A Price Discrimination Model of Trade Promotions

Technical Appendix

Tony Haitao Cui Jagmohan S. Raju Z. John Zhang

May 2007

Technical Appendix

TA. Competitive Model

In this section, we will show that the insight that the manufacturer can use trade promotions to price discriminate between the dominant retailer and the competitive fringe is not model specific. Same insight also applies in an alternative demand model

$$\begin{cases} Q_1 = a_1 - b_1 p_1 + c_1 p_2 \\ Q_2 = a_2 - b_2 p_2 + c_2 p_1 \end{cases}, \tag{TA.1}$$

where $\min\{b_1, b_2\} > \max\{c_1, c_2\}$ (Sayman, Hoch and Raju 2002). In the model, each retailer's retail price could influence the other retailer's demand and the influence impact is different for different retailers.

It is easy to show that the manufacturer would like to offer a lower wholesale price to retailer 1 than to retailer 2 if retailer 1 has a higher price sensitivity than retailer 2. For example, retailer 1 has a higher price sensitivity than retailer 2 when $a_1 = a_2$, $b_1 = b_2$, and $c_1 < c_2$. Again, although the manufacturer cannot directly charge different retailers with different wholesale prices, the manufacturer can use trade promotions to price discriminate between retailers if retailer 1's holding cost is lower than retailer 2's. Because of the limited space, we omit the process of solving the problem, but provide the following results.[1] Figures 1-4 show the results graphically. [2]

Lemma 1 *If the demands of two retailers are given by Equation (TA.1), retailer 1's unit holding cost is* $0 < h_1 < \infty$*, retailer 2's unit holding cost* $h_2 = \infty$*, and the manufacturer will charge a promotional wholesale price* w_p *in the first period and a normal wholesale price* w *for periods* 2

[1] The detailed analytical analysis and proofs are available from authors upon request. The numerical study for $a_1 = a_2 = 1$, $b_1 = b_2 = 0.5$, $c_1 = 0.01$, $h_2 = \infty$, $h_1 \in \{0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.105, 0.11\}$, and $c_2 \in \{0.1, 0.2, 0.3, 0.4, 0.45, 0.49, 0.495\}$ is also available from authors upon request.

[2] We use the percentage increases in the manufacturer's and retailers' profits − the incremental amount of profit over the benchmark profit as a percentage of the benchmark profit − as the y-coordinates, because the benchmark profit is different for different combination of h_1 and c_2 and percentage increases provide a better interpretation than absolute increases.

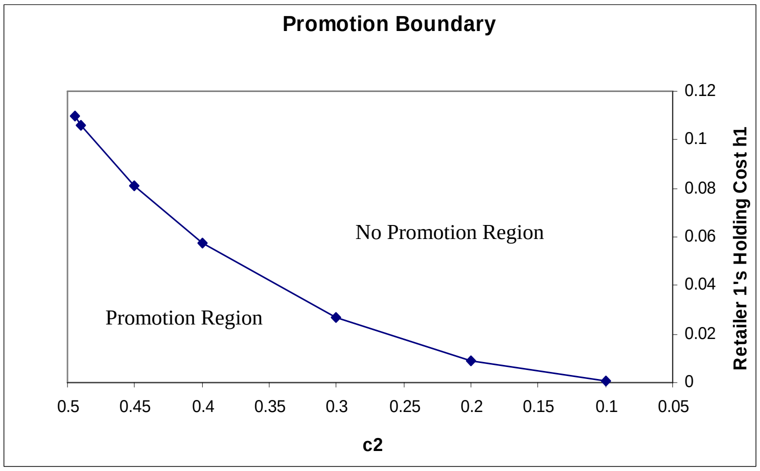

Figure 1: Manufacturer's Promotion Boundary (Competitive Demand Model)

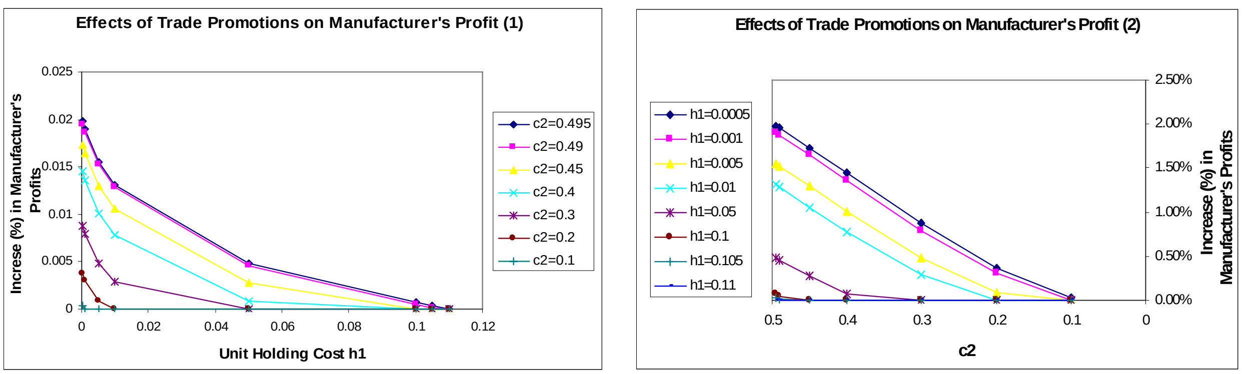

Figure 2: Comparative Static for Manufacturer's Profit Gains (Competitive Demand Model)

to N within an N-period promotion cycle, then retailers' retail prices and their sales in period $i = 2, ..., N$ are given by,

$$\begin{cases} p_1^i = \frac{2a_1b_2+a_2c_1+b_2c_1w-\frac{2}{N+1-i}b_2Q_1^i-\frac{N-i}{2}h_1b_1b_2+\frac{N-i}{4}h_1c_1c_2}{2b_1b_2-c_1c_2} \\ p_2^i = \frac{a_1b_2c_2+a_2b_1b_2+b_1{b_2}^2w-\frac{1}{N+1-i}b_2c_2Q_1^i-\frac{N-i}{4}h_1b_1b_2c_2+\frac{N-i}{8}h_1c_1{c_2}^2}{b_2(2b_1b_2-c_1c_2)} \\ \bar{Q}_1^i = \frac{\frac{1}{N+1-i}b_2Q_1^i+\frac{N-i}{4}h_1b_1b_2-\frac{N-i}{8}h_1c_1c_2}{b_2} \\ \bar{Q}_2^i = \frac{a_1b_2c_2+a_2b_1b_2-b_1{b_2}^2w-\frac{N-i}{4}h_1b_1b_2c_2-\frac{1}{N+1-i}b_2c_2Q_1^i+\frac{N-i}{8}h_1c_1{c_2}^2+b_2c_1c_2w}{2b_1b_2-c_1c_2} \end{cases} \quad \text{(TA.2)}$$

where for $i = 3, ..., N$ we have,

$$\begin{cases} p_1^i - p_1^{i-1} = \frac{h_1}{2} \\ p_2^i - p_2^{i-1} = \frac{h_1c_2}{4b_2} \\ \bar{Q}_1^{i-1} - \bar{Q}_1^i = \frac{h_1(2b_1b_2-c_1c_2)}{4b_2} \\ \bar{Q}_2^i - \bar{Q}_2^{i-1} = \frac{h_1c_2}{4} \end{cases} \quad \text{(TA.3)}$$

