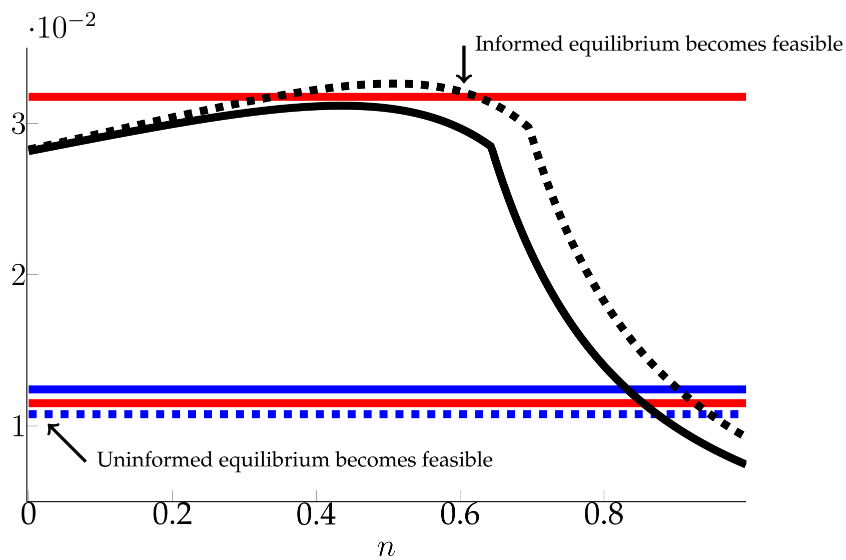

Figure 12: Contagion on the Equilibrium Type

In contrast, an increase in the probability of default in the source country also reduces the incentives to become informed in the uninformed equilibrium, making the uninformed equilibrium less likely for a given level of information cost.

These results are also relevant in interpreting the mapping of fundamentals to sovereign debt prices, as changes in sovereign prices in a country may not even be the result of changes in the fundamentals of that country. Furthermore, the strength of contagion, and then the sensitivity of a country's sovereign prices to foreign fundamentals, also depend on the other country's equilibrium, not even on the own equilibrium. Finally, a change in other country's fundamentals can change the nature of equilibrium in a given country, changing the sensitivity of the sovereign spreads to own fundamentals as well.

4 Conclusions

We constructed a simple model of portfolio choice with information acquisition, where the portfolio is composed by sovereign debt of different countries and information is about the fundamentals of the countries that determine their probabilities of default.

For a single country we have shown that the participation of informed investors (informed equilibrium) is more likely when the country is highly indebted and when there is more certainty about its fundamentals. An equilibrium in which a country raises funds from informed investors is inferior, as investors obtain less utility and the country faces higher and more volatile prices, then higher debt burden.

Given that an informed and a uninformed equilibrium may coexist, small changes in fundamentals can generate large changes in the sovereign debt experience. If the selection of equilibrium is hysteresis (the country remains in a given equilibrium as long as it is sustainable) then the sovereign price of two countries with the same fundamentals but different past can have very different experiences.

For many countries, contagion does not require fundamental linkages or common factors, just a common pool of investors that react to changes in fundamentals of each country and rebalance the portfolio. This contagion is stronger when there is an informed equilibrium at the source of the shock. Furthermore, shocks in one country may change the informational equilibrium under which other countries raise funds.

Our results show why it is not straightforward to interpret changes in sovereign debt prices as informative about the country's fundamentals, as they depend not only on the country's own fundamentals, but also on the country's informational equilibrium (and thus, potentially on past fundamentals), other countries' fundamentals and other countries' informational equilibria.

We have highlighted the main forces behind information acquisition (which determines the sensitivity of sovereign prices to fundamentals) and contagion (which determines the sensitivity of sovereign prices to others' fundamentals). There are many reasons why we may expect these forces to be also quantitatively relevant.

Just to mention a few magnifying forces. First, the probability of default is endogenous and depends on sovereign prices. There is a feedback effect across countries: an exogenous increase in default probability in one country induces a reduction of prices in several other countries, increasing the probabilities of default in all those countries, further reduction of prices, and so on. Second, fundamental linkages across countries naturally magnify contagion. Third, if there is time varying prudence, for example because of time varying risk-aversion or time varying wealth. Fourth, market segmentation can concentrate contagion in certain regions, buffering others. Finally, how a shock in a country changes the informational equilibrium in other countries depend on the structure of the costs to acquire information: if a country attracts informed investors and then makes easier for them to acquire information about other similar countries, then it is more likely that those other countries also attract informed investors.

References

Alter, Adrian, and Andreas Beyer. 2014. "The Dynamics of Spillover Effects During the European Sovereign Debt Turmoil." *Journal of Banking and Finance* 42:134–153.

Beirne, John, and Marcel Fratzscher. 2013. "The Pricing of Sovereign Risk and Contagion During the European Sovereign Debt Crisis." ECB Working Paper 1625.

Broner, Fernando, Gaston Gelos, and Carmen Reinhart. 2004. "When in Peril, Retrench: Testing the Portfolio Channel of Contagion." NBER Working Paper 10941.

Calvo, Guillermo, and Enrique Mendoza. 1999. "Rational Contagion and the Globalization of Securities Markets." *Journal of International Economics* 51:79–113.

Cole, Hal, and Patrick Kehoe. 1996. "Reputation Spillover Across Relationships: Reviving Reputation Models of Debt." Working Paper 5486, National Bureau of Economic Research.

De Santis, Roberto. 2012. "The Euro Area Sovereign Debt Crisis: Safe Haven, Credit Rating Agencies and the Spread of the Fever from Greece, Ireland and Portugal." ECB Working Paper 1419.

Goldstein, I, and A Pauzner. 2004. "Contagion of Self-Fulfilling Financial Crises due to Diversification of Investment Portfolios." *Journal of Economic Theory*, no. 119:151–183.

Gross, Marco, and Christoffer Kok Sorensen. 2013. "Measuring Contagion Potential Among Sovereign and BAnks Using a Mixed-Cross-Section GVAR." ECB Working Paper 1570.

Grossman, Sanford, and Joseph Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70 (3): 393–408 (June).

Kyle, Albert, and Wei Xiong. 2001. "Contagion as a Wealth Effect." *Journal of Finance* 56 (4): 1401–1440.

Lizarazo, Sandra. 2013. "Default Risk and Risk Averse International Investors." *Journal of International Economics* 89:317–330.

Reinhart, Carmen, and Kenneth Rogoff. 2009. *This Time is Different: Eight Centuries of Financial Folly*. Princeton: Princeton University Press.

Yuan, Kathy. 2005. "Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crises, Contagion and Confusion." *Journal of Finance* 60 (1): 379–411.