

THE LENGTH OF PRODUCT LINE IN DISTRIBUTION CHANNELS*

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Abstract

This paper studies a manufacturer's optimal decisions on extending its product line when the manufacturer sells through either a centralized channel or a decentralized channel. We show that a manufacturer may provide a longer product line for consumers in a decentralized channel than in a centralized channel if the market is fully covered. In addition, a manufacturer's decisions on the length of its product line may not always be optimal from a social welfare perspective in either a centralized or a decentralized channel. Under certain conditions, a decentralized channel can provide the product line length which is socially optimal whereas a centralized channel cannot.

(Keywords: *Product Line, Distribution Channel, Consumer Heterogeneity, Game Theory.*)

1. INTRODUCTION

The length of the product line is one of the most important product decisions by marketing managers.¹ This decision on product line length can have significant implications for the profitability of manufacturers that are selling in centralized or decentralized channels (Eliashberg and Steinberg 1993; Villas-Boas 1998; Allain and Waelbroeck 2006; Netessine and Taylor 2007). The length of product lines by manufacturers can also affect consumer and social welfare, leading to public policy concerns (Root 1972; Barwise and Meehan 2005; Allain and Waelbroeck 2006). For instance, Allain and Waelbroeck (2006) find that manufacturers do not extend their product lines enough, and that longer product lines can improve social welfare by providing a better fit between product offerings and consumer needs (Lancaster 1979; Aaker 1996). However, manufacturers are also often criticized for excessive product proliferation through long product lines (Barwise and Meehan 2005). According to Mary Stutzman of Find/VSP, a New York City marketing firm, over-the-counter

¹In this paper, we define the length of a product line as the number of products in a product line. In the package goods industry, the proportion of line and brand extensions is about 80% to 90% of new product offerings (Neff 2005). Similarly, Aaker (1991) reports that 89% of all new consumer nondurable products are line extensions.

drug line extensions may be reaching a saturation point, and long product lines by pharmaceutical manufacturers may not benefit consumers or retailers (Gannon 1994).

In this paper, we study the optimal decisions for a manufacturer on extending its product line when the manufacturer sells through either a centralized channel or a decentralized channel. In addition, we also explore the consequences of longer product lines on consumer and social welfare. We try to answer the following questions: When should a manufacturer have a greater incentive to expand its product line, in a centralized channel or a decentralized channel? How do market characteristics affect a manufacturer's decision on extending its product line? How does a manufacturer's extension of its product line affect consumer and social welfare in a centralized channel and a decentralized channel? To answer these questions, we develop a game-theoretical model to characterize the impact of channel structure on line extension decisions of a manufacturer when the manufacturer sells directly to consumers and when the manufacturer sells through an independent retailer.

1.1. Summary of Main Results

This study generates the following interesting findings. First, we show that a manufacturer, selling in a fully covered market, may provide a longer product line to consumers when the manufacturer sells through an independent retailer than when the manufacturer sells directly to consumers. This result differs from the results in some of the previous literature showing that a manufacturer should provide a shorter product line in a decentralized channel than in a centralized channel (Eliashberg and Steinberg 1993; Netessine and Taylor 2007). The intuition behind this finding is as follows. In a decentralized channel, the manufacturer suffers from channel inefficiencies associated with the double marginalization problem. A longer product line allows the manufacturer to better match its products with consumer tastes. The better matching reduces consumers' heterogeneity on the mismatching between consumer tastes and the manufacturer's product offerings, and more importantly the retailer's leverage of serving heterogeneous consumers for lower wholesale prices.

Consequently, the manufacturer may have more incentives to provide a longer product line in a decentralized channel than in a centralized channel to mitigate the double marginalization problem.

Second, the length of product line by a manufacturer in a centralized channel or in a decentralized channel may not be optimal from a social welfare perspective. A manufacturer may provide either a longer or shorter product line in both a centralized channel and a decentralized channel relative to a social planner who maximizes social welfare. This may help explain the anecdotal findings that product line extensions by manufacturers can be either insufficient or excessive in both types of channels. More interestingly, we find that a manufacturer in a decentralized channel can provide a product line length which is socially optimal from the perspective of a social planner that maximizes social welfare, but the manufacturer in a centralized channel cannot. In other words, channel decentralization can restore a social planner's decision on product line length. This is interesting since a decentralized channel, which is well-known for its inefficiency in maximizing the channel profit, can provide an efficient length of product line from a social welfare perspective; whereas a centralized channel, which is well-known for its efficiency in maximizing the channel profit, may provide an inefficient length of product line.

The rationale is as follows. Channel decentralization leads to the problem of double marginalization, and this can lead to the situation where the market is fully covered when the manufacturer sells through a centralized channel but not fully covered when the manufacturer sells through a decentralized channel. A longer product line not only mitigates the double marginalization problem, but also provides an extra benefit of more market coverage for the manufacturer in a decentralized channel than in a centralized channel. This extra benefit to the manufacturer is also consistent with the social welfare perspective since more market coverage improves consumer welfare. Therefore, both the manufacturer's incentive and the social planner's incentive on product line extension are more closely aligned in reducing the channel inefficiencies in a decentralized channel. Thus, the manufacturer's decision on the length of its product line is more likely to coincide with the social optimum in a decentralized channel than in a centralized channel.

1.2. Related Literature

This paper is related to two streams of literature. The first one is the literature on product line extensions. Longer product lines may help firms achieve higher total demand and market shares (Kotler 2002), target different customer segments (Villas-Boas 2004), and obtain more retail space or better utilization of manufacturing capacity (Lancaster 1979; Quelch and Kenny 1994; Aaker 1996). In addition, more products can also satisfy consumers' needs for "something different" (Klemperer 1992; Randall et al. 1998), and help a firm preempt the market entry of a competitor (Schmalensee 1978). This paper is particularly related to previous research on the effects of market structure on firms' product line extensions. Inderst and Shaffer (2007) find that the length of product line will be reduced after a horizontal merger between non-competing retailers in a distribution channel. Using a linear demand model, Allain and Waelbroeck (2006) study how the division of the product launching cost between the manufacturer and the retailer could affect product line length, and show that a manufacturer in a centralized channel offers a longer product line than in a decentralized channel when the product launching cost is mostly supported by the manufacturer. If a retailer can share a significant cost of the product line extension in a decentralized channel, a manufacturer may provide a longer product line due to the *de facto* cost reduction. In this paper, we contribute to the literature by showing that even when the manufacturer bears the entire cost of extending the product line, the manufacturer may still be willing to offer a longer product line in a decentralized channel than in a centralized channel.

Our paper is also closely related to Villas-Boas (1998) who studies the effects of channel structure on the optimal design of the product line and finds that a manufacturer may offer products that are more differentiated from each other in a decentralized channel than in a centralized channel. In Villas-Boas (1998), the retailer may find it more profitable not to serve the lower margin segment since the profit made from serving this segment may not offset the opportunity loss from serving only the higher margin segment. A strategic manufacturer, who takes into account the strategic behavior by the retailer, will intentionally distort the product quality levels so that the quality

difference between different products is larger in a decentralized channel. The current paper differs from Villas-Boas (1998) in the following ways. First, Villas-Boas (1998) mainly deals with the *design* issue of product line given the fixed number of products in the product line, *i.e.*, how much should products in a line *differ* from each other, while our model focuses on the *length* issue of product line, *i.e.*, should a firm *extend* a single product to a product line with multiple products. Second, Villas-Boas (1998) studies the case where consumers are vertically differentiated. In this paper, we examine the case where consumers are horizontally differentiated and where the number of products is smaller than the number of consumer types, and focus on the optimal number of products in a product line. We provide a new mechanism of beneficial product line extension for a manufacturer: mitigating the double marginalization problem in a decentralized channel. We contribute to the marketing literature by showing that a manufacturer in a decentralized channel can provide a product line length which is optimal from a social welfare perspective but a manufacturer in a centralized channel cannot, and channel decentralization can restore a social planner's decision on product line extension.

The second related stream of research is the one on distribution channel coordination. Due to the well-known problem of double marginalization, a decentralized channel often results in demand recession which leads to lower firm profits and social welfare. A good deal of research focuses on how results in a decentralized channel can be restored to those in a centralized channel through quantity discounts or two-part tariffs (Jeuland and Shugan 1983; Moorthy 1987; Ingene and Parry 1995; Raju and Zhang 2005), franchising and contracting (Lal 1990; Iyer 1998), pull promotion (Gerstner and Hess 1995), bargaining (Iyer and Villas-Boas 2003; Dukes et al. 2006), trade promotions and forward buying (Cui et al. 2008; Desai, et al. 2008), or channel members' concerns of fairness (Cui et al. 2007).² This paper contributes to the distribution channel literature by showing that a decentralized channel, which is well-known for its inefficiency in maximizing the channel profit, can provide a product line length which is consistent with a social planner's deci-

²This is only a representative list.

sion from a social welfare perspective. However, a centralized channel, which is well-known for its efficiency, cannot. Therefore, a decentralized channel, instead of a centralized channel, can be an efficient structure of distribution channel when the decision on product line length is taken into consideration. Consequently, channel coordination may lead to a new type of inefficiency.

The rest of the paper is organized as follows. We first lay out the model setup in the beginning of Section 2. In Sections 2.1 and 2.2, we study the effects of product line extension in a centralized channel and a decentralized channel respectively. In Section 2.3, we focus on how distribution channel structure affects consumer and social welfare. We conclude in Section 3. All proofs of the propositions are given in the Appendix or can be found in the online Technical Appendix.

2. HORIZONTAL PRODUCT LINE EXTENSION

Consider a distribution channel where a manufacturer sells products through a retailer to consumers. Consumers are located uniformly on a Hotelling line bounded between 0 and 1, and the market size is normalized to be one (Hotelling 1929). Each consumer buys at most one product with the reservation price V for her ideal product. When the manufacturer sells only one product, the location of the product is assumed to be at the middle point ($x = \frac{1}{2}$) of the Hotelling line. A consumer located at x incurs a mismatching disutility of $t|x - \frac{1}{2}|$ where t is the unit mismatch cost which measures the magnitude of disutility a consumer incurs from the mismatching between her ideal products and product offerings by the manufacturer. In this paper, we focus the analysis on the most interesting case of $t \leq V$.³ The manufacturer has the option to extend the product line by adding another product with a fixed cost F . We consider only two products in an extended product line to illustrate the main results and mechanisms.^{4 5} When the manufacturer has two products,

³The case of $t > V$ is available in the online Technical Appendix.

⁴The main results do not change qualitatively when the product line is extended to $n(n \geq 3)$ products. See the analysis of a product line with $n(n \geq 3)$ products in the online Technical Appendix.

⁵In the paper, we consider a firm's product extension decisions given the firm has already had a product in the market. That is, we focus on the product extension decisions of an "incumbent" firm instead of the product introduction decision of a new entry firm.

the two products are located at $\frac{1}{4}$ and $\frac{3}{4}$ optimally.⁶ The fixed cost F captures the repositioning cost of the first product and the additional costs such as the cost of R&D of adding the second product.⁷ The marginal production cost for the manufacturer is normalized to zero without further loss of generality. We first analyze the effect of extending the product line in a centralized channel, then we study the product line extension in a decentralized channel.

2.1. Horizontal Product Line Extension in A Centralized Channel

The centralized case with only one product provides a benchmark case for the effect of product line extension in channels. Given any retail price p_{ch1} , the surplus for a consumer located at x is given by $V - p_{ch1} - t|x - \frac{1}{2}|$ and the demand of the product is $D_{ch1} = 2\frac{V-p_{ch1}}{t}$.⁸ The optimal retail price and profit for the manufacturer in the centralized channel are given by $p_{ch1} = V - \frac{t}{2}$ and $\Pi_{ch1} = V - \frac{t}{2}$. When the manufacturer extends the product line by introducing the second product with a fixed cost of F , the optimal retail price and total channel profit are given by $p_{ch2} = V - \frac{t}{4}$ and $\Pi_{ch2} = V - \frac{t}{4} - F$.

Now, we compare the manufacturer's profit before and after product line extension in the centralized channel. We hereafter assume that the manufacturer will not extend the product line if the manufacturer is indifferent between extending the product line and not extending the product line. The manufacturer will profitably extend the product line if the fixed cost is sufficiently low, $F < \frac{t}{4}$. As the consumer mismatch cost t increases, product line extensions are likely to be more profitable and the manufacturer is more likely to extend the product line. This is because as the mismatch cost increases, an additional product will help reduce the mismatch cost for consumers, which enables more consumers to purchase the product or be willing to pay a higher price.

⁶The formal proofs for both centralized and decentralized channels are given in the online Technical Appendix.

⁷Strictly speaking, F is the normalized fixed cost per consumer since the market size has been normalized to be one.

⁸In order to differentiate between different cases, we use subscripts $\{\cdot\}_{ijk}$ to refer to the measure $\{\cdot\}$ in an i ($i \in \{c : \text{centralized}; d : \text{decentralized}\}$) channel with j ($j \in \{h : \text{horizontal}; v : \text{vertical}; b : \text{both horizontal and vertical}\}$) consumer distribution and k ($k \in \{1, 2\}$) products. For instance, the variable p_{ch1} refers to the retail price in a *centralized* channel with *horizontally* distributed consumers and *one* product offered by the manufacturer.

Next, we look at the optimal product line extension decision from a social welfare perspective. When there is only one product in the market, total consumer surplus is given by $CS_{ch1} = \frac{t}{4}$. After product line extension, there are two products in the market and total consumer surplus is given by $CS_{ch2} = \frac{t}{8}$. We calculate social welfare by adding up the firm's profit and consumer welfare. Social welfare before product line extension is given by $SW_{ch1} = V - \frac{t}{4}$. After product line extension, social welfare is given by $SW_{ch2} = V - \frac{t}{8} - F$. Therefore, product line extension is beneficial from the society's point of view when $F < \frac{t}{8}$.

Finally, we compare the optimal product line extension decisions from the manufacturer's perspective and from the social welfare perspective respectively, and find that in a centralized channel, a social planner who maximizes social welfare will not extend the product line, and a manufacturer who maximizes own profit will extend the product line if $\frac{t}{8} \leq F < \frac{t}{4}$. A longer product line allows the manufacturer to better segment the market with two products, although the market will be fully covered with only one product in this case. Such better segmentation gives the manufacturer more monopoly power over consumers and allows the manufacturer to charge a higher price when its product line is extended ($p_{ch2} = V - \frac{t}{4} > p_{ch1} = V - \frac{t}{2}$). Consumers are always worse off after the manufacturer extends its product line ($CS_{ch2} = \frac{t}{8} < CS_{ch1} = \frac{t}{4}$). Since the manufacturer maximizes own profit and the social planner maximizes the sum of the manufacturer's profit and consumer welfare, the social planner is less likely to extend the product line. When $\frac{t}{8} \leq F < \frac{t}{4}$, the benefit of product line extension can offset the fixed cost F for the manufacturer ($\Pi_{ch2} - \Pi_{ch1} = \frac{t}{4} - F$). However, the profit gain for the manufacturer is smaller than the loss of consumer welfare ($CS_{ch2} - CS_{ch1} = \frac{t}{8}$). Thus, in this range, the manufacturer will and the social planner will not extend the product line.