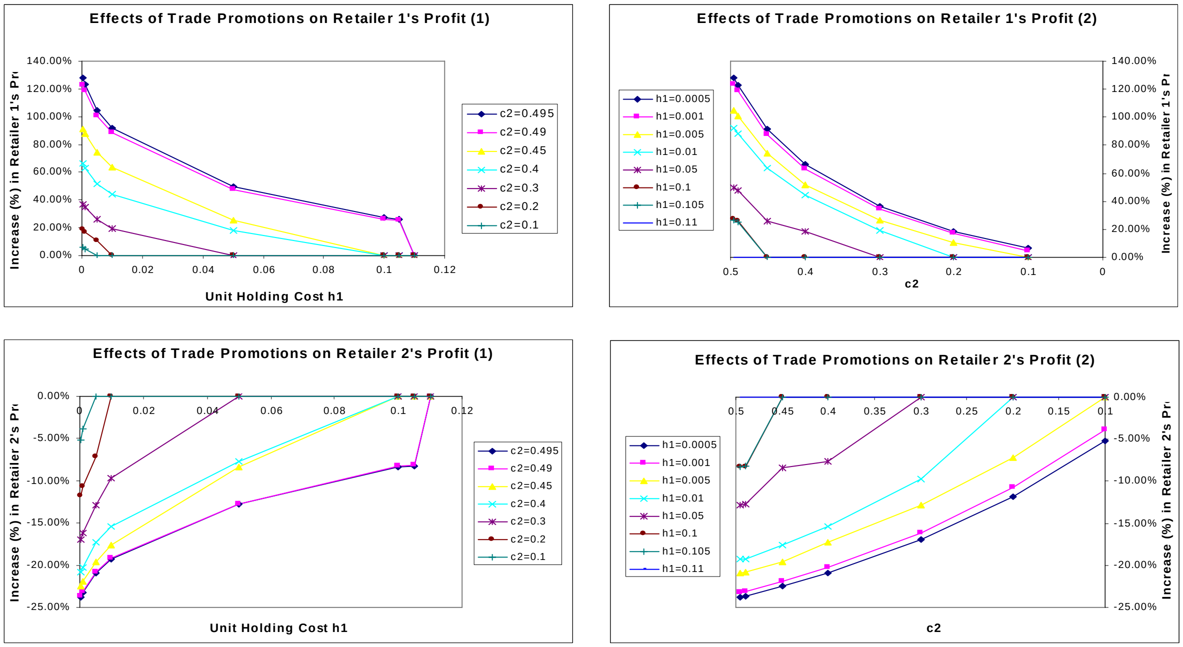

Figure 3: Comparative Static for Retailers' Profit Changes (Competitive Demand Model)

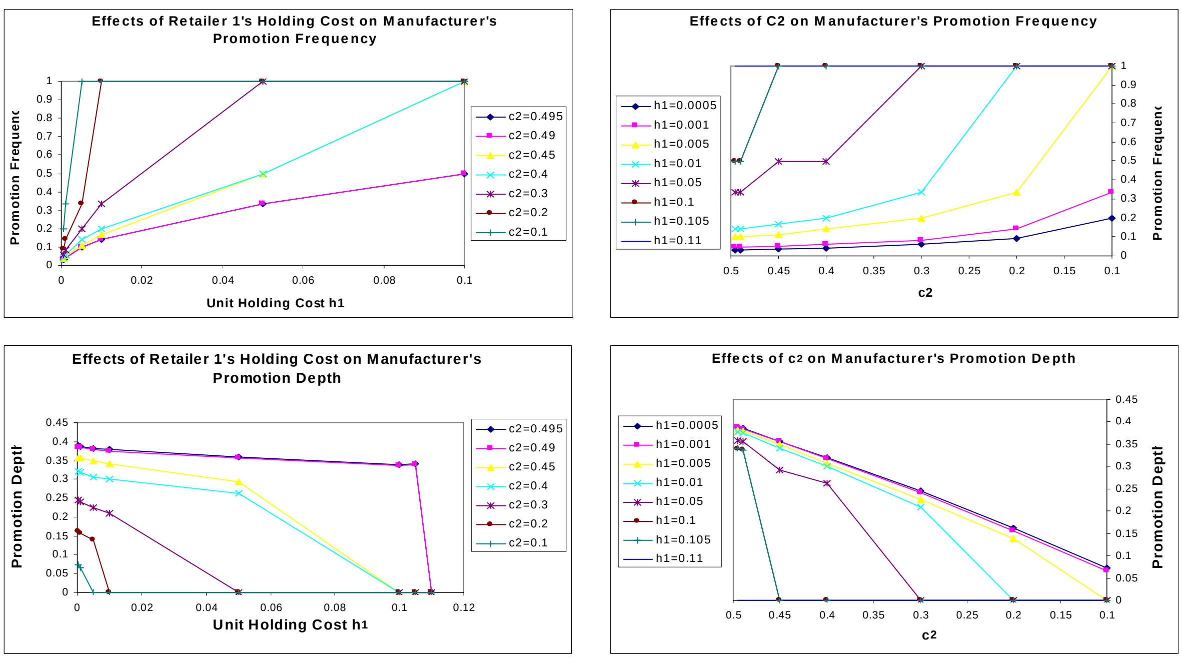

Figure 4: Effects of h_1 and c_2 on Manufacturer's Promotion Frequency and Depth (Competitive Demand Model)

and in the first period,

$$\begin{cases} p_1^1 = \frac{2a_1b_2+a_2c_1+b_2c_1w_p-2b_2(Q_1^1-Q_1^2)}{2b_1b_2-c_1c_2} \\ p_2^1 = \frac{a_1c_2+a_2b_1+b_1b_2w_p-c_2(Q_1^1-Q_1^2)}{2b_1b_2-c_1c_2} \\ \bar{Q}_1^1 = Q_1^1 - Q_1^2 \\ \bar{Q}_2^1 = \frac{b_2[a_1c_2+a_2b_1-b_1b_2w_p-c_2(Q_1^1-Q_1^2)+c_1c_2w_p]}{2b_1b_2-c_1c_2} \end{cases} \quad \text{(TA.4)}$$

Further, retailer 1's total ordering quantity Q_1^1, retailer 2's ordering quantities in the first period (Q_2^1), and retailer 2's ordering quantity from periods 2 to N (Q_2^T) are given by,

$$\begin{cases} Q_1^1 = \frac{4a_1b_2N+2a_2c_1N+2b_2c_1w_p+2b_2c_1w(N-1)}{8b_2} - \frac{N(2b_1b_2-c_1c_2)[h_1(N-1)+2w_p]}{8b_2} \\ \bar{Q}_2^1 = \frac{2a_1b_2c_2+4a_2b_1b_2-a_2c_1c_2-4b_1b_2{}^2w_p+2b_1b_2c_2w_p+3b_2c_1c_2w_p-c_1c_2{}^2w_p}{4(2b_1b_2-c_1c_2)} \\ \bar{Q}_2^2 = \frac{2a_1b_2c_2+2a_2b_1b_2+b_2wc_1c_2}{4(2b_1b_2-c_1c_2)} + \frac{a_2+c_2w_p+h_1c_2-2b_2w}{4} \\ Q_2^T = \sum_{i=0}^{N-2}(\bar{Q}_2^2 + i \times \frac{h_1c_2}{4}) = (N-1)[\bar{Q}_2^2 + \frac{h_1c_2(N-2)}{8}] \end{cases} \quad \text{(TA.5)}$$

and the manufacturer chooses wholesale prices w_p (the promotional price in the first period) and w (the normal wholesale price from periods 2 to N) to maximize its profit Π_M^T, where

$$\Pi_M^T = w_p \times (Q_1^1 + \bar{Q}_2^1) + w \times Q_2^T \quad \text{(TA.6)}$$

The optimal N, which maximizes the manufacturer's average profit per period in a promotion cycle, is found by searching feasible range which is upper limited by the constraint $w_p + (N-1)h_1 \leq w$.

Result 1 *If retailer 1 has a higher price sensitivity and a lower unit holding cost than retailer 2, the manufacturer could price discriminate between retailers by offering trade promotions.*

Result 2 *Promotions may increase the manufacturer's profit and the benefits of trade promotions for the manufacturer decreases when h_1 increases or c_2 decreases.*

Result 3 *Promotions may increase retailer 1's profit, and its gain from promotions decreases when h_1 increases or c_2 decreases. Promotions may reduce retailer 2's profit and its profit decreases when h_1 decreases or c_2 increases.*

Result 4 *Promotion frequency increases when h_1 increases or c_2 decreases. Promotion depth decreases when h_1 increases or c_2 decreases.*

The results confirm the intuition we discussed in the dominant retailer model: When the retailer(s) with higher price sensitivity has lower holding costs than the retailer(s) with lower price sensitivity, trade promotions can help the manufacturer price discriminate between retailers and improve the manufacturer's profit.

TB. Effects of Trade Promotions

In Technical Appendix TB, several graphs about the effects of trade promotions on the manufacturer's profit, retailers' profits, social welfare, and promotion frequency and depth are listed.

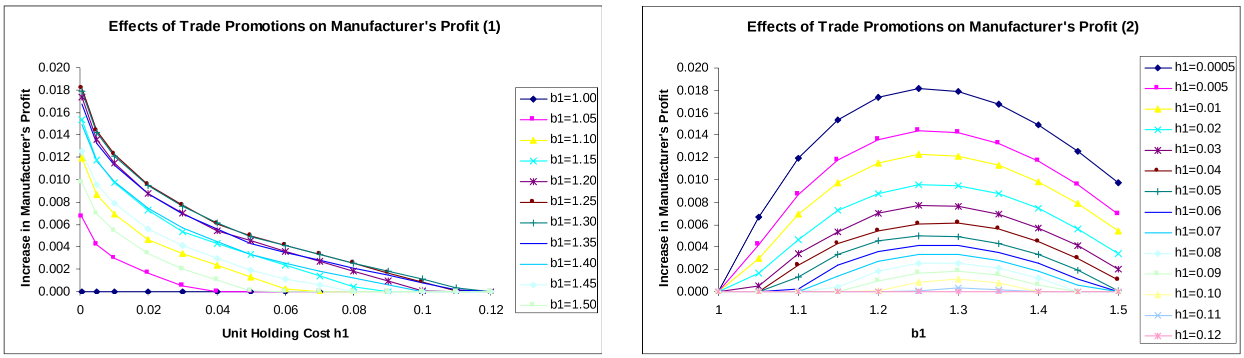

Figure 5: Comparative Static for Manufacturer's Profit Gains

TC. Finite Holding Costs for Both Dominant Retailer and Competitive Fringe

When the manufacturer offers trade promotions and both retailers' holding costs are finite, both retailers can make forward buying as the dominant retailer does in the basic model. But the dominant retailer may take advantage of trade promotions at a higher degree than the competitive fringe through forward buying if the dominant retailer has a lower holding cost than the competitive fringe. The dominant retailer's forward buying is more efficient than the competitive fringe's. This is because the dominant retailer, with lower inventory holding costs than the competitive fringe, pays a lower average effective acquisition cost and can make forward buying for more periods than the competitive fringe.

The manufacturer, however, has different interest in different retailers' forward buying behaviors. The manufacturer would like to induce the dominant retailer to forward buy in order for the dominant retailer to set a low retail price. The competitive fringe's forward buying, on the other hand, will reduce the manufacturer's profit since such forward buying dilutes price discrimination between the dominant retailer and competitive fringe. Furthermore, the forward buying from the