2.2. Horizontal Product Line Extension in A Decentralized Channel

In this section, we analyze the product line extension decisions in a decentralized channel. We model the manufacturer's and the retailer's decisions in a two-stage game. In the first stage, the

manufacturer decides whether to extend the product line, and decides on its wholesale price w_{dh1} . In the second stage, the retailer decides on the retail price given the wholesale price w_{dh1} . We focus on the case where the manufacturer is more powerful than the retailer such that the manufacturer is able to make a take-it-or-leave-it offer of products to sell to the retailer. Everything else remains the same as in the centralized channel. We solve the game using backward induction to guarantee subgame perfection.

When there is only one product for sale, the equilibrium prices for the manufacturer and the retailer are:

$$w_{dh1} = \begin{cases} \frac{V}{2}, & \text{if } \frac{V}{2} < t \leq V; \\ V - t, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad p_{dh1} = \begin{cases} \frac{3V}{4}, & \text{if } \frac{V}{2} < t \leq V; \\ V - \frac{t}{2}, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad (2.1)$$

The profits for the manufacturer (Π_{dh1}) and the retailer (π_{dh1}) are:

$$\Pi_{dh1} = \begin{cases} \frac{V^2}{4t}, & \text{if } \frac{V}{2} < t \leq V; \\ V - t, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad \pi_{dh1} = \begin{cases} \frac{V^2}{8t}, & \text{if } \frac{V}{2} < t \leq V; \\ \frac{t}{2}, & \text{if } t \leq \frac{V}{2}. \end{cases} \quad (2.2)$$

Now, we analyze the case where the manufacturer extends the product line by adding another product with a fixed cost of F . The equilibrium prices and profits for both the manufacturer and the retailer for $t \leq V$ are as follows:

$$w_{dh2} = V - \frac{t}{2}, \quad p_{dh2} = V - \frac{t}{4}, \quad \Pi_{dh2} = V - \frac{t}{2} - F, \quad \pi_{dh2} = \frac{t}{4}. \quad (2.3)$$

We compare the manufacturer's profits before and after product line extension. Product line extension is profitable for the manufacturer if the fixed cost is sufficiently low, $F < \frac{t}{2}$ if $t \leq \frac{V}{2}$ and $F < V - \frac{t}{2} - \frac{V^2}{4t}$ if $\frac{V}{2} < t \leq V$.

We then compare the optimal product line extension decisions in the centralized channel and in the decentralized channel, and summarize the main result in the following proposition.

PROPOSITION 1: *When the market is fully covered ($t \leq \frac{V}{2}$), the manufacturer extends its product line in a decentralized channel but not in a centralized channel if $\frac{t}{4} \leq F < \frac{t}{2}$.*

Proposition 1 shows that a manufacturer in a decentralized channel may provide a longer product line than in a centralized channel. Therefore, channel decentralization may cause a manufacturer to extend its product line. The intuition of this interesting result is as follows. When selling in a decentralized channel, the manufacturer faces the classic problem of double marginalization.⁹ When t is sufficiently small ($t \leq \frac{V}{2}$), the manufacturer is forced to keep its wholesale price ($w_{dh1} = V - t$) low enough to encourage the retailer to sell to all consumers including those located at the extreme end of the Hotelling line ($x = 1$). A longer product line reduces the overall consumer heterogeneity on the mismatching between consumer tastes and the manufacturer's product offerings, and also the retailer's leverage of serving heterogeneous consumers for lower wholesale prices. Therefore, a longer product line can help the manufacturer mitigate the double marginalization problem in a decentralized channel, and such a benefit is not present when the manufacturer sells in a centralized channel. Consequently, the manufacturer has a greater incentive to extend its product line in a decentralized channel than in a centralized channel.

To better understand the intuition, recall that the manufacturer in the centralized channel can increase its price for $p_{ch2} - p_{ch1} = \frac{t}{4}$ after the manufacturer extends the product line. In a decentralized channel, after the manufacturer extends the product line, the retailer can again increase the retail price for $p_{dh2} - p_{dh1} = \frac{t}{4}$. However, the manufacturer increases its wholesale price by $w_{dh2} - w_{dh1} = \frac{t}{2} > p_{ch2} - p_{ch1} = \frac{t}{4}$. Therefore, the extension of the product line in a decentralized channel actually increases the manufacturer's price and profit more than it does in a centralized channel. The increase in price and profit for the manufacturer reflects the extra benefit from mitigating the double marginalization problem in the decentralized channel when the market is always fully covered ($t \leq \frac{V}{2}$). When the manufacturer sells only one product, for $\frac{V}{2} < t \leq V$, the market is not fully covered in a decentralized channel but is fully covered in a centralized channel.

⁹This holds even when t is sufficiently small ($t \leq \frac{V}{2}$) and the whole market is always fully covered.

In this case, product line extension provides an additional benefit of market expansion for the manufacturer, and the manufacturer in a centralized channel does not but the manufacturer in a decentralized channel does extend the product line when $\frac{t}{4} \leq F < V - \frac{t}{4} - \frac{V^2}{2t}$.

Proposition 1 complements the findings in the literature that channel decentralization always shortens product lines when only the manufacturer bears the cost of extending a product line (Eliashberg and Steinberg 1993; Allain and Waelbroeck 2006; Netessine and Taylor 2007). Allain and Waelbroeck (2006), for instance, show that channel decentralization has a negative effect on a manufacturer's incentive to extend the product line, if the manufacturer bears the entire cost of extending the product line. If the retailer in the decentralized channel shares a significant amount of the fixed cost of product line extension, the manufacturer may have a stronger incentive to extend its product line. In this case, it is important to note that the driving force behind additional product line extensions for a manufacturer after channel decentralization is the cost reduction described in Allain and Waelbroeck (2006). In this paper, however, there is no cost reduction and the manufacturer bears the entire cost of product line extension. The reason for the manufacturer to be more likely to extend the product line is because of the extra benefit of product line extension on mitigating the double marginalization problem, which is present in a decentralized channel but absent in a centralized channel.

To further confirm the effect of product line extension on mitigating the double marginalization problem, we examine the case when the market is not fully covered ($t > V$).¹⁰ In this case, the impact of product line extension on mitigating the double marginalization problem is low since the manufacturer cannot increase its wholesale prices without reducing market coverage. That is, the wholesale price and the product line extension decisions for the manufacturer are bound more by considerations of market coverage than by mitigating the double marginalization problem in the decentralized channel. The manufacturer is thus always less likely to extend the product line in a decentralized channel than in a centralized channel.

¹⁰See the detailed analysis in the online Technical Appendix.

Next, we study the effect of product line extension on consumer surplus and social welfare in a decentralized channel. When there is only one product in the market, total consumer surplus is given by $CS_{dh1} = \frac{t}{4}$ if $t \leq \frac{V}{2}$ and $CS_{dh1} = \frac{V^2}{16t}$ if $\frac{V}{2} < t \leq V$. After product line extension, total consumer surplus for $t \leq V$ is given by $CS_{dh2} = \frac{t}{8}$. Consumer surplus decreases for $t \leq \frac{V}{\sqrt{2}}$ and increases for $\frac{V}{\sqrt{2}} < t \leq V$. Recall that consumers are always worse off when the manufacturer extends the product line in a centralized channel. In contrast, when the manufacturer sells through a decentralized channel, consumers may benefit from product line extension. The benefit comes from the market expansion effect of a longer product line by the manufacturer in a decentralized channel since the market is not fully covered when the manufacturer has only one product but fully covered after the manufacturer extends the product line for $\frac{V}{2} < t \leq V$.

We now look at the effect of product line extension on social welfare by considering the profits of both the manufacturer and the retailer, and consumer welfare, together. Social welfare before product line extension is given by $SW_{dh1} = V - \frac{t}{4}$ if $t \leq \frac{V}{2}$ and $SW_{dh1} = \frac{7V^2}{16t}$ if $\frac{V}{2} < t \leq V$. After product line extension, social welfare is given by $SW_{dh2} = V - \frac{t}{8} - F$ for $t \leq V$. Therefore, a social planner will extend the product line if the fixed cost is sufficiently low, $F < \frac{t}{8}$ if $t \leq \frac{V}{2}$ and $F < V - \frac{t}{8} - \frac{7V^2}{16t}$ if $\frac{V}{2} < t \leq V$.

We compare the product line extension decision for a manufacturer with the product line extension decision for a social planner in a decentralized channel. We find that in a decentralized channel, the manufacturer extends the product line but the social planner does not when $t \leq \frac{V}{2}$ if $\frac{t}{8} \leq F < \frac{t}{2}$ or when $\frac{V}{2} < t \leq \frac{V}{\sqrt{2}}$ if $V - \frac{t}{8} - \frac{7V^2}{16t} \leq F < V - \frac{t}{2} - \frac{V^2}{4t}$. When $\frac{V}{\sqrt{2}} < t \leq V$ if $V - \frac{t}{2} - \frac{V^2}{4t} \leq F < V - \frac{t}{8} - \frac{7V^2}{16t}$, the manufacturer does not extend but the social planner does extend the product line.

2.3. Channel Structure and Welfare-Improving Product Line Extension

The previous analysis lists the threshold values of the fixed cost F for an optimal product line extension for the manufacturer and a social planner, respectively. The comparison of the threshold

values of F suggests that the manufacturer's decision on product line extension is not always consistent with the decision of a social planner who maximizes social welfare. Consequently, one interesting question to ask is: in which channel structure is a manufacturer more likely to make a welfare-improving product line extension decision? We try to answer this question by comparing the conditions of beneficial product line extension for a manufacturer and for a social planner in both a centralized channel and a decentralized channel. We summarize the main results in the following proposition.

PROPOSITION 2: *The manufacturer in a decentralized channel can provide the socially optimal product line length while a manufacturer in a centralized channel cannot when $\frac{t}{8} \leq F < \min\{V - \frac{t}{8} - \frac{7V^2}{16t}, \frac{t}{4}\}$ if $\frac{V}{2} < t \leq V$. Under this condition, channel decentralization by a manufacturer can help restore a social planner's decision on product line extension.*

Interestingly, Proposition 2 shows that a decentralized channel, which is well-known for its inefficiency in maximizing the channel profit due to the double marginalization problem, can be more efficient than a centralized channel on product line extension from a social welfare perspective. Therefore, channel decentralization by a manufacturer can have important implications for achieving social optimum from a social welfare perspective. In Proposition 2, we refer restoring a social planner's decision on product line to the situation where the manufacturer's decision on its product line extension in a decentralized channel is the same as a social planner's decision in the decentralized channel, and for the *same* parameter space, the manufacturer's decision is not consistent with a social planner's line extension decision in a centralized channel.¹¹

The rationale behind the effect of channel decentralization on restoring a social planner's decision comes from the difference in market coverage between a centralized channel and a decentralized channel. When $\frac{V}{2} < t \leq V$ and the manufacturer sells in a centralized channel, the market is fully

¹¹As shown in the online Technical Appendix, in a market where consumers are vertically differentiated or where consumers are both horizontally and vertically differentiated (Moorthy 1984,1988; Desai 2001), channel decentralization can still restore a social planner's decision on product line extension.

covered. In this case, the only incentive to extend the product line for the manufacturer and a social planner is to reduce the mismatching cost for consumers. However, the social benefit of extending the product line is a reduction in mismatching costs to the average consumers, while the private benefit to the manufacturer pertains only to the marginal consumers located at the extreme end of the Hotelling line ($x = 1$). Therefore, the manufacturer's and the social planner's incentives of extending the product line are not necessarily aligned. When the manufacturer sells in a decentralized channel, the market is not fully covered due to the double-marginalization problem. In this case, a longer product line by the manufacturer not only mitigates the double marginalization problem, but also provides an extra benefit of market expansion to the manufacturer in a decentralized channel. This market expansion effect benefits social welfare as well since more consumers will be covered, and the incentives for both the manufacturer and the social planner are more closely aligned in reducing the inefficiencies in the decentralized channel. Therefore, the decision to extend the product line by the manufacturer is more likely to be consistent with that from a social welfare perspective.

To further understand the intuition, recall that after product line expansion, consumer welfare always decreases in a centralized channel when $t \leq V$ but can increase in a decentralized channel when $\frac{V}{\sqrt{2}} < t \leq V$. In addition, the manufacturer is more likely to extend the product line in a decentralized channel than in a centralized channel when $\frac{t}{2} < t \leq V$. Therefore, product line extension by the manufacturer is more likely to benefit consumer welfare in a decentralized channel than in a centralized channel. Although a manufacturer maximizes its own profit when deciding the product line extension, the positive spillover effect of product line extension on consumer welfare is consistent with a social welfare perspective. When $\frac{V}{2} < t < \frac{V}{\sqrt{2}}$, consumer welfare decreases after the manufacturer extends the product line in a decentralized channel, but the benefit to firms from market expansion dominates the loss to consumer welfare. In this situation, product line extension by the manufacturer again improves social welfare. This is in contrast to the centralized channel where the market expansion effect is absent.

To further confirm the additional benefit of the market coverage effect of product line extensions in a decentralized channel, we examine the case of $t \leq \frac{V}{2}$ and $t > V$. When $t \leq \frac{V}{2}$, the market is always fully covered even when the manufacturer has only one product in a decentralized channel, and product line extension does not bring in the benefit of more market coverage. Therefore, channel decentralization cannot restore a social planner's decision on line extensions. For $t > V$, the market is not fully covered when the manufacturer has only one product even in a centralized channel. In this case, product line extension by the manufacturer has the market coverage effect in both centralized and decentralized channels. However, the market expansion effect is larger in a decentralized channel than in a centralized channel, and channel decentralization can restore a social planner's decision on product line extension when $V < t \leq (4-2\sqrt{2})V$ and $V - \frac{t}{8} - \frac{3V^2}{4t} \leq F < \frac{V^2}{4t}$.¹²

It is important to acknowledge that channel decentralization does not always restore a social planner's decision on product line extension. Under certain conditions, channel decentralization can also generate a situation that is further away from the social planner's decision. For instance, when $t \leq \frac{V}{2}$ and $\frac{t}{4} \leq F < \frac{t}{2}$, the decision of the manufacturer in a centralized channel is but the decision of the manufacturer in a decentralized channel is not consistent with the decision of the social planner. In this case, channel decentralization can actually lead to a distortion of the manufacturer's decision on product line extension from the social planner's decision.¹³

3. CONCLUSION

In this paper, we study the optimal product line extension decisions for a manufacturer when the manufacturer sells through either a centralized channel or a decentralized channel. The results in this paper have significant implications for marketing managers and social planners. First, we show

¹²See the analysis in the online Technical Appendix. When $t > \frac{8-\sqrt{26}}{2}V$, channel decentralization can still restore a social planner's decision on product line extension but through a different mechanism. In that case, the manufacturer is less likely to extend the product line in a decentralized channel than in a centralized channel. This can be consistent with a social planner's decision due to the fact that the manufacturer will extend the product line while a social planner will not in a centralized channel.