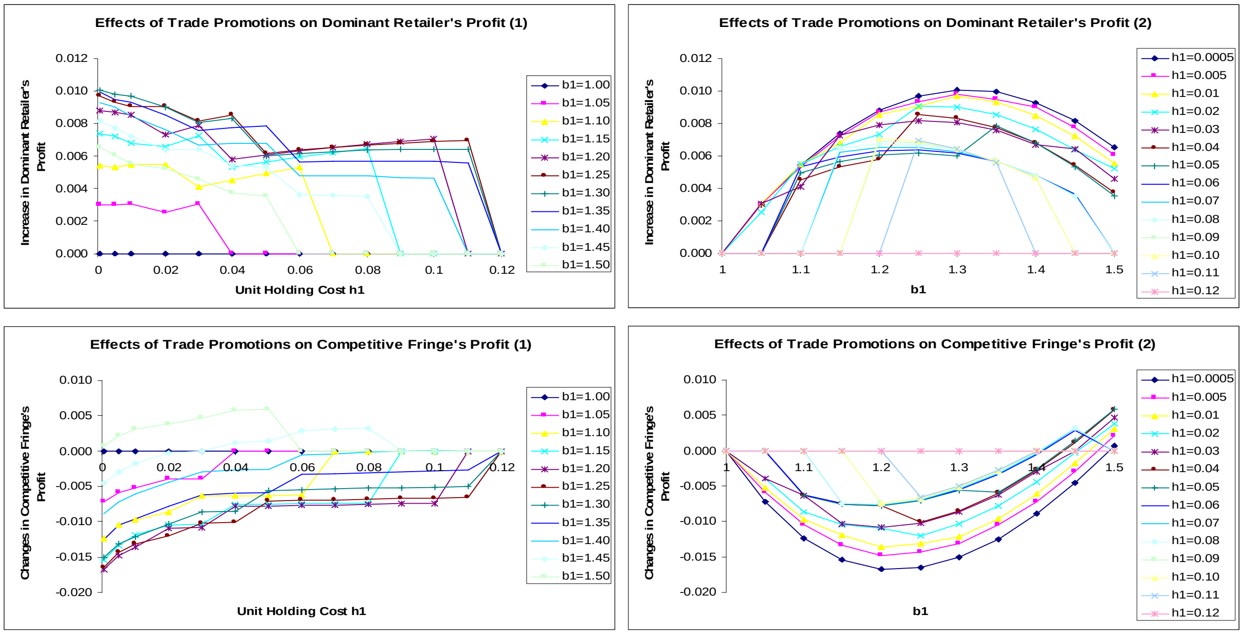

Figure 6: Comparative Static for Retailers' Profit Gains

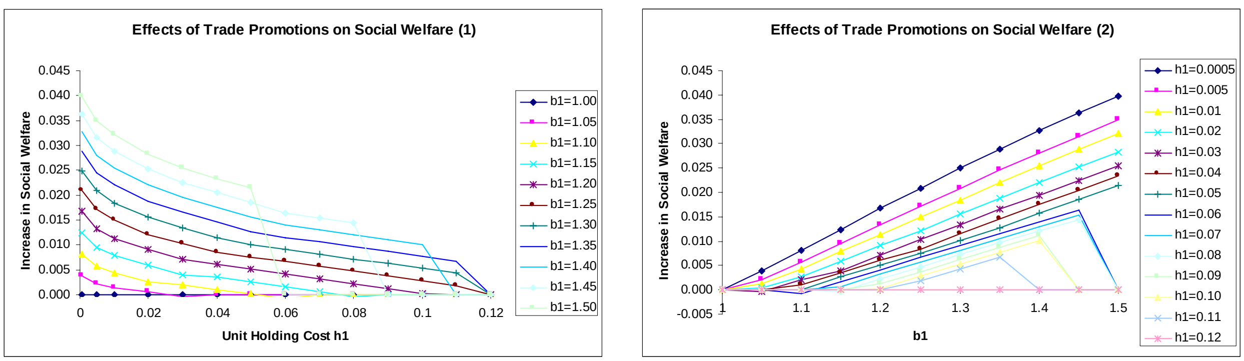

Figure 7: Comparative Static for Social Welfare Gains

competitive fringe will bring inventory costs to the channel and the occurrence of such inventory costs is not offering any benefits to either the manufacturer or the channel. Therefore, the manufacturer would like to induce the competitive fringe not to forward buy.[3] The manufacturer can then set a low wholesale price in the first period within any price cycle to induce the dominant retailer to make forward buying, but charge higher wholesale prices in the following periods to squeeze surplus

[3]In the paper, we do not consider the manufacturer's inventory holding costs. If the manufacturer's inventory holding cost is counted and the inventory holding cost is lower for the manufacturer than for the dominant retailer, it is unclear whether the system gains from transferring the inventory to the dominant retailer. If the manufacturer considers inventory holding cost, however, it will have more incentives to induce the dominant retailer to make forward buying. The social welfare will obviously be lower.

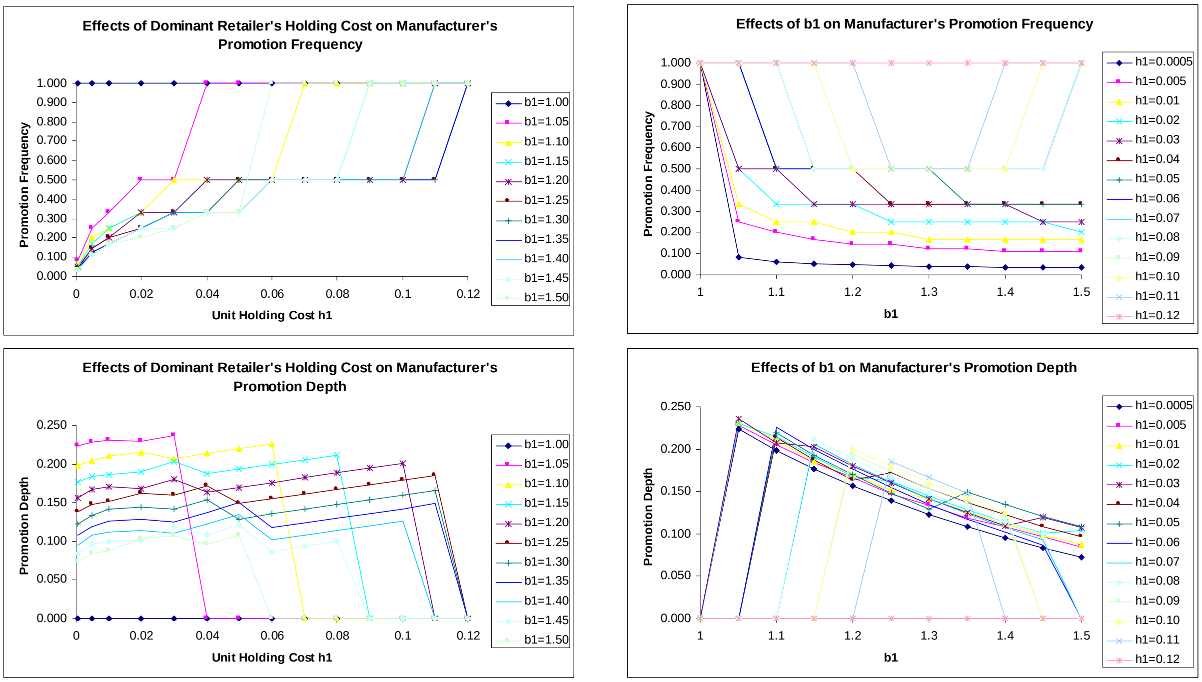

Figure 8: Comparative Static for Manufacturer's Promotion Frequency and Depth

from the competitive fringe while keeping the competitive fringe slightly better-off by not making forward buying. In this way, the manufacturer is providing the dominant retailer an opportunity to order products at a promotional price, and, at the same time, preventing the competitive fringe from carrying inventory across periods. Lemma 2 states the manufacturer's optimal strategies.

Lemma 2 *If both retailers have positive unit holding costs and the dominant retailer's unit holding cost is smaller than the competitive fringe's* ($0 < h_1 < h_2 < \infty$)*, then the manufacturer will offer the lowest wholesale price in the first period and charge an increasing wholesale price in each of the following N-1 periods within any N-period pricing cycle. The wholesale price* w_i *in the* i^{th} *period* ($2 \le i \le N$)*, the retail price* p^j ($1 \le j \le N$)*, and the manufacturer's average profit per period* Π_{h_1} *are respectively given by*

$$\begin{cases} w_i = \begin{cases} w_1 + h_2(i-1) & i = 2, ..., n \\ \frac{a + b_1 w_1 + b_1 h_1 (i-1)}{2b_1} & i = n+1, ..., N \end{cases} \\ p^j = \frac{a + b_1 w_1 + b_1 h_1 (j-1)}{2b_1} & j = 1, ..., N \\ \Pi_{h_1,h_2} = \frac{1}{N}\{w_1 \sum_{i=1}^{N}(a - b_1 p^i) + \sum_{j=1}^{n}(b_1 - b)p^j[w_1 + h_2(j-1)] \\ \qquad + \sum_{l=n+1}^{N}(b_1 - b)p^l \cdot p^l\} \end{cases} , \qquad \text{(TC.1)}$$

where $n = \texttt{min}\{\texttt{int}[\frac{a-b_1w_1}{b_1(2h_2-h_1)}+1], N\}$*, and we also have*

$$\begin{cases} w_i = p^i & i = n+1, ..., N \\ p^i - p^{i-1} = \frac{h_1}{2} & i = 2, ..., N \end{cases}. \tag{TC.2}$$