¹³Other conditions for this distortion include (1) $\frac{V}{2} < t < \frac{V}{\sqrt{2}}$ and $\max\{V - \frac{t}{8} - \frac{7V^2}{16t}, \frac{t}{4}\} \leq F < V - \frac{t}{2} - \frac{V^2}{4t}$; or (2) $\frac{V}{\sqrt{2}} < t \leq V$ and $V - \frac{t}{2} - \frac{V^2}{4t} \leq F < V - \frac{t}{8} - \frac{7V^2}{16t}$.

that a manufacturer may want to provide a longer product line in a decentralized channel than in a centralized channel. The longer product line can help the manufacturer mitigate the well-known double marginalization problem in a decentralized channel. Second, we also show that a manufacturer's decision on the length of its product line in a centralized channel or in a decentralized channel may not be optimal from the perspective of a social planner who maximizes social welfare. Interestingly, channel decentralization can restore a social planner's product line decision in that the manufacturer's decision in a decentralized channel on line extension is consistent with the social planner's, but the manufacturer's decision in a centralized channel is not consistent with the social planner's. This implies that a decentralized channel, which is well-known for its inefficiency in maximizing the channel profit, can provide an efficient product line from the perspective of social welfare.¹⁴

We would also like to point out the limitations of this paper and topics for future research. First, we abstract away from the possibility that each consumer purchases more products after a manufacturer extends the product line. Getting people to buy more products could be a very important reason for manufacturers to extend their product lines, which is not modeled here. The current setup applies to cases where the incremental utility of buying a second product for consumers is sufficiently low. If consumers purchase more than one product when the product line is longer, the manufacturer will enjoy an additional benefit from increased demands beyond the strategic benefits studied in this paper. It is shown in the paper that a longer product line can help a manufacturer mitigate the double marginalization problem in a decentralized channel since an extended product line reduces consumers' heterogeneity in mismatching. The possibility for each consumer to purchase more products after a line extension can increase such a benefit for the manufacturer, and the manufacturer may be more likely to extend the product line in a decentralized channel than in a centralized channel.

¹⁴The results will not change qualitatively in a competitive context or if the retailer can strategically adopt products to maximize its own profit (Dukes et al. 2009). The detailed analysis can be found in the online Technical Appendix.

Second, this paper mainly focuses on the cases where the maximum number of products in an extended product line is equal to two. Although the model extension in the online Technical Appendix extends our results to an n product case, considering more than two products may sometimes change the results qualitatively (Villas-Boas 1998). Future research can further examine this issue in a context other than the one analyzed here. This is especially important when firms can offer customization for their products.

Third, the base model of this paper studies a monopoly manufacturer's product line extension decisions when selling through a monopoly retailer. Although competition at the manufacturer level is examined in the model extension, a full-fledge study of competition at both the manufacturer and the retailer levels is needed to help us fully understand the effects of distribution channel structures on product line extensions. In addition, competing retailers may carry different subsets of the same product lines from the manufacturers. A word of caution on this is that a model with competition at both levels may be too complicated to make the model tractable.

Fourth, in this paper, we have focused on the effects of distribution channel structure on a manufacturer's product line extension decisions. A relevant issue is the effects of product line length on a manufacturer's choice of distribution channel structure. A manufacturer with a long product line may not have the infrastructure and capability to sell all the products by itself, and thus may have to rely on intermediaries. All these issues are worthy of deliberate considerations for marketing managers, and we leave the analysis of these issues for future research.

Appendix

Proof of Proposition 1: When consumers are horizontally heterogeneous, a manufacturer in a centralized channel will extend the product line for a low F , $F < \frac{t}{4}$. When the channel is decentralized, the manufacturer will extend the product line when $F < \frac{t}{2}$ if $t \leq \frac{V}{2}$ or when

$F < V - \frac{t}{2} - \frac{V^2}{4t}$ if $\frac{V}{2} < t \leq V$. For $t \leq \frac{V}{2}$, the manufacturer in a centralized channel will extend the product line if the value of F is below $\frac{t}{4}$, which is smaller than the threshold value of

$F, \frac{t}{2}$, for the manufacturer in a decentralized channel to extend the product line. Therefore, the manufacturer in a decentralized channel will extend the product line while the manufacturer in a centralized channel will not when $t \leq \frac{V}{2}$ and $\frac{t}{4} < F \leq \frac{t}{2}$. The proofs for other conditions of t follow the same logic.

Proof of Proposition 2: The manufacturer in a centralized channel will extend its product line when $t \leq V$ if $F < \frac{t}{4}$. The social planner who maximizes social welfare will extend the product line when $t \leq V$ if $F < \frac{t}{8}$ in the channel. Thus, the manufacturer will extend the product line for $\frac{t}{8} \leq F < \frac{t}{4}$ while a social planner would not. Similarly, we can show that the manufacturer in a decentralized channel will extend the product line when $t \leq \frac{V}{2}$ if $\frac{t}{8} \leq F < \frac{t}{2}$ but a social planner would not. Since the range of t within which the manufacturer will but a social planner will not extend the product line in the decentralized channel contains the range in the centralized channel, the decentralization will not help restore the social planner's choice for $t \leq \frac{V}{2}$.

When $\frac{V}{2} < t \leq \frac{V}{\sqrt{2}}$, the manufacturer in a centralized channel will extend its product line although a social planner will not extend the product line if $\frac{t}{8} \leq F < \frac{t}{4}$. The manufacturer in a decentralized channel will extend the product line while a social planner will not, however, only if $V - \frac{t}{8} - \frac{7V^2}{16t} \leq F < V - \frac{t}{2} - \frac{V^2}{4t}$. That is, both the manufacturer and the social planner in the decentralized channel will extend the product line for $\frac{t}{8} \leq F < \min\{V - \frac{t}{8} - \frac{7V^2}{16t}, \frac{t}{4}\}$, but only the manufacturer in the centralized channel would like to extend the product line for $\frac{t}{8} \leq F < \frac{t}{4}$. Therefore, the decentralization of the channel will restore the social planner's choice when $\frac{V}{2} < t \leq \frac{V}{\sqrt{2}}$ if $\frac{t}{8} \leq F < \min\{V - \frac{t}{8} - \frac{7V^2}{16t}, \frac{t}{4}\}$. When $\frac{V}{\sqrt{2}} < t \leq V$, the manufacturer in a centralized channel will extend the product line while a social planner will not extend if $\frac{t}{8} \leq F < \frac{t}{4}$. Both the manufacturer and the social planner in a decentralized channel, however, will choose to extend the product line for $\frac{t}{8} \leq F < \frac{t}{4}$. That is, the decentralization of the channel will restore the social planner's choice when $\frac{V}{\sqrt{2}} < t \leq V$ if $\frac{t}{8} \leq F < \frac{t}{4}$. Since $\frac{t}{4} < V - \frac{t}{8} - \frac{7V^2}{16t}$ for $\frac{V}{\sqrt{2}} < t \leq V$, we have the conclusion that the decentralization of the channel will restore the social planner's choice when $\frac{V}{2} < t \leq V$ if $\frac{t}{8} \leq F < \min\{V - \frac{t}{8} - \frac{7V^2}{16t}, \frac{t}{4}\}$.

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The Length of Product Line in Distribution Channels

Online Technical Appendix

1. HORIZONTAL PRODUCT LINE EXTENSION WHEN $t > V$

1.1. Product Line Extension in A Centralized Channel

When the manufacturer has only one product, the optimal retail price and the profit for the manufacturer in the centralized channel are given by $p_{ch1} = \frac{V}{2}$ and $\Pi_{ch1} = \frac{V^2}{2t}$. When the consumer mismatch cost is high ($t > V$), the market is not fully covered and consumers located beyond $x = \frac{1}{2} + \frac{V}{2t}$ and $x = \frac{1}{2} - \frac{V}{2t}$ are left out of the market. When the manufacturer extends the product line by introducing the second product with a fixed cost of F , the optimal retail price and the channel profit are as follows:

$$p_{ch2} = \begin{cases} \frac{V}{2}, & \text{if } t > 2V; \\ V - \frac{t}{4}, & \text{if } V < t \leq 2V; \end{cases} \quad \Pi_{ch2} = \begin{cases} \frac{V^2}{t} - F, & \text{if } t > 2V; \\ V - \frac{t}{4} - F, & \text{if } V < t \leq 2V. \end{cases} \quad (\text{TA1})$$

The manufacturer will profitably extend the product line if the fixed cost is sufficiently low as follows:

$$\begin{cases} F < V - \frac{t}{4} - \frac{V^2}{2t}, & \text{if } V < t \leq 2V; \\ F < \frac{V^2}{2t}, & \text{if } t > 2V. \end{cases} \quad (\text{TA2})$$

Next, we calculate consumer welfare by adding up individual consumer surplus. When there is only one product in the market, total consumer surplus is given by $CS_{ch1} = \frac{V^2}{4t}$. After product line

extension, total consumer surplus is given by:

$$CS_{ch2} = \begin{cases} \frac{V^2}{2t}, & \text{if } t > 2V; \\ \frac{t}{8}, & \text{if } V < t \leq 2V. \end{cases} \quad (\text{TA3})$$

Social welfare before product line extension is given by $SW_{ch1} = \frac{3V^2}{4t}$. After product line extension, social welfare is given by:

$$SW_{ch2} = \begin{cases} \frac{3V^2}{2t} - F, & \text{if } t > 2V; \\ V - \frac{t}{8} - F, & \text{if } V < t \leq 2V. \end{cases} \quad (\text{TA4})$$

Therefore, product line extension is beneficial from society's point of view when

$$\begin{cases} F < V - \frac{t}{8} - \frac{3V^2}{4t}, & \text{if } V < t \leq 2V; \\ F < \frac{3V^2}{4t}, & \text{if } t > 2V. \end{cases} \quad (\text{TA5})$$

We compare the optimal product line extension decisions from the manufacturer's perspective and from a social planner's perspective. When $V < t \leq \sqrt{2}V$ if $V - \frac{t}{8} - \frac{3V^2}{4t} \leq F < V - \frac{t}{4} - \frac{V^2}{2t}$, the social planner does not but the manufacturer does extend the product line. When $\sqrt{2}V < t \leq 2V$ if $V - \frac{t}{4} - \frac{V^2}{2t} \leq F < V - \frac{t}{8} - \frac{3V^2}{4t}$ or when $t > 2V$ if $\frac{V^2}{2t} \leq F < \frac{3V^2}{4t}$, the social planner does but the manufacturer does not extend the product line.

1.2. Product Line Extension in A Decentralized Channel

When there is only one product for sale, the equilibrium prices and profits for the manufacturer and the retailer are

$$w_{dh1} = \frac{V}{2}, p_{dh1} = \frac{3V}{4}, \Pi_{dh1} = \frac{V^2}{4t}, \pi_{dh1} = \frac{V^2}{8t}. \quad (\text{TA6})$$

When the manufacturer extends the product line by adding another product with a fixed cost

of F , the equilibrium prices and profits for the manufacturer and the retailer are

$$w_{dh2} = \frac{V}{2}, p_{dh2} = \frac{3V}{4}, \Pi_{dh2} = \frac{V^2}{2t} - F, \pi_{dh2} = \frac{V^2}{4t}. \quad (\text{TA7})$$

Product line extension is profitable for the manufacturer if $F < \frac{V^2}{4t}$. Next, we compare the optimal product line extension decisions in a centralized channel and in a decentralized channel, and find that the manufacturer extends the product line while a social planner will not in a centralized channel but not in a decentralized channel when $V < t \leq 2V$ if $\frac{V^2}{4t} \leq F < V - \frac{t}{4} - \frac{V^2}{2t}$ or when $t > 2V$ if $\frac{V^2}{4t} \leq F < \frac{V^2}{2t}$.

Next, we study the effect of product line extension on consumer surplus and social welfare in a decentralized channel. When there is only one product in the market, total consumer surplus (CS_{dh1}) is given by $CS_{dh1} = \frac{V^2}{16t}$. After product line extension, total consumer surplus (CS_{dh2}) is given by $CS_{dh2} = \frac{V^2}{8t}$. Social welfare before product line extension is given by $SW_{dh1} = \frac{7V^2}{16t}$. After product line extension, total social welfare is given by $SW_{dh2} = \frac{7V^2}{8t} - F$. Therefore, a social planner will extend the product line if $F < \frac{7V^2}{16t}$. We compare the product line extension decision for a manufacturer with the decision for a social planner in a decentralized channel, and find that the manufacturer does not but the social planner does extend the product line when $t > V$ if $\frac{V^2}{4t} \leq F < \frac{7V^2}{16t}$.

We now compare the conditions of beneficial product line extension to a manufacturer and to a social planner in both a centralized channel and a decentralized channel, and summarize the main results in the following proposition.

PROPOSITION TA 1: *Channel decentralization can help restore a social planner's decision when $V < t \leq (4 - 2\sqrt{2})V$ if $V - \frac{t}{8} - \frac{3V^2}{4t} \leq F < \frac{V^2}{4t}$, when $\frac{8 - \sqrt{26}}{2}V < t \leq 2V$ if $\max\{\frac{7V^2}{16t}, V - \frac{t}{4} - \frac{V^2}{2t}\} \leq F < V - \frac{t}{8} - \frac{3V^2}{4t}$, or when $t > 2V$ if $\frac{V^2}{2t} \leq F < \frac{3V^2}{4t}$.*

2. PRODUCT LINE EXTENSIONS WITH n PRODUCTS ($n \geq 3$)

In the base model, the maximum length of the product line is two. In this section, we extend the base model and allow the manufacturer to introduce n products in the product line. When the manufacturer extends the product line from one product to $n \geq 3$ products with a fixed cost of F for each additional product, the optimal locations for all products are given by $(\frac{1}{2n}, \frac{3}{2n}, \dots, \frac{2n-1}{2n})$.¹ The derivation details are given in the Appendix, and we report the main results in the following proposition.

PROPOSITION TA 2: *The manufacturer does not extend the product line in a centralized channel but does extend the product line in a decentralized channel when $t \leq \frac{V}{2}$ if $\frac{t}{2n} \leq F < \frac{t}{n}$ and when $\frac{V}{2} < t \leq V$ if $\frac{t}{2n} \leq F < \frac{1}{n-1} \left(V - \frac{t}{n} - \frac{V^2}{4t} \right)$. Channel decentralization restores a social planner's decision on product line extension when $\frac{V}{2} < t \leq V$ if $\frac{t}{4n} \leq F < \min\left\{ \frac{1}{n-1} \left(V - \frac{t}{4n} - \frac{7V^2}{16t} \right), \frac{t}{2n} \right\}$.*

Proof: The centralized case with only one product is already solved. When the manufacturer extends the product line from one product to $n \geq 3$, the optimal retail price and total channel profit are given by $p_{chn} = V - \frac{t}{2n}$ and $\Pi_{chn} = V - \frac{t}{2n} - (n-1)F$. The manufacturer will profitably extend the product line if the fixed cost is sufficiently low $F < \frac{t}{2n}$. After product line extension, total consumer surplus and social welfare are given by $CS_{chn} = \frac{t}{4n}$ and $SW_{chn} = V - \frac{t}{4n} - (n-1)F$. Therefore, product line extension is beneficial from the society's point of view when $F < \frac{t}{4n}$.

The decentralized case with only one product is also solved in the context. When the manufacturer extends the product line from one product to $n \geq 3$ products with a fixed cost of F for each additional product, the optimal locations for both products are given by $(\frac{1}{2n}, \frac{3}{2n}, \dots, \frac{2n-1}{2n})$. The optimal prices and total channel profit are given by $w_{dhn} = V - \frac{t}{n}$, $p_{dhn} = V - \frac{t}{2n}$, $\Pi_{dhn} = V - \frac{t}{n} - (n-1)F$, and $\pi_{dhn} = \frac{t}{2n}$. The manufacturer will profitably extend the product line if the

¹Here we assume the manufacturer makes the extension decision once, i.e., to extend the product line from one product to n products directly. An alternative way is to assume that the manufacturer extends the product line by adding one product each time. That is, there are $n-1$ times of extension decisions. The results will not change qualitatively. We analyze the current setting since it makes the analysis more tractable.

fixed cost is sufficiently low: when $F < \frac{t}{n}$ if $t \leq \frac{V}{2}$ or when $F < \frac{1}{n-1} \left(V - \frac{t}{n} - \frac{V^2}{4t} \right)$ if $\frac{V}{2} < t \leq V$.