With a finite unit holding cost, the competitive fringe can also make forward buying for some periods (denoted by periods $i = 2, ..., n \le N$), if both the promotional wholesale price w_1 and unit hold cost h_2 are low enough. Knowing this, the manufacturer will be willing to charge a low wholesale price (but still higher than the promotional wholesale price w_1) in these periods to "match" the competitive fringe's effective acquisition cost, which is given by the sum of the promotional wholesale price w_1 and the holding cost accumulated by h_2 per period, *i.e.*, $w_1 + h_2(i-1)$. The manufacturer will stop such "matchings" from the period in which the competitive fringe's effective acquisition cost starts being higher than the retail price, in which case the manufacturer will charge a wholesale price equal to retail price to squeeze all surplus from the competitive fringe. That is, $w_i = p^i$ for any $i = n+1, ..., N$. Here the value of n is given by the very last period in a promotion cycle, in which period the competitive fringe's effective acquisition cost is lower than the retail price (so it is optimal for the competitive fringe to make forward buying if the wholesale price in the period is higher than its effective acquisition cost). Therefore, n is equal to the highest integer i satisfying $w_1 + h_2(i-1) \le p^i = \frac{a+b_1w_1+b_1h_1(i-1)}{2b_1}$, which gives $n = \texttt{min}\{\texttt{int}[\frac{a-b_1w_1}{b_1(2h_2-h_1)}+1], N\}$ since n should obviously be no larger than N.

Given the length of pricing cycle N, the manufacturer sets a promotional wholesale price w_1 to maximize the average profit Π_{h_1,h_2}[4]. To determine the optimal length of pricing cycle N, the manufacturer will solves the following optimization problem as in the basic model:

$$\begin{aligned} \max_N \quad & \Pi_{h_1,h_2}(N) \\ s.t. \quad & n = \texttt{min}\{\texttt{int}[\frac{a-b_1w_1}{b_1(2h_2-h_1)}+1], N\} \\ & w_1 = \texttt{argmax}\, \Pi_{h_1,h_2}(w_1, N) \end{aligned}$$

[4]Since there is no closed-form solution for w_1 when both h_1 and h_2 are finite, we search the optimal wholesale price w_1 in the range of $\{0, \frac{a}{b_1}\}$ by a step of $\epsilon = 0.01$ in the numerical studies below.

$$w_1 + h_1(N-1) \le w_N \tag{TC.3}$$

$$w_N = \mathtt{min}\{\frac{a + b_1 w_1 + b_1 h_1(N-1)}{2b_1}, w_1 + h_2(N-1)\}$$

$$N \ge 1.$$

It is straightforward to show that the effective unit acquisition cost for the dominant retailer is lower than that for the competitive fringe in period $i = 2, ..., N$ when the competitive fringe has a higher unit holding cost than the dominant retailer. This leads to the following proposition.

Proposition 1 *The manufacturer can use trade promotions to price discriminate between retailers with different but finite inventory holding costs.*

Proof of Proposition 1. In period $i = 1, ..., n \le N$, the competitive fringe pays unit wholesale price $w_i = w_1 + h_2(i-1)$ which is larger than the dominant retailer's effective acquisition cost $w_1 + h_1(i-1)$ for any $h_1 < h_2$. In period $i = n+1, ..., N$, the competitive fringe pays unit wholesale price $w_i = p^i = \frac{a+b_1w_1+b_1h_1(i-1)}{2b_1}$ which is also larger than the dominant retailer's effective acquisition cost $w_1 + h_1(i-1)$ for any $a - b_1 p^i > 0$. So the price discrimination mechanism still works when both retailers have finite but different unit inventory holding costs. Q.E.D.

Similar to the basic model, we have the following results and corresponding graphs are given below.[5]

Result 5 *For $0 < h_1 < h_2 < \infty$, the increased profits from trade promotions over single wholesale price for the manufacturer are decreasing in the dominant retailer's holding cost h_1 and maximized for medium b_1 for given (h_1, h_2).*

Result 6 *For $0 < h_1 < h_2 < \infty$, the benefits from trade promotions for the dominant retailer are decreasing in its holding cost h_1 and have an inverted-U relationship with b_1. The competitive*

[5] For the ease of comparison, all graphs are plotted for $h_1 = 0.0005$, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, 0.11, 0.12 and $h_2 = 0.01$, 0.1, 0.5, 1.0 but the manufacturer will not offer trade promotions for any $h_1 \ge h_2$ as shown in graphs.

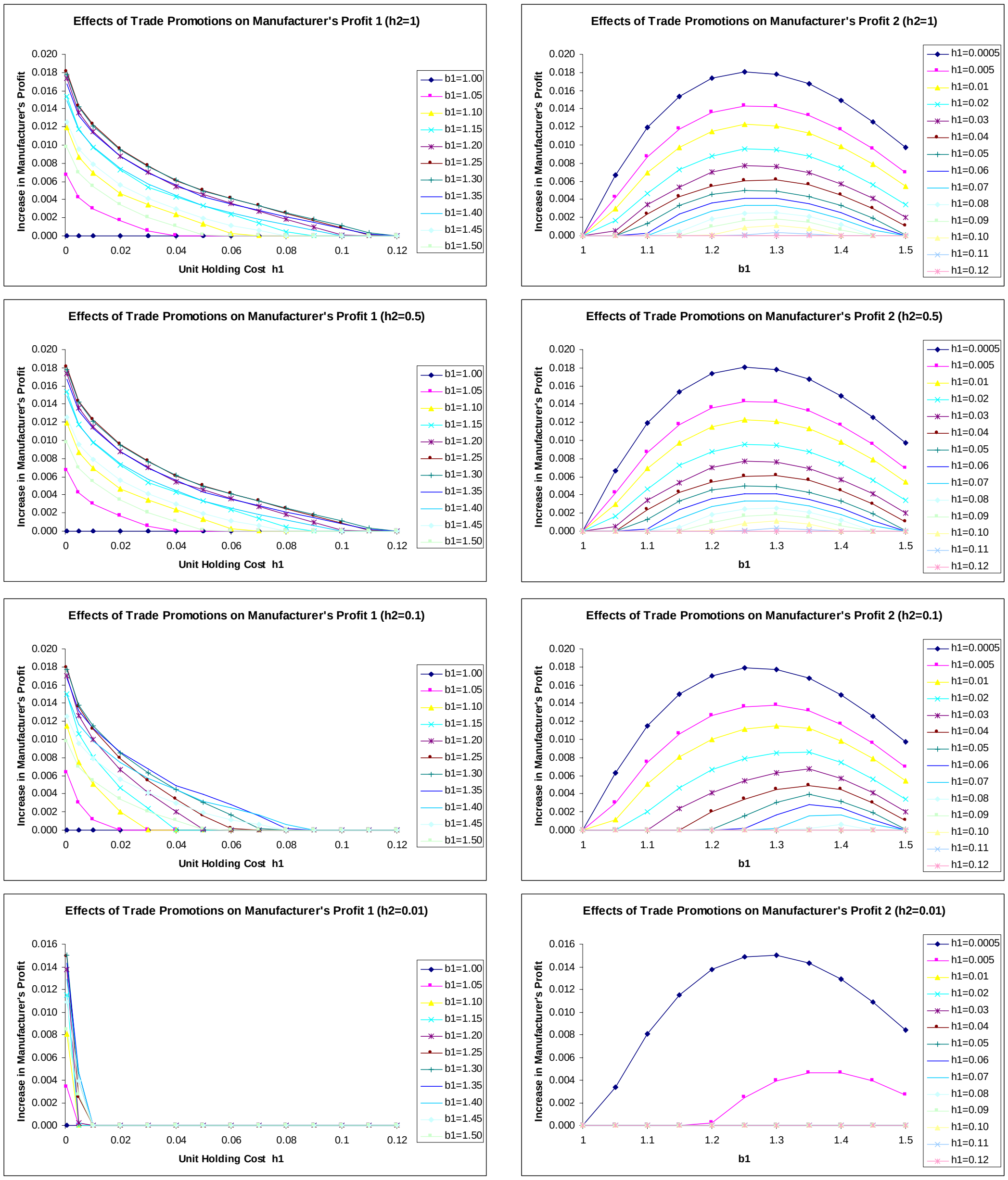

Figure 9: Effects of Trade Promotions on Manufacturer's Profit with Different h_2

fringe retailers can become either worse off or better off due to trade promotions. Their gain from trade promotions increases with h_1 *and has a U-shaped relationship with* b_1*.*

Result 7 *For* $0 < h_1 < h_2 < \infty$*, trade promotions can increase social welfare. The increase in social welfare is smaller when the dominant retailer's holding cost* h_1 *is larger or* b_1 *is smaller.*

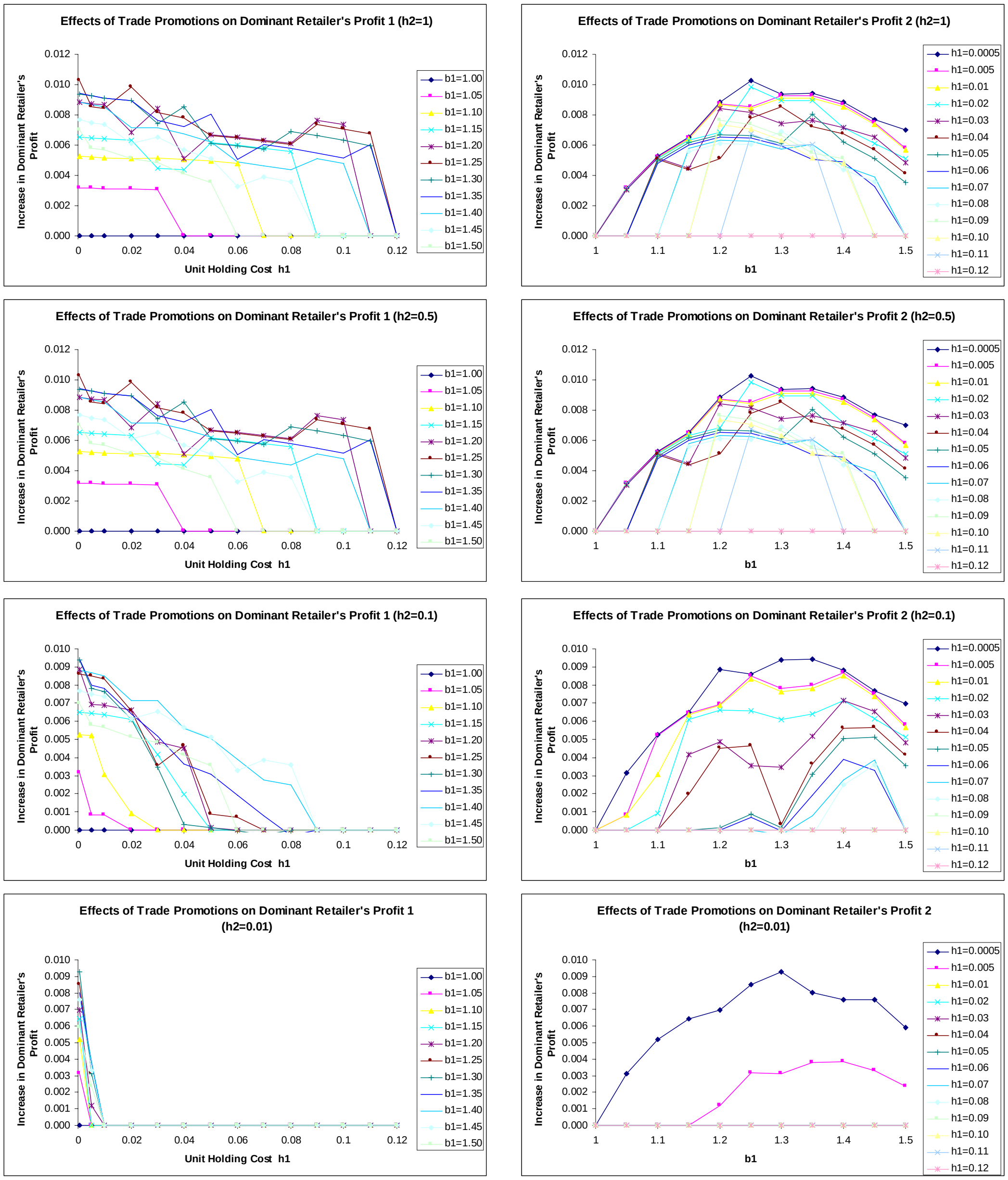

Figure 10: Comparative Static for Dominant Retailer's Profit Gains with Different h_2

Result 8 *For $0 < h_1 < h_2 < \infty$, the manufacturer's both promotion frequency and promotion depth are increasing in the dominant retailer's unit holding cost h_1 and decreasing in the dominant retailer's price sensitivity parameter b_1.*

Above analysis and results show that the manufacturer may still have incentives to offer trade promotions to retailers to price discriminate between dominant retailer and competitive fringe if the competitive fringe has a finite but higher inventory holding cost than the dominant retailer.