We compare the optimal product line extension decisions in the centralized channel and in the decentralized channel, and find that the manufacturer will extend the product line in a decentralized channel but not in a centralized channel when $t \leq \frac{V}{2}$ if $\frac{t}{2n} \leq F < \frac{t}{n}$, when $\frac{V}{2} < t \leq V$ if $\frac{t}{2n} \leq F < \frac{1}{n-1} \left(V - \frac{t}{n} - \frac{V^2}{4t} \right)$.

After product line extension, total consumer surplus and social welfare are given by $CS_{dhn} = \frac{t}{4n}$ and $SW_{dhn} = V - \frac{t}{4n} - (n-1)F$. Therefore, product line extension is beneficial from the society's point of view when $F < \frac{t}{4n}$ if $t \leq \frac{V}{2}$ or when $F < \frac{1}{n-1} \left(V - \frac{t}{4n} - \frac{7V^2}{16t} \right)$ if $\frac{V}{2} < t \leq V$.

By comparing the conditions of beneficial product line extension to a manufacturer and to a social planner in both a centralized channel and a decentralized channel, we can show that decentralization could restore a social planner's product line extension decision when $\frac{V}{2} < t \leq V$ if $\frac{t}{4n} \leq F < \min \left\{ \frac{1}{n-1} \left(V - \frac{t}{4n} - \frac{7V^2}{16t} \right), \frac{t}{2n} \right\}$.

Q.E.D.

Proposition TA 2 shows that the results in the base model also hold for a general case where n products can be introduced in the product line. Note that the parameter space depends on the length n of the product line. This is because the manufacturer can better segment the markets with more products when the product line is longer (*i.e.*, n increases). At the same time, the overall fixed cost of product line extension nF increases as well.

3. PROOF OF OPTIMAL PRODUCT LOCATIONS IN BOTH CENTRALIZED AND DECENTRALIZED CHANNELS WHEN CONSUMERS ARE HORIZONTALLY HETEROGENEOUS

We first prove that the optimal product locations in a decentralized channel are given by $\frac{1}{4}$ and $\frac{3}{4}$ when consumers are horizontally heterogeneous. Since the proof for a centralized channel is very similar to and simpler than the proof for a decentralized channel, it is omitted here for the sake of brevity.

We prove the optimal locations of $\frac{1}{4}$ and $\frac{3}{4}$ in a decentralized channel in two steps. First, when the unit mismatch cost t is large enough such that both products will each act as a local monopoly product, the number of consumers covered by product 1 located at $\frac{1}{4}$ is given by $\bar{x}_{lr} - \bar{x}_{ll} = \frac{2(V-p_1)}{t}$ where $\bar{x}_{ll} = \frac{4p_1+t-4V}{4t}$ and $\bar{x}_{lr} = \frac{4V-4p_1+t}{4t}$ are the locations of marginal consumers. The optimal retail price p_1 for the retailer is solved from maximizing the retailer's profit from product 1 given the wholesale price w_1 , and the price is given by $p_1 = \frac{1}{2}(V+w_1)$. Maximizing its profit, the manufacturer will choose $w_1^* = \frac{V}{2}$. The markets of both products will be separate for any $\bar{x}_{lr} = \frac{V}{2t} < \frac{1}{2}$, *i.e.*, $t > V$. Note that there will be multiple optimal location sets for any $t > V$ and $(\frac{1}{4}, \frac{3}{4})$ is only one of the many optimal location sets.

Now let's consider the optimal locations when the unit mismatch cost t is small enough such that the whole market is fully covered in equilibrium. Assume the locations of both products are given by l_1 and l_2 respectively. We first prove that given l_1 , the optimal location of product 2 is given by $\frac{1+2l_1}{2}$. In order to prove such a result, we need to consider four cases: $(l_1 \leq \frac{1}{4}, l_2 \geq \frac{1+2l_1}{2})$, $(l_1 \leq \frac{1}{4}, l_2 \leq \frac{1+2l_1}{2})$, $(\frac{1}{4} \leq l_1 \leq \frac{1}{2}, l_2 \geq \frac{1+2l_1}{2})$, and $(\frac{1}{4} \leq l_1 \leq \frac{1}{2}, 2l_1 \leq l_2 \leq \frac{1+2l_1}{2})$.² We will provide the proof for the first case and the proofs of the other three cases are easy to derive by following the similar logic. We first prove the following lemma in order to solve the optimal locations.

LEMMA TA 1: *Given $l_1 \leq \frac{1}{4}$ and $l_2 \geq \frac{1+2l_1}{2}$, the optimal product location for product 2 is given by*

²The four cases are overlapping at $(l_1, l_2) = (\frac{1}{4}, \frac{1+2l_1}{2})$ since this is the optimal solution as shown below and we would like to make sure this candidate location set is considered in each of the four cases.

$l_2^* = \frac{1+2l_1}{2}$ when the market is fully covered due to small t .

Proof: Given any $l_1 \leq \frac{1}{4}$, the monopoly zone of product 1 is given by $\bar{x}_{1r} - \bar{x}_{1l} = (\frac{V-p_1}{t} + l_1) - (l_1 - \frac{V-p_1}{t})$. Since the market is fully covered, we must have $\bar{x}_{1l} = 0$ and \bar{x}_{1r} is thus given by $\bar{x}_{1r} = 2l_1$. Therefore, for any $l_2 \geq \frac{1+2l_1}{2}$, the optimal location l_2^* is given by the middle point of the uncovered market, *i.e.*, $l_2^* = 2l_1 + \frac{1-2l_1}{2} = \frac{1+2l_1}{2}$. The marginal consumers of product 2 are given by $\bar{x}_{2l} = l_2 - \frac{V-p_2}{t}$ and $\bar{x}_{2r} = \frac{V-p_2}{t} + l_2$.

Given $l_1 \leq \frac{1}{4}$ and $l_2^* = \frac{1+2l_1}{2}$, the retailer will choose retail prices p_1 and p_2 to maximize its profit $\pi = (p_1 - w_1)(\bar{x}_{1r} - \bar{x}_{1l}) + (p_2 - w_2)(\bar{x}_{2r} - \bar{x}_{2l})$ and the optimal retail prices are given by

$$p_1 = \frac{V+w_1}{2}, \quad p_2 = \frac{V+w_2}{2}. \quad (\text{TA8})$$

Expecting the optimal retail prices, the manufacturer will choose wholesale prices w_1 and w_2 to maximize its profit $\Pi = w_1(\bar{x}_{1r} - \bar{x}_{1l}) + w_2(\bar{x}_{2r} - \bar{x}_{2l}) = -8tl_1^2 + 4tl_1 + V - t$. Since Π is concave in l_1 , the first-order condition leads to the optimal location l_1^* and it is easy to derive the optimal l_2^* and wholesale prices:

$$l_1^* = \frac{1}{4}, \quad l_2^* = \frac{3}{4}, \quad w_1^* = w_2^* = V - \frac{t}{2}. \quad (\text{TA9})$$

The proofs for the other three cases of $(l_1 \leq \frac{1}{4}, l_2 \geq \frac{1+2l_1}{2})$, $(\frac{1}{4} \leq l_1 < \frac{1}{2}, 2l_1 < l_2 \leq \frac{1+2l_1}{2})$, and $(\frac{1}{4} \leq l_1 < \frac{1}{2}, l_2 \geq \frac{1+2l_1}{2})$ follow the similar logic.

Q.E.D.

4. PRODUCT LINE EXTENSION WITH VERTICAL CONSUMER HETEROGENEITY

In this section, we study the case of vertical product line extension.³ Consumers are vertically and uniformly located on a Hotelling line, and consumers' locations are captured by $\theta \in [0, 1]$. The consumer surplus is given by $U = \theta q - p$ where q is product quality and p is the retail price of the product. The manufacturer's cost of producing a product with quality q is τq^2 . This is a standard setup of vertical consumer heterogeneity, in line with extant marketing literature (Moorthy 1984, 1988; Desai 2001).⁴ We again start with analyzing the effects of product line extension with a fixed cost of F in a centralized channel, followed by the analysis in a decentralized channel.⁵ For the sake of conciseness, we omit the details and report the main results in the following proposition.

PROPOSITION TA 3: *When consumers are vertically heterogeneous, a manufacturer does (not) extend its product line in a centralized (decentralized) channel when $\frac{1}{675\tau} < F < \frac{2}{675\tau}$. Channel decentralization restores a social planner's decision when $\frac{2}{675\tau} < F < \frac{1}{225\tau}$.*

Proof: In the centralized channel with vertically heterogeneous consumers, the manufacturer will extend the product line for $F < \frac{2}{675\tau}$. In the decentralized channel, the manufacturer will extend the product line for $F < \frac{1}{675\tau} < \frac{2}{675\tau}$. Since $\frac{1}{675\tau} < \frac{2}{675\tau}$, the manufacturer is less likely to extend the product line in a decentralized channel than in a centralized channel. In a centralized channel, the social planner would like to extend the product line if $F < \frac{1}{225\tau}$ as shown in the paper. The manufacturer, however, would like to extend the product line only if $F < \frac{2}{675\tau}$. It is then very easy to get the results as shown.

In a decentralized channel, the social planner will not extend the product line if $F \geq \frac{7}{2700\tau}$ as shown in the paper. The manufacturer will not extend the product line if $F \geq \frac{1}{675\tau}$. Therefore,

³The analysis of the case where consumers are both horizontally and vertically differentiated is provided in the next section.

⁴An alternative setup of consumer surplus is $U = V + \theta q - p$ where V is the base value for the product with quality equal to zero. This setup may lead to the uninteresting result of $p^* = V$ and $q^* = 0$ when V is sufficiently large.

⁵To be consistent with the case of horizontal consumer heterogeneity, the cost of product line extension F is assumed to be fixed so as to capture the repositioning cost and other fixed costs such as R&D. The analysis of the case where F is a function of product quality q , however, may generate additional insights and we leave this for future research.

both the manufacturer and the social planner will not extend the product line for $F \geq \frac{7}{2700\tau}$. In a centralized channel, however, the manufacturer will not extend the product line while a social planner will extend for $\frac{2}{675\tau} < F < \frac{1}{225\tau}$. Therefore, the channel decentralization will help restore the social planner's decision for $\frac{2}{675\tau} < F < \frac{1}{225\tau}$.

Q.E.D.

Proposition TA 3 shows that the manufacturer will be less likely to extend the product line in a centralized channel but will not extend in a decentralized channel when consumers are vertically heterogeneous. The intuition here is similar to the case where consumers are horizontally distributed and the market is not fully covered ($t > V$). When consumers are vertically heterogeneous, the market is never fully covered since some consumers do not have much value for the product due to the characteristics of the vertical market. Thus, the benefit of improving the manufacturer's monopoly power over the retailer is weak once the manufacturer extends the product line since the manufacturer's wholesale price is also bound by considerations of market coverage. Due to the double marginalization problem in the decentralized channel, the manufacturer will find it less profitable to extend the product line.

Consistent with the case when consumers are horizontally heterogeneous, channel decentralization by a manufacturer can also restore the product line extension by a social planner who maximizes social welfare. The intuition is again similar to the case of horizontal consumer heterogeneity with the market not fully covered. In the region $\frac{2}{675\tau} < F < \frac{1}{225\tau}$, when the channel is centralized, the social planner will extend the product line, but the manufacturer will not. When the channel is decentralized, both the manufacturer and the social planner will not extend the product line.

5. PRODUCT LINE EXTENSION WITH BOTH HORIZONTAL AND VERTICAL CONSUMER HETEROGENEITY

In this section, we study the optimal product line extensions when consumers are both horizontally and vertically heterogeneous (Desai 2001). Consumers are located on the surface of a square with a size of 1×1 . The consumer located at (x, θ) incurs a disutility of $t|x - x_0|$ for purchasing a product located $|x - x_0|$ away horizontally from the consumer, and the consumer's evaluation of a product with quality q is equal to θq . That is, consumer (x, θ) 's surplus from buying a product located at x_0 is given by $U(x, \theta, q, p) = \theta q - t|x - x_0| - p$. Note that this model reverts back to the vertical model for $t = 0$ and to the horizontal model for $\theta q = V$.⁶ The marginal cost for the manufacturer to produce a product of quality q is assumed to be equal to τq^2 .⁷

5.1. A Centralized Channel With One Product

When the manufacturer has only one product, the optimal horizontal location of the product is $x = \frac{1}{2}$ since this location will generate the lowest mismatching costs for consumers on average. For any quality level q_{cb1} , the demand is given by:

$$D_{cb1} = \int_0^{\frac{1}{2}} \left[1 - \frac{t(\frac{1}{2} - x) + p_{cb1}}{q_{cb1}} \right] dx + \int_{\frac{1}{2}}^1 \left[1 - \frac{t(x - \frac{1}{2}) + p_{cb1}}{q_{cb1}} \right] dx = 1 - \frac{4p_{cb1} + t}{4q_{cb1}}. \quad (\text{TA10})$$

The manufacturer's profit Π_{cb1} is given by

$$\Pi_{cb1} = (p_{cb1} - \tau q_{cb1}^2) D_{cb1} = (p_{cb1} - \tau q_{cb1}^2) \left(1 - \frac{4p_{cb1} + t}{4q_{cb1}} \right). \quad (\text{TA11})$$

⁶Strictly speaking, the horizontal model is not a special case of the general model since q is a decision variable in the general model and consumers are heterogeneous on q . However, the model can still revert back to the horizontal model if consumers become homogeneous with product quality and product quality q becomes exogenous ($\theta q = V$) with $\tau = 0$. Since the current model is the closest to the models in literature and generates cleaner results, we assume such a consumer utility function as used in the paper.

⁷We focus on the cases where the high end consumers in the market are fully covered. That is, the consumers with $\theta = 1$ will get non-negative utility from consuming a product. This is satisfied for any $0 < t\tau \leq \frac{3}{8}$. For simplicity, we define the market as fully covered if high end consumers are fully covered and as partially covered (or not fully covered) otherwise.

The manufacturer chooses price p_{cb1} and quality q_{cb1} to maximize its profit, and the optimal quality and price are given by

$$q_{cb1} = \frac{1+\sqrt{1+3t\tau}}{6\tau}, p_{cb1} = \frac{4-3t\tau+4\sqrt{1+3t\tau}}{36\tau}, \Pi_{cb1} = \frac{(1-3t\tau+\sqrt{1+3t\tau})^2}{54\tau(1+\sqrt{1+3t\tau})}. \quad (\text{TA12})$$

5.2. A Centralized Channel With Two Products

Now, we study the case where the manufacturer offers two differentiated products in a centralized channel. It is assumed that the manufacturer decides on optimal locations followed by decisions on price and quality. We first solve the case where both products can be both horizontally and vertically differentiated, and show that in such a case it is always optimal for the manufacturer to horizontally differentiate products only. We call such a case “*Horizontally Differentiated Product*” below. Next, we solve the case where both products are restricted to be vertically differentiated only.⁸ We compare the manufacturer’s profits in both cases to derive the optimal strategy for the manufacturer.