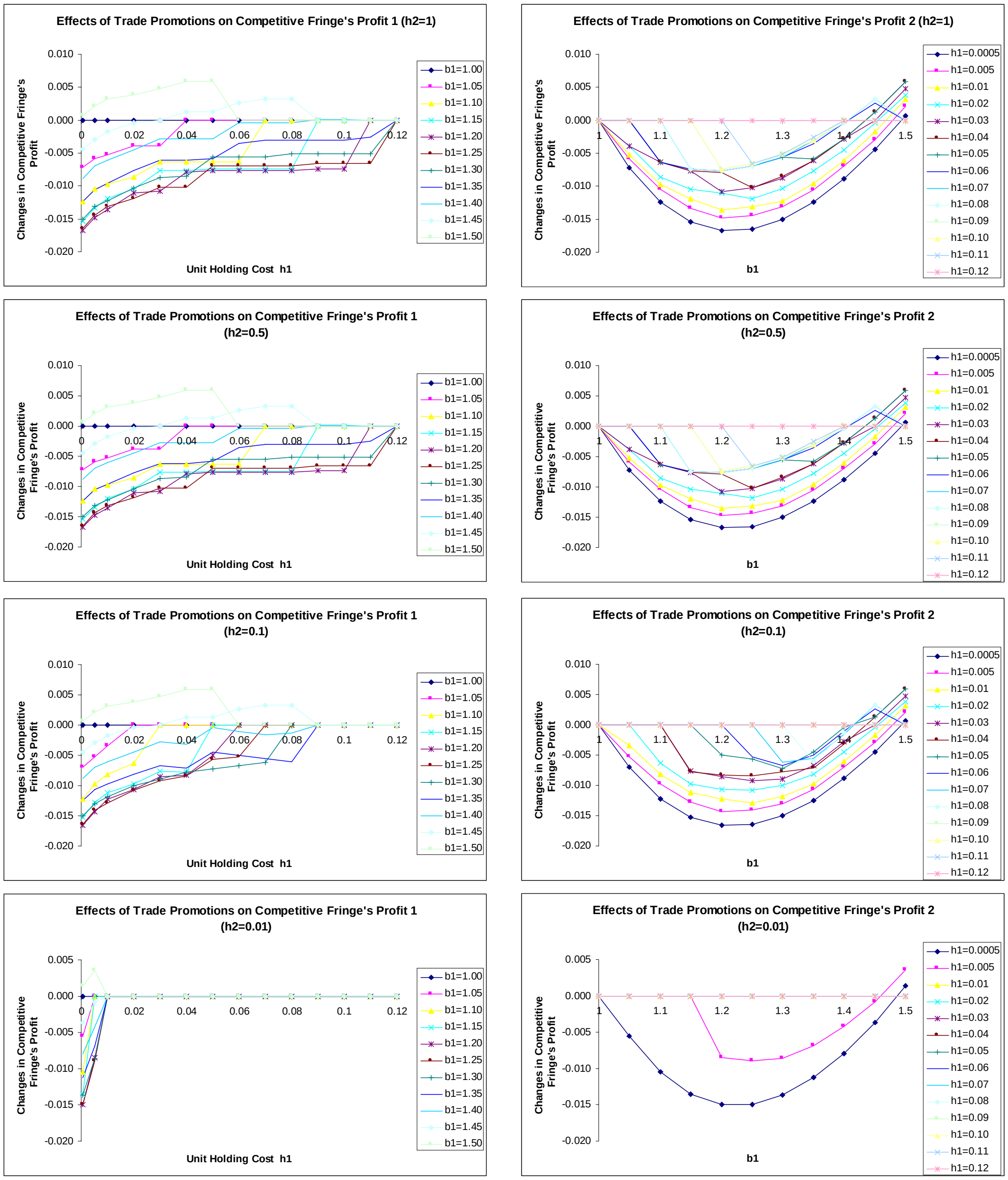

Figure 11: Comparative Static for Competitive Fringe's Profit Changes with Different h_2

The benefits for the manufacturer by offering trade promotions come from the different degrees to which retailers can take advantage of, because of the differential inventory holding costs.