Horizontally Differentiated Products

The manufacturer makes decisions on the following variables of product i ($i = 1, 2$): location l_{cbb2i} , price p_{cbb2i} , and quality q_{cbb2i} . We calculate the optimal quality levels and optimal locations, and have the following results.^{9 10}

LEMMA TA 2: *In a centralized channel where both products can be both horizontally and vertically differentiated, the optimal prices, qualities, and locations of both products are given below.*

⁸We need to solve the vertical differentiation case separately since the decisions on locations are made before decisions on price and quality.

⁹We use subscripts $\{\cdot\}_{iblk}$ to refer to the measure $\{\cdot\}$ in an i ($i \in \{c : \text{centralized}; d : \text{decentralized}\}$) channel with *both* vertically and horizontally distributed consumers for k ($k \in \{1, 2\}$) products when products (if $k = 2$) are l ($l \in \{h : \text{horizontally}; v : \text{vertically}\}$) differentiated. For instance, the variable q_{cbh2} refers to the quality of the product in a *centralized* channel with *both* horizontally and vertically distributed consumers and with *horizontally* differentiated products offered by the manufacturer.

¹⁰In Lemma TA 2 below, we list the results for both partially covered market and fully covered market. In the analysis afterwards, we focus on the case where the market is fully covered by both products.

For $t\tau > \frac{16}{25}$, the market is partially covered by both products, and there are multiple optimal locations, including locations $(\frac{1}{4}, \frac{3}{4})$. We have:

$$l_{cbh21} = \frac{1}{4}, l_{cbh22} = \frac{3}{4}, q_{cbh21} = q_{cbh22} = \frac{2}{5\tau}, \quad (\text{TA13})$$

$$p_{cbh21} = p_{cbh22} = \frac{6}{25\tau}, \Pi_{cbh2} = \frac{32}{3125t\tau^2} - F.$$

For $t\tau \leq \frac{16}{25}$, the market is fully covered by both products, and the unique optimal locations are given by $(\frac{1}{4}, \frac{3}{4})$. We have:

$$l_{cbh21} = \frac{1}{4}, l_{cbh22} = \frac{3}{4}, q_{cbh21} = q_{cbh22} = \frac{2+\sqrt{4+6t\tau}}{12\tau}, \quad (\text{TA14})$$

$$p_{cbh21} = p_{cbh22} = \frac{8-3t\tau+4\sqrt{4+6t\tau}}{72\tau}, \Pi_{cbh2} = \frac{(2-3t\tau+\sqrt{4+6t\tau})^2}{108\tau(2+\sqrt{4+6t\tau})} - F.$$

Proof: We prove the lemma in two steps. First, we solve for the case where the market is not fully covered by both products. Second, we solve the case where the market is fully covered by both products.

First, when the market is not fully covered by both products, we assume the consumers located at $(0, 1)$ and $(1, 1)$ are covered by some product and get zero utility. That is, these two consumers are marginal consumers. It is assumed in a way to simplify the analysis while we will get same solutions by assuming different marginal consumers. Given both products' locations (l_{cbh21}, l_{cbh22}) and marginal consumers $(0, 1)$ and $(1, 1)$, the demand for both products are respectively given by,

$$\begin{cases} D_{cbh21} = \int_0^{l_{cbh21}} (1 - \theta_{L1}) dx + \int_{l_{cbh21}}^1 (1 - \theta_{R1}) dx = \frac{(q_{cbh21} - p_{cbh21})^2}{tq_{cbh21}} \\ D_{cbh22} = \int_{2l_{cbh22}-1}^{l_{cbh22}} (1 - \theta_{L2}) dx + \int_{l_{cbh22}}^1 (1 - \theta_{R2}) dx = \frac{(q_{cbh22} - p_{cbh22})^2}{tq_{cbh22}} \end{cases} \quad (\text{TA15})$$

where θ_{ki} represents the function of locations of the marginal consumers who are located on the k side of product i ($k = L, R$ and $i = 1, 2$), and $l_{cbh21} = \frac{q_{cbh21} - p_{cbh21}}{t}$, $\theta_{L1} = \frac{p_{cbh21} + t(l_{cbh21} - x)}{q_{cbh21}}$, $\theta_{R1} = \frac{p_{cbh21} + t(x - l_{cbh21})}{q_{cbh21}}$, $l_{cbh22} = \frac{p_{cbh22} - q_{cbh22} + t}{t}$, $\theta_{L2} = \frac{p_{cbh22} + t(l_{cbh22} - x)}{q_{cbh22}}$, and $\theta_{R2} = \frac{p_{cbh22} + t(x - l_{cbh22})}{q_{cbh22}}$.

The manufacturer's profit is given by

$$\Pi_{cbh2} = (p_{cbh21} - \tau q_{cbh21}^2)D_{cbh21} + (p_{cbh22} - \tau q_{cbh22}^2)D_{cbh22} - F \quad (\text{TA16})$$

Solving for the manufacturer's optimization problem gives the following results:

$$l_{cbh21} = \frac{4}{25t\tau}, l_{cbh22} = 1 - \frac{4}{25t\tau}, q_{cbh21} = q_{cbh22} = \frac{2}{5\tau}, \quad (\text{TA17})$$

$$p_{cbh21} = p_{cbh22} = \frac{6}{25\tau}, \Pi_{cbh2} = \frac{32}{3125t\tau^2} - F.$$

The constraint that the market is not fully covered by both products is satisfied for any $t\tau > \frac{16}{25}$. In the analysis above, we assume consumers $(0, 1)$ and $(1, 1)$ are marginal consumers to make it easy to solve the model. It is easy to show that any locations of $(\frac{4}{25t\tau} \leq l_{cbh21} \leq 1 - 3\frac{4}{25t\tau}, 3\frac{4}{25t\tau} \leq l_{cbh22} \leq 1 - \frac{4}{25t\tau})$, including $(l_{cbh21} = \frac{1}{4}, l_{cbh22} = \frac{3}{4})$, are optimal.

Next, we consider the case where the market is fully covered by both products. The analysis is considerably more complicated than the first case and we will proceed in two steps. In the first step, we solve the model where the manufacturer chooses same prices and qualities for both products. In the second step, we show that the solutions from the first model are also optimal for the case where the manufacturer can choose different prices and qualities for both products. We now proceed to the first step where the prices and qualities are assumed to be same for both products.

Assume the locations of both products are given by (l_{cbh21}, l_{cbh22}) respectively, and the quality of the second product is no lower than the first product, *i.e.*, $q_{cbh22} \geq q_{cbh21}$. The locations of

marginal consumers who get zero utility from consuming a product are given by

$$\theta_{L1} = \frac{p_{cbh21} + t(l_{cbh21} - x)}{q_{cbh21}}, \theta_{R1} = \frac{p_{cbh21} + t(x - l_{cbh21})}{q_{cbh21}}, \theta_{L2} = \frac{p_{cbh22} + t(l_{cbh22} - x)}{q_{cbh22}}, \theta_{R2} = \frac{p_{cbh22} + t(x - l_{cbh22})}{q_{cbh22}}. \quad (\text{TA18})$$

The locations of marginal consumers between the two products are given by function $\theta_0(x)$,

$$\theta_0(x) = \frac{p_{cbh22} - p_{cbh21} + t(l_{cbh21} + l_{cbh22}) - 2tx}{q_{cbh22} - q_{cbh21}}. \quad (\text{TA19})$$

The marginal consumer who gets zero utility from consuming either product l_3 and the marginal consumer who has a $\theta = 1$ are respectively given by

$$\begin{cases} l_3 = \frac{q_{cbh21}p_{cbh22} - q_{cbh22}p_{cbh21} + t(q_{cbh21}l_{cbh22} + q_{cbh22}l_{cbh21})}{t(q_{cbh21} + q_{cbh22})} \\ l_4 = \frac{p_{cbh22} - p_{cbh21} + t(l_{cbh21} + l_{cbh22}) - (q_{cbh22} - q_{cbh21})}{2t} \end{cases}. \quad (\text{TA20})$$

The demand for both products are respectively given by,

$$\begin{cases} D_{cbh21} = \int_0^{l_{cbh21}} (1 - \theta_{L1}) dx + \int_{l_{cbh21}}^{l_4} (1 - \theta_{R1}) dx + \int_{l_4}^{l_3} (\theta_0 - \theta_{R1}) dx \\ D_{cbh22} = \int_{l_4}^{l_3} (1 - \theta_0) dx + \int_{l_3}^{l_{cbh22}} (1 - \theta_{L2}) dx + \int_{l_{cbh22}}^1 (1 - \theta_{R2}) dx \end{cases}. \quad (\text{TA21})$$

Note that $q_{cbh22} \geq q_{cbh21}$ ensures that $\theta_0(x)$ is a non-increasing function of x , *i.e.*, $l_4 \leq l_3$. The manufacturer's profit is given by

$$\Pi_{cbh2} = (p_{cbh21} - \tau q_{cbh21}^2) D_{cbh21} + (p_{cbh22} - \tau q_{cbh22}^2) D_{cbh22} - F. \quad (\text{TA22})$$

Assuming the manufacturer chooses same prices and qualities for both products, we have the following results:

$$l_{cbh21} = \frac{1}{4}, l_{cbh22} = \frac{3}{4}, q_{cbh21} = q_{cbh22} = \frac{2+\sqrt{4+6t\tau}}{12\tau}, \quad (\text{TA23})$$

$$p_{cbh21} = p_{cbh22} = \frac{8-3t\tau+4\sqrt{4+6t\tau}}{72\tau}, \quad \Pi_{cbh2} = \frac{(2-3t\tau+\sqrt{4+6t\tau})^2}{108\tau(2+\sqrt{4+6t\tau})} - F.$$

In order to fully cover the market, the following condition must be satisfied: $t\tau \leq \frac{16}{25}$.

Last, it is easy to show that the solutions above will satisfy both the first-order conditions and second-order conditions of the optimization problem when the manufacturer can choose different prices and qualities for both products.¹¹ Thus, the optimal solutions for the case where the market is fully covered by both products in the centralized channel are given by equation (TA23).

Q.E.D.

The manufacturer will be willing to extend the product line if the fixed cost is sufficiently low:

$$F < \frac{(2-3t\tau+\sqrt{4+6t\tau})^2}{108\tau(2+\sqrt{4+6t\tau})} - \frac{(1-3t\tau+\sqrt{1+3t\tau})^2}{54\tau(1+\sqrt{1+3t\tau})}. \quad (\text{TA24})$$

Interestingly, the product quality is lower after the product line extension. This is summarized in the following proposition:

PROPOSITION TA 4: *Product quality will be lower after the manufacturer extends the product line horizontally in a centralized channel where consumers are both horizontally and vertically differentiated.*

Proof: The result is easy to show given $\frac{1+\sqrt{1+3t\tau}}{6\tau} > \frac{2+\sqrt{4+6t\tau}}{12\tau}$ for any $t > 0$ and $\tau > 0$.

¹¹Due to the complexity of the optimization problem, we cannot get the solutions directly by solving for the full model. Here we show that the solutions given above satisfy both the first-order conditions and second-order conditions of the optimization problem.

Q.E.D.

The intuition behind Proposition TA 4 is as follows. Before the product line is extended, consumers on average incur a high mismatch cost, and the manufacturer has to increase the product quality to make the product attractive to consumers. After product line extension, the products will fit consumers' tastes better, and the manufacturer is therefore less concerned with lower market coverage by a product. The manufacturer thus has less an incentive to maintain a high quality.

Vertically Differentiated Products

Now we study the case where the manufacturer offers two vertically differentiated products that are both located at l_{cbv2} . The manufacturer will choose the qualities, prices, and locations of both products to maximize its total profit. We calculate the optimal quality levels and have the following results.

$$l_{cbv21} = \frac{1}{2}, q_{cbv21} = \frac{1+\sqrt{1+15t\tau}}{10\tau}, q_{cbv22} = \frac{11+\sqrt{1+15t\tau}}{30\tau}, p_{cbv21} = \frac{3+3\sqrt{1+15t\tau}}{50\tau} - \frac{t}{20}, \quad (\text{TA25})$$

$$p_{cbv22} = \frac{113+13\sqrt{1+15t\tau}}{450\tau} - \frac{7t}{60}, \Pi_{cbv2} = \frac{27-190t\tau+675t^2\tau^2+(27-80t\tau-75t^2\tau^2)\sqrt{1+15t\tau}}{675\tau(1-5t\tau+\sqrt{1+15t\tau})} - F.$$

The reason for the manufacturer to locate both products at $l_{cbv2} = \frac{1}{2}$ is as follows. Suppose both products are instead located at $\tilde{l} < \frac{1}{2}$. There are three cases to consider: 1). the market is not fully covered by both products and consumer $(0, 1)$ is not served; 2). the market is not fully covered by both products and consumer $(0, 1)$ is served; and 3). the market is fully covered by both products. In case 1), $l_{cbv2} = \frac{1}{2}$ is one of multiple optimal solutions, since the manufacturer can locate its products at $x = \frac{1}{2}$ without hurting its demand because of the partial coverage of the market. In case 2), given the prices and qualities of both products, the manufacturer can always increase its demand by moving the location of the products towards $x = \frac{1}{2}$ until consumer $(1, 1)$ is served by some product, which leads to our case 3). In case 3), since $\tilde{l} < \frac{1}{2}$, the number of marginal consumers

to the right side of the products, *i.e.*, $x > \tilde{l}$, is larger than the number of marginal consumers to the left side of the products, *i.e.*, $x < \tilde{l}$. By moving product location to right by ϵ , the manufacturer will be able to increase its demand, which leads to a higher profit. The manufacturer can continue to do so until the number of marginal consumers to the right side of the products is equal to the number to the left, *i.e.*, until when $l_{cbv2} = \frac{1}{2}$. Thus, $l_{cbv2} = \frac{1}{2}$ is always an optimal location for both products.

Given the optimal solutions as shown in equation (TA25), the manufacturer will be willing to extend the product line if the fixed cost is sufficiently low:

$$F < \frac{27-190t\tau+675t^2\tau^2+(27-80t\tau-75t^2\tau^2)\sqrt{1+15t\tau}}{675\tau(1-5t\tau+\sqrt{1+15t\tau})} - \frac{(1-3t\tau+\sqrt{1+3t\tau})^2}{54\tau(1+\sqrt{1+3t\tau})}. \quad (\text{TA26})$$

Horizontal vs. Vertical Product Line Extensions

When deciding to extend the product line horizontally or vertically, the manufacturer compares the profits between the two scenarios. The results are given below:

$$\begin{cases} \Pi_{cbv2} \geq \Pi_{cbh2}, & \text{if } t\tau \leq 0.0559; \\ \Pi_{cbh2} > \Pi_{cbv2}, & \text{if } t\tau > 0.0559. \end{cases} \quad (\text{TA27})$$

That is, the manufacturer will horizontally extend the product line for a large $t\tau$ and vertically extend the product line otherwise if the fixed cost is low enough in a centralized channel when consumers are both horizontally and vertically heterogeneous. This is because when $t\tau$ is large, either the consumer mismatching disutility is large or the cost of providing higher product quality is large. In either case, the manufacturer will optimally choose horizontal product line extension to help reduce consumer mismatching cost or avoid the high cost of providing high quality products. When $t\tau$ is small, either consumer mismatching disutility is low or the cost of increasing the product quality is lower. In either case, the manufacturer will find vertical product line extension more profitable.

Now we study the effects of product line extension on consumer welfare and social welfare. When there is only one product in the market, total consumer surplus is given by $CS_{cb1} = \frac{4-45t\tau+(4+21t\tau)\sqrt{1+3t\tau}}{432\tau}$. If there are two horizontally differentiated products in the market after product line extension, total consumer surplus is given by $CS_{cbh2} = \frac{32-24t\tau+63t^2\tau^2+(16-24t\tau)\sqrt{4+6t\tau}}{864\tau(2+\sqrt{4+6t\tau})}$. If there are two vertically differentiated products in the market after product line extension, total consumer surplus is given by $CS_{cbv2} = \frac{208-795t\tau+(8+195t\tau)\sqrt{1+15t\tau}}{10800\tau}$.

Social welfare can be calculated by adding the firm's profit and total consumer surplus. When there is only one product in the market, social welfare is given by $SW_{cb1} = \frac{8-12t\tau+45t^2\tau^2+(8-24t\tau)\sqrt{1+3t\tau}}{144\tau(1+\sqrt{1+3t\tau})}$. When there are two horizontally differentiated products in the market, we have social welfare given by $SW_{cbh2} = \frac{32-24t\tau+45t^2\tau^2+(16-24t\tau)\sqrt{4+6t\tau}}{288\tau(2+\sqrt{4+6t\tau})} - F$. When there are two vertically differentiated products in the market, the social welfare is $SW_{cbv2} = \frac{216-1520t\tau+5900t^2\tau^2+(216-640t\tau-725t^2\tau^2)\sqrt{1+15t\tau}}{3600\tau(1-5t\tau+\sqrt{1+15t\tau})} - F$.

When deciding to extend the product line horizontally or vertically, the social planner will compare the social welfare of horizontally extended product line with that of vertically extended product line. We have $SW_{cbh2} > SW_{cbv2}$. The social planner will extend the product line horizontally when the fixed cost is sufficiently low:

$$F < \frac{32-24t\tau+45t^2\tau^2+(16-24t\tau)\sqrt{4+6t\tau}}{288\tau(2+\sqrt{4+6t\tau})} - \frac{8-12t\tau+45t^2\tau^2+(8-24t\tau)\sqrt{1+3t\tau}}{144\tau(1+\sqrt{1+3t\tau})}. \quad (\text{TA28})$$

The comparison between the line extension decisions by the social planner and by the manufacturer shows that the firm and the social planner may have conflicting incentives on extending the product line. If the extension cost F is small enough such that the firm would like to extend the product line, it will vertically extend the line when $t\tau$ is small ($t\tau \leq 0.0559$), but the social planner would like to horizontally extend the product line.

We compare the threshold values of F for the social planner to extend the product line with the threshold values of F for the manufacturer to be willing to extend the product line. We find

an interesting result: a manufacturer is more likely to extend the product line for $t\tau \leq 0.0404$ compared to a social planner. For a larger $t\tau$, the social planner is more willing to extend the product line than the firm.

PROPOSITION TA 5: *When consumers are both horizontally and vertically heterogeneous and products are horizontally differentiated, the manufacturer is more (less) willing to extend the product line than a social planner for a small (large) $t\tau$ in a centralized channel.*

Proof: Equation (TA24) shows the threshold value of F under which the manufacturer in a centralized channel with both horizontally and vertically heterogeneous consumers and horizontally differentiated products will be willing to extend the product line. Equation (TA28) shows the threshold value of F under which the social planner in the same channel will be willing to extend the product line. It is easy to derive the results of Proposition TA 5 through the comparison of the two threshold values.

Q.E.D.

5.3. A Decentralized Channel With One Product

When the manufacturer has only one product, the optimal horizontal location of the product is $x = \frac{1}{2}$ since such a location will minimize the mismatch disutility for consumers. For any quality level q_{db1} , the demand is given by:

$$D_{db1} = \int_0^{\frac{1}{2}} \left[1 - \frac{t(\frac{1}{2}-x)+p_{db1}}{q_{db1}}\right] dx + \int_{\frac{1}{2}}^1 \left[1 - \frac{t(x-\frac{1}{2})+p_{db1}}{q_{db1}}\right] dx = 1 - \frac{4p_{db1}+t}{4q_{db1}}. \quad (\text{TA29})$$

The optimal retail price p_{db1} to maximize the retailer's profit $\pi_{db1} = (p_{db1} - w_{db1})D_{db1}$ is given by $p_{db1} = \frac{4w+4q-t}{8}$. The manufacturer's profit is given by $\Pi_{db1} = (w_{db1} - \tau q_{db1}^2)D_{db1}$ and the optimal quality and wholesale price are as follows:

$$q_{db1} = \frac{1+\sqrt{1+3t\tau}}{6\tau}, \quad w_{db1} = \frac{4-3t\tau+4\sqrt{1+3t\tau}}{36\tau}, \quad p_{db1} = \frac{5-6t\tau+5\sqrt{1+3t\tau}}{36\tau}. \quad (\text{TA30})$$

Firms' profits are given by

$$\Pi_{db1} = \frac{(1-3t\tau+\sqrt{1+3t\tau})^2}{108\tau(1+\sqrt{1+3t\tau})}, \quad \pi_{db1} = \frac{(1-3t\tau+\sqrt{1+3t\tau})^2}{216\tau(1+\sqrt{1+3t\tau})}. \quad (\text{TA31})$$

5.4. A Decentralized Channel With Two Products

Next, we study the case where the manufacturer offers two differentiated products in a decentralized channel. It is assumed that the manufacturer decides on product locations followed by its decisions on wholesale price and product quality. Given the locations, wholesale prices, and qualities of both products, the retailer set retail prices. We first solve the case where both products can be both horizontally and vertically differentiated, and show that in such a case it is always optimal for the manufacturer to horizontally differentiate products only. Following the practice before, we call such a case “*Horizontally Differentiated Product*”. Next, we solve the case where both products are restricted to be vertically differentiated only. We compare the manufacturer’s profits in both cases to derive the optimal strategy for the manufacturer.

Horizontally Differentiated Products

The manufacturer makes decisions on the following variables of product i ($i = 1, 2$): location l_{dbb2i} , wholesale price w_{dbb2i} , and quality q_{dbb2i} . We calculate the optimal quality levels and optimal locations, and have the following results.

LEMMA TA 3: *In a decentralized channel where both products can be both horizontally and vertically differentiated and the market is fully covered, the optimal prices, qualities, and locations of both*

products are given below.

$$\begin{aligned}
l_{dbh21} &= \frac{1}{4}, \quad l_{dbh22} = \frac{3}{4}, \quad q_{dbh21} = q_{dbh22} = \frac{2+\sqrt{4+6t\tau}}{12\tau}, \quad w_{dbh21} = w_{dbh22} = \frac{8-3t\tau+4\sqrt{4+6t\tau}}{72\tau}, \\
p_{dbh21} = p_{dbh22} &= \frac{10-6t\tau+5\sqrt{4+6t\tau}}{72\tau}, \quad \Pi_{dbh2} = \frac{(2-3t\tau+\sqrt{4+6t\tau})^2}{216\tau(2+\sqrt{4+6t\tau})} - F, \quad \pi_{dbh2} = \frac{(2-3t\tau+\sqrt{4+6t\tau})^2}{432\tau(2+\sqrt{4+6t\tau})}.
\end{aligned} \tag{TA32}$$

Proof: We assume the manufacturer chooses same wholesale prices w_{dbh2} and qualities q_{dbh2} for both products and the retailer chooses same retail prices p_{dbh2} .¹² Denote the locations of both products as (l_{dbh21}, l_{dbh22}) respectively. The locations of marginal consumers who get zero utility from consuming a product are given by

$$\theta_{L1} = \frac{p_{dbh21}+t(l_{dbh21}-x)}{q_{dbh21}}, \quad \theta_{R1} = \frac{p_{dbh21}+t(x-l_{dbh21})}{q_{dbh21}}, \quad \theta_{L2} = \frac{p_{dbh22}+t(l_{dbh22}-x)}{q_{dbh22}}, \quad \theta_{R2} = \frac{p_{dbh22}+t(x-l_{dbh22})}{q_{dbh22}}. \tag{TA33}$$

Given equal wholesale prices, retail prices, and the quality levels of both products, the marginal consumers l_3 between both products are given by

$$l_3 = \frac{l_{dbh22}+l_{dbh21}}{2}. \tag{TA34}$$

The demand for both products are respectively given by,

$$\begin{cases} D_{dbh21} = \int_0^{l_{dbh21}} (1 - \theta_{L1}) dx + \int_{l_{dbh21}}^{l_3} (1 - \theta_{R1}) dx \\ D_{dbh22} = \int_{l_3}^{l_{dbh22}} (1 - \theta_{L2}) dx + \int_{l_{dbh22}}^1 (1 - \theta_{R2}) dx \end{cases}. \tag{TA35}$$

The manufacturer's and retailer's profits are given by

$$\begin{cases} \Pi_{dbh2} = (w_{dbh2} - \tau q_{dbh2}^2)(D_{dbh21} + D_{dbh22}) - F \\ \pi_{dbh2} = (p_{dbh2} - w_{dbh2})(D_{dbh21} + D_{dbh22}) \end{cases} \tag{TA36}$$

¹²The model without such constraints does not have any closed form solution.

The manufacturer chooses (l_{dbh21}, l_{dbh22}) , w_{dbh2} , and q_{dbh2} to maximize Π_{dbh2} , and the retailer sequentially chooses p_{dbh2} to maximize its profit π_{dbh2} . Solving the sequential game above will lead to the results given in equation (TA32). In order to fully cover the market, the following condition must be satisfied: $t\tau \leq \frac{3}{8}$.

Q.E.D.

Vertically Differentiated Products

When the two products are vertically differentiated in a decentralized channel, we have

$$q_{dbv21} = \frac{1+\sqrt{1+15t\tau}}{10\tau}, \quad q_{dbv22} = \frac{11+\sqrt{1+15t\tau}}{30\tau}, \quad w_{dbv21} = \frac{6-5t\tau+6\sqrt{1+15t\tau}}{100\tau}, \quad (TA37)$$

$$w_{dbv22} = \frac{226-105t\tau+26\sqrt{1+15t\tau}}{900\tau}, \quad p_{dbv21} = \frac{8-15t\tau+8\sqrt{1+15t\tau}}{100\tau}, \quad p_{dbv22} = \frac{278-165t\tau+28\sqrt{1+15t\tau}}{900\tau}.$$

Firms' profits are given by

$$\Pi_{dbv2} = \frac{27-190t\tau+675t^2\tau^2+(27-80t\tau-75t^2\tau^2)\sqrt{1+15t\tau}}{1350\tau(1-5t\tau+\sqrt{1+15t\tau})} - F, \quad (TA38)$$

$$\pi_{dbv2} = \frac{27-190t\tau+675t^2\tau^2+(27-80t\tau-75t^2\tau^2)\sqrt{1+15t\tau}}{2700\tau(1-5t\tau+\sqrt{1+15t\tau})}.$$

Horizontal vs. Vertical Product Line Extensions

When making decisions on product line extension, the manufacturer compares the profits between the two scenarios above. The results are listed below:

$$\begin{cases} \Pi_{dbv2} > \Pi_{dbh2}, & \text{if } t\tau \leq 0.0559; \\ \Pi_{dbh2} \geq \Pi_{dbv2}, & \text{if } t\tau > 0.0559. \end{cases} \quad (TA39)$$

Again, the manufacturer will horizontally extend the product line for a large $t\tau$ and vertically extend the product line otherwise. Interestingly, the manufacturer's decisions of extending the

product line are the same for both decentralized and centralized channels, *i.e.*, it will vertically extend the product line for any $t\tau \leq 0.0559$.

Social Planner: Horizontal and Vertical Product Line Extensions in a Decentralized Channel

Next, we study the effect of product line extension on consumer welfare and social welfare. When there is only one product in the market, total consumer surplus is given by $CS_{db1} = \frac{1-18t\tau+(1+12t\tau)\sqrt{1+3t\tau}}{432\tau}$. If there are two horizontally differentiated products in the market after product line extension, total consumer surplus is given by $CS_{dbh2} = \frac{4-3t\tau+18t^2\tau^2+(2-3t\tau)\sqrt{4+6t\tau}}{432\tau(2+\sqrt{4+6t\tau})}$. If there are two vertically differentiated products in the market after product line extension, total consumer surplus is given by $CS_{cbv2} = \frac{52-255t\tau+(2+105t\tau)\sqrt{1+15t\tau}}{10800\tau}$.

Social welfare can be calculated by adding the firm's profit and total consumer surplus. When there is only one product in the market, social welfare is given by $SW_{db1} = \frac{14-21t\tau+90t^2\tau^2+(14-42t\tau)\sqrt{1+3t\tau}}{432\tau(1+\sqrt{1+3t\tau})}$. When there are two horizontally differentiated products in the market, social welfare is given by $SW_{dbh2} = \frac{28-21t\tau+45t^2\tau^2+(14-21t\tau)\sqrt{4+6t\tau}}{432\tau(2+\sqrt{4+6t\tau})} - F$. When there are two vertically differentiated products in the market, social welfare is $SW_{dbv2} = \frac{378-2660t\tau+10950t^2\tau^2+(378-1120t\tau-1425t^2\tau^2)\sqrt{1+15t\tau}}{10800\tau(1-5t\tau+\sqrt{1+15t\tau})} - F$.

In order to decide to extend the product line or not, the social planner will need to compare the social welfare of horizontally extended product line with that of vertically extended product line. The results are given below:

$$\begin{cases} SW_{dbv2} \geq SW_{dbh2}, & \text{if } t\tau \leq 0.0590; \\ SW_{dbh2} > SW_{dbv2}, & \text{if } t\tau > 0.0590. \end{cases} \quad (\text{TA40})$$

With a sufficiently low fixed cost, the social planner will extend the product line vertically when $t\tau$ is small and will extend the product line horizontally when $t\tau$ is large. The threshold values of

F for the social planner to extend the product line are given below.

$$\begin{cases} F < \frac{378-2660t\tau+10950t^2\tau^2+(378-1120t\tau-1425t^2\tau^2)\sqrt{1+15t\tau}}{10800\tau(1-5t\tau+\sqrt{1+15t\tau})} - \frac{14-21t\tau+90t^2\tau^2+(14-42t\tau)\sqrt{1+3t\tau}}{432\tau(1+\sqrt{1+3t\tau})}, & \text{if } t\tau \leq 0.0590; \\ F < \frac{28-21t\tau+45t^2\tau^2+(14-21t\tau)\sqrt{4+6t\tau}}{432\tau(2+\sqrt{4+6t\tau})} - \frac{14-21t\tau+90t^2\tau^2+(14-42t\tau)\sqrt{1+3t\tau}}{432\tau(1+\sqrt{1+3t\tau})}, & \text{if } t\tau > 0.0590. \end{cases} \quad (\text{TA41})$$

Interestingly, the decentralization of a channel may mitigate the confliction between the social planner and the manufacturer on line extension decisions. If the extension cost F is small enough such that the manufacturer in a decentralized channel would like to extend the product line, it will vertically extend the line when $t\tau$ is small, which is consistent with the social planner's decision but inconsistent with the manufacturer's decision in a centralized channel. This is summarized below.