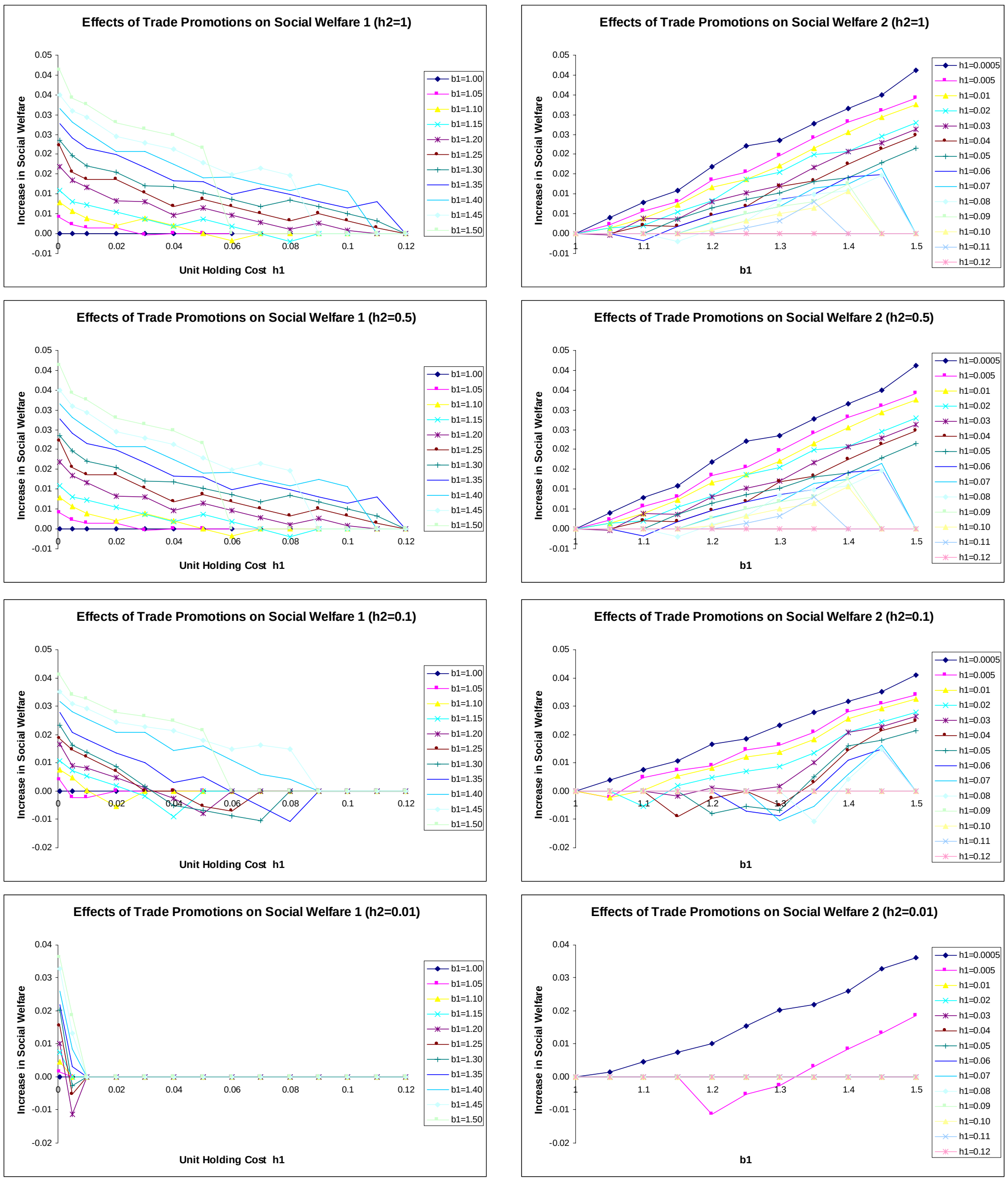

Figure 12: Comparative Static for Social Welfare Gains with Different h_2

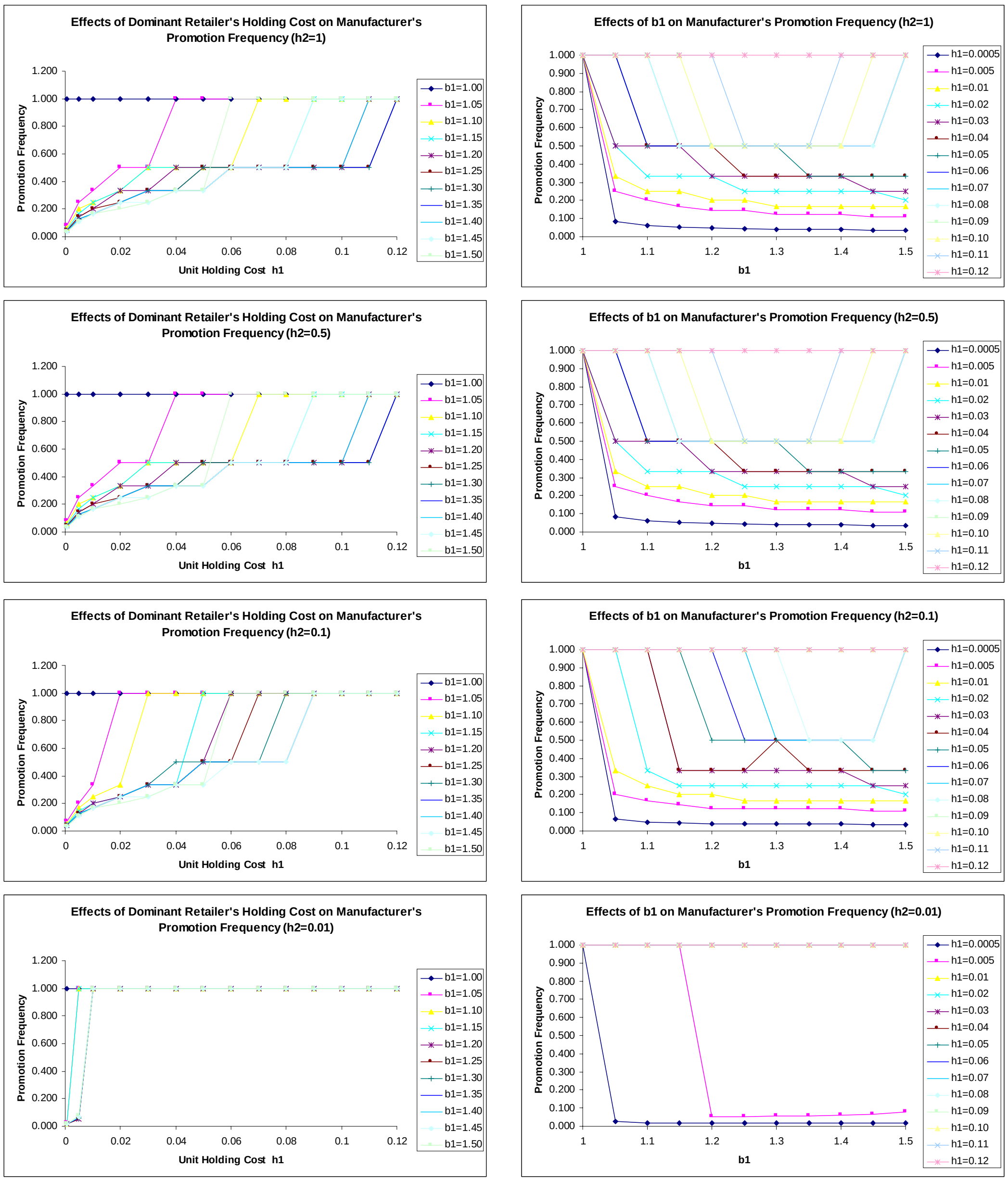

Figure 13: Comparative Static for Manufacturer's Promotion Frequency with Different h_2

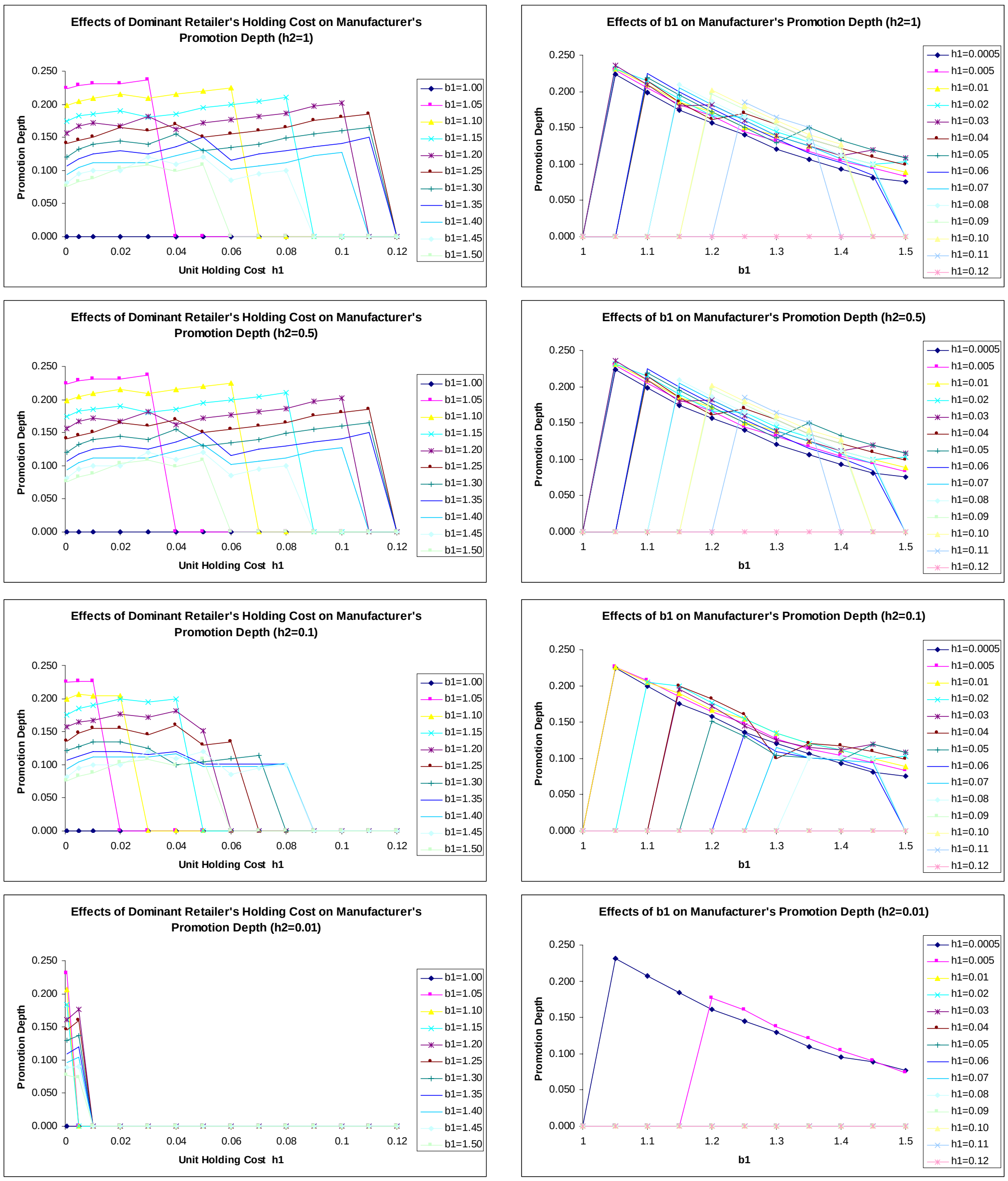

Figure 14: Comparative Static for Manufacturer's Promotion Depth with Different h_2