PROPOSITION TA 6: *When consumers are both horizontally and vertically heterogeneous, the manufacturer will vertically extend the product line for small $t\tau$ ($t\tau \leq 0.0559$) in both centralized and decentralized channels, and the decentralization of the channel will restore a social planner's decisions on product line extension for $t\tau \leq 0.0559$, compared with a centralized channel.*

Proof: Equation (TA27) shows that the manufacturer in a centralized channel will be willing to vertically extend the product line for a small $t\tau$ ($t\tau \leq 0.0559$) but the social planner would like to horizontally extend the product line always, when consumers are both vertically and horizontally heterogeneous and F is small enough. Equations (TA39) and (TA40) show, however, that both the manufacturer and social planner in a decentralized channel will be willing to vertically extend the product line for a small $t\tau$ ($t\tau \leq \min\{0.0559, 0.0590\} = 0.0559$). That is, the decentralization of the channel will restore a social planner's decisions on product line extension for a small $t\tau$ when consumers are both horizontally and vertically heterogeneous.

Q.E.D.

6. PRODUCT LINE EXTENSION WITH DUOPOLY COMPETITION

In this section, we consider the case where the manufacturer is competing against another firm. It is assumed that a firm Q is located at the right end of the Hotelling line and is competing with the manufacturer for horizontally heterogeneous consumers. We further assume that firm Q is competing with the manufacturer in one segment of size $\frac{8}{11} < \alpha \leq 1$ (labeled as segment 1) and the manufacturer has a loyal segment of size $1 - \alpha$ (labeled as segment 2).¹³ Each consumer in segment 1 has a value of V for a product from either firm. Each consumer in segment 2 has a value of V for the manufacturer's product and has a value of zero for firm Q 's product. Thus the manufacturer is a monopoly in segment 2 and competes with firm Q in segment 1. Firm Q sets its retail price q and has a zero marginal cost. Other settings are similar to those in the base model. We first solve the case of a centralized channel in which the manufacturer competes with firm Q on retail price directly.

6.1. A Centralized Channel With One Product

When the manufacturer has only one product, the product is located at the left end of the Hotelling line as is commonly assumed in the literature (d'Aspremont, et al. 1979). The marginal consumer in segment 1 is located at $\bar{x} = \frac{q_{ch1} - p_{ch1}^C + t}{2t}$ where q_{ch1} is the price set by firm Q and p_{ch1}^C is the retail price set by the manufacturer. The manufacturer's and firm Q 's profit functions are given by

$$\begin{cases} \Pi_{ch1}^{MC} = \alpha \cdot p_{ch1}^C \cdot \bar{x} + (1 - \alpha) \cdot p_{ch1}^C \cdot 1 \\ \Pi_{ch1}^{QC} = \alpha \cdot q_{ch1} \cdot (1 - \bar{x}) \end{cases}, \quad (\text{TA42})$$

Solving the optimization problems for both firms leads to the following optimal prices

$$\begin{cases} p_{ch1}^C = \frac{(4-\alpha)t}{3\alpha} \\ q_{ch1} = \frac{(2+\alpha)t}{3\alpha} \end{cases}. \quad (\text{TA43})$$

¹³We focus on the cases where $\frac{8}{11} < \alpha \leq 1$ and V is not extraordinarily large such that the solutions are interior.

Firms' profits and social welfare are given by

$$\begin{cases} \Pi_{ch1}^{MC} = \frac{(4-\alpha)^2 t}{18\alpha} \\ \Pi_{ch1}^{QC} = \frac{(2+\alpha)^2 t}{18\alpha} \\ SW_{ch1}^C = V - \frac{(4+10\alpha-5\alpha^2)t}{36\alpha} \end{cases}. \quad (\text{TA44})$$

For any $t \leq \frac{3\alpha V}{2(2+\alpha)}$, the market is fully covered. In addition, for $\alpha > \frac{2}{5}$, both firms will sell in segment 1, *i.e.*, $0 < \bar{x} < 1$.¹⁴

6.2. A Centralized Channel With Two Products

When the manufacturer extends the product line to two products, the original product is assumed to remain at the left end of the Hotelling line and the optimal location of the new product is given by $0 < L < 1$. Both products' prices are given by p_{ch21}^C and p_{ch22}^C , respectively. Given $L > 0$, the marginal consumer between the manufacturer's two products is located at $\bar{x} = \frac{p_{ch22}^C - p_{ch21}^C + tL}{2t}$ in both segments, and the marginal consumer between the manufacturer and firm Q in segment 1 is located at $\bar{y} = \frac{tL + q_{ch2} - p_{ch22}^C + t}{2t}$.¹⁵ The manufacturer's and firm Q 's profit functions are given by

$$\begin{cases} \Pi_{ch2}^{MC} = \alpha[p_{ch21}^C \cdot \bar{x} + p_{ch22}^C \cdot (\bar{y} - \bar{x})] + (1 - \alpha)[p_{ch21}^C \cdot \bar{x} + p_{ch22}^C \cdot (1 - \bar{x})] - F \\ \Pi_{ch2}^{QC} = \alpha[q_{ch2} \cdot (1 - \bar{y})] \end{cases}. \quad (\text{TA45})$$

¹⁴The case where firm Q is a monopoly in segment 1 while the manufacturer is a monopoly in segment 2 is less interesting in the current model, since we are interested in how competition between firms affects the manufacturer's and a social planner's product line extension strategies.

¹⁵Since it is easier for the manufacturer to change prices than the position of a product, we assume that the manufacturer makes decisions in two stages: in the first stage, the manufacturer decides on L ; in the second stage, given L both the manufacturer and firm Q simultaneously decide on retail prices to compete for end consumers. We solve the model using backward induction.

Both firms simultaneously maximize their respective profits by choosing optimal prices, and the prices and profits are given by

$$\left\{ \begin{array}{l} p_{ch21}^C = \frac{(8-2\alpha+5\alpha L)t}{6\alpha} \\ p_{ch22}^C = \frac{(4-\alpha+\alpha L)t}{3\alpha} \\ q_{ch2} = \frac{(2+\alpha-\alpha L)t}{3\alpha} \\ \Pi_{ch2}^{MC} = \frac{[(4\alpha^2+9\alpha)L^2-8(\alpha^2-4\alpha)L+4(\alpha^2-8\alpha+16)]t}{72\alpha} - F \\ \Pi_{ch2}^{QC} = \frac{(2+\alpha-\alpha L)^2 t}{18\alpha} \end{array} \right. . \quad (\text{TA46})$$

Since $\frac{\partial \Pi_{ch2}^{MC}}{\partial L} > 0$ for any $0 \leq L \leq 1$, the manufacturer chooses an as large L as possible such that firm Q finds it non-profitable to deviate from the current price to a deeper price in order to steal all consumers of the new product in segment 1. The highest price \tilde{q} to steal all consumers of the new product in segment 1 is given by $\tilde{q} = p_{ch2}^C - t(1-L) = \frac{4(1-\alpha+\alpha L)t}{3\alpha}$, and firm Q 's profit from deviation is given by $\tilde{\Pi}_{ch2}^{QC} = \frac{t}{3}(4-L)(1-\alpha+\alpha L)$. The highest L to prevent firm Q from deviating from $q_{ch2} = \frac{(2+\alpha-\alpha L)t}{3\alpha}$ is thus given by $L^* = \frac{1}{7\alpha}[16\alpha - 1 - 3\sqrt{9\alpha^2 + 12\alpha - 3}]$. It is easy to show that the manufacturer has no incentive to deviate from such an L^* . The optimal prices and marginal consumers are thus given by

$$\left\{ \begin{array}{l} p_{ch21}^C = \frac{t}{14\alpha}(17 + 22\alpha - 5\sqrt{9\alpha^2 + 12\alpha - 3}) \\ p_{ch22}^C = \frac{t}{7\alpha}(9 + 3\alpha - \sqrt{9\alpha^2 + 12\alpha - 3}) \\ q_{ch2} = \frac{t}{7\alpha}(5 - 3\alpha + \sqrt{9\alpha^2 + 12\alpha - 3}) \\ \bar{x}_{ch2}^C = \frac{1}{28\alpha}(-1 + 16\alpha - 3\sqrt{9\alpha^2 + 12\alpha - 3}) \\ \bar{y}_{ch2}^C = \frac{1}{14\alpha}(-5 + 17\alpha - \sqrt{9\alpha^2 + 12\alpha - 3}) \end{array} \right. . \quad (\text{TA47})$$

Firms' profits and the social welfare are given by

$$\begin{cases} \Pi_{ch2}^{MC} = \frac{t}{392\alpha^2} [72\alpha^3 + 601\alpha^2 + 388\alpha - 26 - (24\alpha^2 + 168\alpha - 6)\sqrt{9\alpha^2 + 12\alpha - 3}] - F \\ \Pi_{ch2}^{QC} = \frac{t}{98\alpha} (3\alpha - 5 - \sqrt{9\alpha^2 + 12\alpha - 3})^2 \\ SW_{ch2}^C = V - \frac{t}{784\alpha^2} [-360\alpha^3 + 3005\alpha^2 + 1220\alpha - 338 + (120\alpha^2 - 1000\alpha + 78)\sqrt{9\alpha^2 + 12\alpha - 3}] - F \end{cases} \quad (\text{TA48})$$

Given both the manufacturer's profit and social welfare in equation (TA44) with one product and in equation (TA48) with two products, we can solve for the threshold values of fixed cost F under which the manufacturer or the social planner is willing to extend the product line. The threshold values of fixed cost F are therefore solved from $F_{ch}^{MC} = \Pi_{ch2}^{MC} - \Pi_{ch1}^{MC}$ and $F_{ch}^{SC} = SW_{ch2}^C - SW_{ch1}^C$

$$\begin{cases} F_{ch}^{MC} = \frac{t}{3528\alpha^2} [452\alpha^3 + 6977\alpha^2 + 356\alpha - 234 - (216\alpha^2 + 1512\alpha - 54)\sqrt{9\alpha^2 + 12\alpha - 3}] \\ F_{ch}^{SC} = \frac{t}{7056\alpha^2} [2260\alpha^3 - 25085\alpha^2 - 10196\alpha + 3042 - (1080\alpha^2 + 9000\alpha - 702)\sqrt{9\alpha^2 + 12\alpha - 3}] \end{cases} \quad (\text{TA49})$$

For any $t \leq \frac{7\alpha(5-3\alpha-\sqrt{9\alpha^2+12\alpha-3})V}{2(28-42\alpha)}$, the market is fully covered. In addition, for $\alpha > \frac{1}{2}$, we have $0 < \bar{x}_{ch2}^C < L^* < \bar{y}_{ch2}^C < 1$.

6.3. A Decentralized Channel With One Product

In a decentralized channel with a single product by each firm, the manufacturer sells its product through a retailer, which is competing with firm Q for end consumers. The marginal consumer in segment 1 is thus given by $\bar{x} = \frac{q_{dh1} - p_{dh1}^C + t}{2t}$ where q_{dh1} is the price set by firm Q and p_{dh1}^C is the retail price set by the retailer. The retailer's and firm Q 's profit functions are given by

$$\begin{cases} \Pi_{dh1}^{RC} = \alpha(p_{dh1}^C - w_{dh1}^C) \cdot \bar{x} + (1 - \alpha) \cdot (p_{dh1}^C - w_{dh1}^C) \cdot 1 \\ \Pi_{dh1}^{QC} = \alpha[q_{dh1} \cdot (1 - \bar{x})] \end{cases}, \quad (\text{TA50})$$

Solving the optimization problems for both the retailer and firm Q leads to the following optimal prices

$$\begin{cases} p_{dh1}^C = \frac{(4-\alpha)t + 2\alpha w_{dh1}^C}{3\alpha} \\ q_{dh1} = \frac{(2+\alpha)t + \alpha w_{dh1}^C}{3\alpha} \end{cases} \quad (\text{TA51})$$

Given retail prices in equation (TA51), the manufacturer sets its wholesale price w_{dh1}^C to maximize its profit $\Pi_{dh1}^{MC} = \alpha w_{dh1}^C \cdot \bar{x} + (1-\alpha) \cdot w_{dh1}^C \cdot 1$. The optimal wholesale and retail prices and marginal consumer in segment 1 are thus given by

$$\begin{cases} w_{dh1}^C = \frac{(4-\alpha)t}{2\alpha} \\ p_{dh1}^C = \frac{2(4-\alpha)t}{3\alpha} \\ q_{dh1} = \frac{(8+\alpha)t}{6\alpha} \\ \bar{x}_{dh1}^C = \frac{11\alpha-8}{12\alpha} \end{cases} \quad (\text{TA52})$$

Firms' profits and social welfare are given by

$$\begin{cases} \Pi_{ch1}^{RC} = \frac{(4-\alpha)^2 t}{72\alpha} \\ \Pi_{ch1}^{MC} = \frac{(4-\alpha)^2 t}{24\alpha} \\ \Pi_{ch1}^{QC} = \frac{(8+\alpha)^2 t}{72\alpha} \\ SW_{ch1}^C = V - \frac{(64-8\alpha-11\alpha^2)t}{144\alpha} \end{cases} \quad (\text{TA53})$$

For any $t \leq \frac{3\alpha V}{8+\alpha}$, the market is fully covered. In addition, for $\alpha > \frac{8}{11}$, we have $0 < \bar{x}_{dh1}^C < 1$.

6.4. A Decentralized Channel With Two Products

When the manufacturer extends the product line to two products that are sold to end consumers through a retailer, the original product is assumed to remain at the left end of the Hotelling line and the optimal location of the new product is given by $0 < L < 1$. Given $L > 0$, the marginal consumer between the manufacturer's two products in both segments is given by $\bar{x} = \frac{p_{dh22}^C - p_{dh21}^C + tL}{2t}$, and the marginal consumer between the retailer and firm Q in segment 1 is given by $\bar{y} = \frac{tL + q_{dh2} - p_{dh22}^C + t}{2t}$.

The retailer's and firm Q 's profit functions are given by

$$\left\{ \begin{array}{l} \Pi_{dh2}^{RC} = \alpha[(p_{dh21}^C - w_{dh21}^C) \cdot \bar{x} + (p_{dh22}^C - w_{dh22}^C) \cdot (\bar{y} - \bar{x})] \\ \quad + (1 - \alpha)[(p_{dh21}^C - w_{dh21}^C) \cdot \bar{x} + (p_{dh22}^C - w_{dh22}^C) \cdot (1 - \bar{x})] \cdot \\ \Pi_{dh2}^{QC} = \alpha[q_{dh2} \cdot (1 - \bar{y})] \end{array} \right. \quad (\text{TA54})$$

Solving the optimization problems for both the retailer and firm Q leads to the following optimal prices

$$\left\{ \begin{array}{l} p_{dh21}^C = \frac{(8-2\alpha+5\alpha L)t + \alpha(3w_{dh21}^C + w_{dh22}^C)}{6\alpha} \\ p_{dh22}^C = \frac{(4-\alpha+\alpha L)t + 2\alpha w_{dh22}^C}{3\alpha} \\ q_{dh2} = \frac{(2+\alpha-\alpha L)t + \alpha w_{dh22}^C}{3\alpha} \end{array} \right. \quad (\text{TA55})$$

Given retail prices in equation (TA55), the manufacturer sets its wholesale prices w_{dh21}^C and w_{dh22}^C to maximize its profit $\Pi_{dh2}^{MC} = \alpha[w_{dh21}^C \cdot \bar{x} + w_{dh22}^C \cdot (\bar{y} - \bar{x})] + (1 - \alpha)[w_{dh21}^C \cdot \bar{x} + w_{dh22}^C \cdot (1 - \bar{x})] - F$.

The optimal wholesale and retail prices and the manufacturer's profit are thus given by

$$\left\{ \begin{array}{l} w_{dh21}^C = \frac{(4-\alpha+2\alpha L)t}{2\alpha} \\ w_{dh22}^C = \frac{(4-\alpha+\alpha L)t}{2\alpha} \\ p_{dh21}^C = \frac{(32-8\alpha+17\alpha L)t}{12\alpha} \\ p_{dh22}^C = \frac{2(4-\alpha+\alpha L)t}{3\alpha} \\ q_{dh2} = \frac{(8+\alpha-\alpha L)t}{6\alpha} \\ \Pi_{dh2}^{MC} = \frac{((3+2\alpha)\alpha L^2 + 4\alpha(4-\alpha)L + 32 - 16\alpha + 2\alpha^2)t}{48\alpha} - F \end{array} \right. \quad (\text{TA56})$$

Since $\frac{\partial \Pi_{dh2}^{MC}}{\partial L} > 0$ for any $0 \leq L \leq 1$, the manufacturer chooses an as large L as possible such that firm Q finds it non-profitable to deviate from the current price to a deeper price in order to steal all consumers of the new product in segment 1. The highest price \tilde{q} to steal all consumers of the new product in segment 1 is given by $\tilde{q} = p_{dh22}^C - t(1 - L) = \frac{(8-5\alpha+5\alpha L)t}{8\alpha}$, and firm Q 's profit from deviation is given by $\tilde{\Pi}_{dh2}^{QC} = \frac{t}{24}(8 - L)(8 - 5\alpha + 5\alpha L)$. The highest L to prevent firm Q from deviating from $q_{dh2} = \frac{(8+\alpha-\alpha L)t}{6\alpha}$ is thus given by $L^* = \frac{1}{32\alpha}(137\alpha - 8 - 3\sqrt{1225\alpha^2 + 1008\alpha - 448})$.

It is easy to show that the manufacturer has no incentive to deviate from such an L^* . The optimal prices and marginal consumers are thus given by

$$\left\{ \begin{array}{l} w_{dh21}^C = \frac{t}{32\alpha}(56 + 121\alpha - 3\sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ w_{dh22}^C = \frac{3t}{64\alpha}(40 + 35\alpha - \sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ p_{dh21}^C = \frac{t}{128\alpha}(296 + 691\alpha - 17\sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ p_{dh22}^C = \frac{t}{16\alpha}(40 + 35\alpha - \sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ q_{dh2} = \frac{t}{64\alpha}(88 + 35\alpha - \sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ \bar{x}_{dh2}^C = \frac{L^*}{8} = \frac{1}{256\alpha}(137\alpha - 8 - 3\sqrt{1225\alpha^2 + 1008\alpha - 448}) \\ \bar{y}_{dh2}^C = \frac{1}{128\alpha}(-88 + 163\alpha - \sqrt{1225\alpha^2 + 1008\alpha - 448}) \end{array} \right. \quad . \quad (\text{TA57})$$

Firms' profits and the social welfare are given by

$$\left\{ \begin{array}{l} \Pi_{dh2}^{RC} = \frac{t}{16384\alpha^2}[4900\alpha^3 + 22513\alpha^2 + 5744\alpha - 1984 - (140\alpha^2 + 571\alpha - 24)\sqrt{1225\alpha^2 + 1008\alpha - 448}] \\ \Pi_{dh2}^{MC} = \frac{t}{8192\alpha^2}[7350\alpha^3 + 26321\alpha^2 + 6896\alpha - 1984 - (210\alpha^2 + 651\alpha - 24)\sqrt{1225\alpha^2 + 1008\alpha - 448}] - F \\ \Pi_{dh2}^{QC} = \frac{t}{8192\alpha}(35\alpha - 88 - \sqrt{1225\alpha^2 + 1008\alpha - 448})^2 \\ SW_{dh2}^C = V - \frac{t}{32768\alpha^2}[-53900\alpha^3 + 766889\alpha^2 + 223984\alpha - 113088 \\ \quad + (1540\alpha^2 - 22179\alpha + 1368)\sqrt{1225\alpha^2 + 1008\alpha - 448}] - F \end{array} \right. \quad (\text{TA58})$$

The threshold values of fixed cost F under which the manufacturer or the social planner in a decentralized channel is willing to extend the product line are therefore solved from $F_{dh}^{MC} = \Pi_{dh2}^{MC} - \Pi_{dh1}^{MC}$ and $F_{dh}^{SC} = SW_{dh2}^C - SW_{dh1}^C$.

$$\left\{ \begin{array}{l} F_{dh}^{MC} = \frac{t}{24576\alpha^2}[21026\alpha^3 + 87155\alpha^2 + 4304\alpha - 5952 - (630\alpha^2 + 1953\alpha - 72)\sqrt{9\alpha^2 + 12\alpha - 3}] \\ F_{dh}^{SC} = \frac{t}{294912\alpha^2}[462572\alpha^3 - 6918385\alpha^2 - 1884784\alpha + 1017792 \\ \quad - (13860\alpha^2 - 199611\alpha + 12312)\sqrt{9\alpha^2 + 12\alpha - 3}] \end{array} \right. \quad . \quad (\text{TA59})$$

For any $t \leq \frac{32\alpha V}{(88-35\alpha+\sqrt{1225\alpha^2+1008\alpha-448})}$, the market is fully covered. In addition, for $\alpha > \frac{8}{11}$, we have $0 < \bar{x}_{dh2}^C < L^* < \bar{y}_{dh2}^C < 1$.

Figure 1 shows how the manufacturer's and social planner's product line extension strategies change as α changes in both the centralized and decentralized channels. In the figure, label $M_i (S_i)$ refers to the number of products, i , the manufacturer (a social planner) would like to have. The figure suggests that channel decentralization can restore the social planner's extension decision in a competitive context.

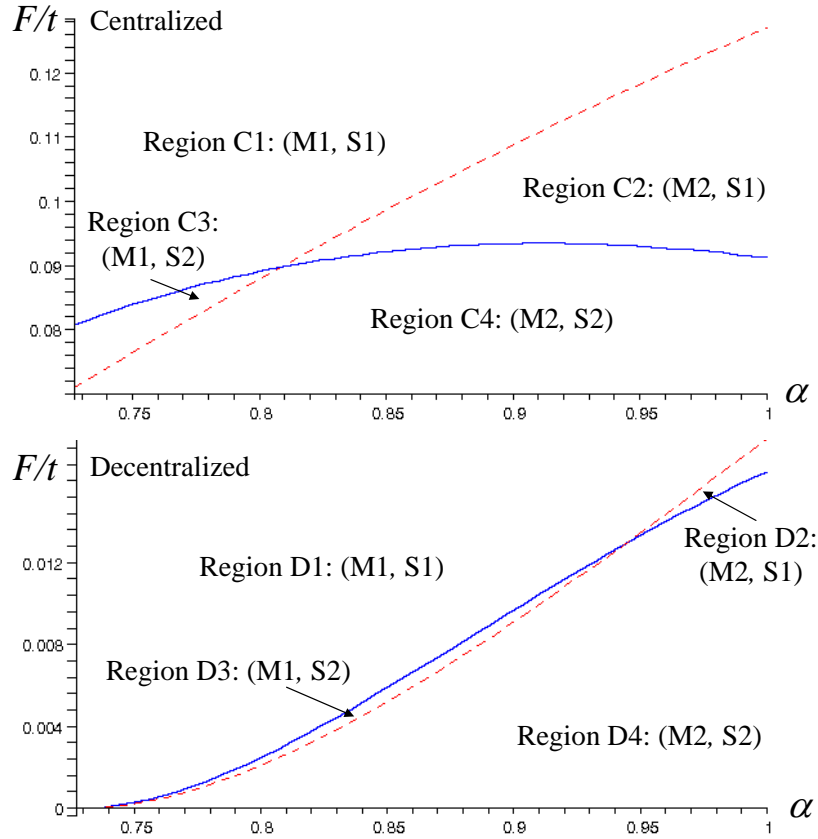


Figure 1: Manufacturer's and Social Planner's Product Line Extension Strategies within Different Channel Structures

PROPOSITION TA 7: *When the manufacturer in a centralized channel competes with a firm for horizontally heterogeneous consumers, the manufacturer will extend the product line while a social planner will not in region C₂ of Figure 1, and the manufacturer will not but a social planner will*

extend the product line in region C_3 . When the manufacturer in a decentralized channel competes with a firm for horizontally heterogeneous consumers, the manufacturer will extend the product line while a social planner will not in region D_2 , while the manufacturer will not but a social planner will extend the product line in region D_3 . Channel decentralization will restore the social planner's decision in regions C_2 and C_3 .

Proof: The threshold values of fixed cost F for the manufacturer or social planner to extend the product line in both centralized and decentralized channels are given by equations (TA49) and (TA59). The results in the proposition can easily be derived from both equations.

Q.E.D.

Proposition TA 7 suggests that the main results still hold when multiple firms are competing in the market. In addition, the study of duopoly competition with different market size α also suggests that the size of the loyal segment for the manufacturer does matter. When the manufacturer has a relatively small loyal segment (*i.e.*, a large α), the manufacturer tends to extend the product line while a social planner will not. When the manufacturer has a relatively large loyal segment (*i.e.*, a small α), the manufacturer tends to offer only one product while a social planner will extend the product line. The reason that parameter α influences the manufacturer's extension strategies is as follows. When the manufacturer has a small loyal segment, both firms compete intensively in the common market. The manufacturer thus has a stronger incentive to extend the product line to encroach upon its competitor's market than when it has a large loyal segment.

7. EFFECTS OF RETAILER'S PRODUCT ADOPTION STRATEGY

In this section, we consider the case where the retailer can strategically adopt products to maximize its own profit when the manufacturer extends its product line (Villas-Boas 1998). We assume the retailer makes product adoption decisions after the manufacturer sets wholesale price(s). Since the manufacturer's decisions and firms' profits are same as in the base model both in the centralized cases and in the decentralized case where the manufacturer offers one product, we list firms' profits and social welfare here. When there is only one product in a centralized channel, the optimal retail price, manufacturer's profit, and social welfare are given by $p_{ch1} = V - \frac{t}{2}$, $\Pi_{ch1} = V - \frac{t}{2}$, and $SW_{ch1} = V - \frac{t}{4}$. When there are two products in a centralized channel, the optimal retail prices, manufacturer's profit, and social welfare are given by $p_{ch21} = p_{ch22} = V - \frac{t}{4}$, $\Pi_{ch2} = V - \frac{t}{4} - F$, and $SW_{ch2} = V - \frac{t}{8} - F$. The comparison suggests that the manufacturer will extend the product line if $F \leq \frac{t}{4}$ and that the social planner will extend the line if $F \leq \frac{t}{8}$.

When there is only one product in a decentralized channel, the optimal prices are given by:

$$w_{dh1} = \begin{cases} \frac{V}{2}, & \text{if } \frac{V}{2} < t \leq V; \\ V - t, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad p_{dh1} = \begin{cases} \frac{3V}{4}, & \text{if } \frac{V}{2} < t \leq V; \\ V - \frac{t}{2}, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad (\text{TA60})$$

The manufacturer's profit (Π_{dh1}) and social welfare (SW_{dh1}) are:

$$\Pi_{dh1} = \begin{cases} \frac{V^2}{4t}, & \text{if } \frac{V}{2} < t \leq V; \\ V - t, & \text{if } t \leq \frac{V}{2}; \end{cases} \quad SW_{dh1} = \begin{cases} \frac{7V^2}{16t}, & \text{if } \frac{V}{2} < t \leq V; \\ V - \frac{t}{4}, & \text{if } t \leq \frac{V}{2}. \end{cases} \quad (\text{TA61})$$

Now lets consider the case where there are two products in a decentralized channel. Section 3 of the Technical Appendix has shown that the optimal locations of both products are given by $\frac{1}{4}$ and $\frac{3}{4}$ when $t \leq V$. The optimal prices are given by:

$$w_{dh21} = w_{dh22} = V - \frac{t}{2}; \quad p_{dh21} = p_{dh22} = V - \frac{t}{4}. \quad (\text{TA62})$$

Firms' profit (Π_{dh2} , π_{dh2}) and social welfare (SW_{dh2}) are given by:

$$\Pi_{dh2} = V - \frac{t}{2} - F; \quad \pi_{dh2} = \frac{t}{4}; \quad SW_{dh2} = V - \frac{t}{8} - F. \quad (\text{TA63})$$

Now let's consider the retailer's product adoption strategy. Given wholesale prices $w_{dh21} = w_{dh22} = V - \frac{t}{2}$, suppose the retailer decides to adopt only one product. Assume the retailer adopts the product located at $\frac{1}{4}$. Adopting the product at $\frac{3}{4}$ leads to same results and the analysis is omitted here.

For $t \leq V$, the manufacturer sets the wholesale prices for both products in such a way that the profit maximizing retailer sets the retail prices to make the consumers located at $(0, \frac{1}{2}, 1)$ as marginal consumers who get a surplus of zero. Thus, the retailer will not increase its retail price from $p_{dh21} = V - \frac{t}{2}$ since doing so will lead to a lower profit from the product. Now let's consider if the retailer has any incentive to reduce its retail price to $\tilde{p}_{dh21} < p_{dh21} = V - \frac{t}{2}$. For any $\tilde{p}_{dh21} < p_{dh21}$, the consumer located at 0 will get a positive surplus while the location of the marginal consumer on the right side of product 1 will be larger than $\frac{1}{2}$. The location of the new marginal consumer is solved from equation $V - \tilde{p}_{dh21} - t(\bar{x} - \frac{1}{4})$, which leads to $\bar{x} = \frac{1}{t}(V - \tilde{p}_{dh21} + \frac{t}{4})$. The retailer's new profit is given by:

$$\tilde{\pi}_{dh2} = (\tilde{p}_{dh21} - w_{dh21}) \cdot \bar{x} = [\tilde{p}_{dh21} - (V - \frac{t}{8})] \cdot \frac{1}{t}(V - \tilde{p}_{dh21} + \frac{t}{4}) \quad (\text{TA64})$$

The partial derivatives of $\tilde{\pi}_{dh2}$ with respect to \tilde{p}_{dh21} are given by:

$$\frac{\partial \tilde{\pi}_{dh2}}{\partial \tilde{p}_{dh21}} = \frac{1}{t}(2V - 2\tilde{p}_{dh21} - \frac{t}{4}); \quad \frac{\partial^2 \tilde{\pi}_{dh2}}{\partial \tilde{p}_{dh21}^2} = -\frac{2}{t} < 0. \quad (\text{TA65})$$

Since $\frac{\partial^2 \tilde{\pi}_{dh2}}{\partial \tilde{p}_{dh21}^2} < 0$ always and $\frac{\partial \tilde{\pi}_{dh2}}{\partial \tilde{p}_{dh21}} > 0$ for $\tilde{p}_{dh21} \leq V - \frac{t}{4}$, the optimal retail price for the retailer is given by $\tilde{p}_{dh21} = V - \frac{t}{4}$. That is, the retailer will have no incentive to deviate from $p_{dh21} = V - \frac{t}{4}$ if the retailer adopts only one product. Since the retailer can double its profit by adopting the

second product located at $\frac{3}{4}$ with a retail price of $p_{dh22} = V - \frac{t}{4}$, the optimal wholesale prices for both products by the manufacturer will be given by Equation (TA62).

This suggests that if the retailer strategically adopts products after the manufacturer decides the wholesale price, the main results from the paper will not change. That is, Propositions 1 and 2 in the paper still hold when the retailer becomes strategic on product adoption.